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Experiments on Transportation Support System using Multi-Vehicle's Formation

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Abstract: This paper considers the transportation support system using multi-vehicle's formation. The objective of our research is to carry the load using four vehicles keeping the formation with information exchange among vehicles. The control law based on a Lyapunov function is proposed based on the virtual structure and the graph Laplacian. In the simulation and experiment, it is shown that four vehicles can reach the reference position by using four wheeled mobile vehicles.

Keywords: multi-vehicle, formation, virtual structure, Lyapunov function, stability

1. INTRODUCTION

Recently, the networked multi-agent system and formation control have been a focus in control problems [1, 2]. In multi-agent system, each agent autonomously converges to a consensus value of their information states based on the information states of its neighbors. The consensus algorithms have been studied, consensus problems can be applied to formation control problems. Though, formation control problems are expected at various field, in particular, the formation control of nonholonomic multi-vehicle system based on virtual structure is proposed [3, 4].

The formation control can be utilized to the transportation system to conserve the energy in a production line of a factory. If large load can be carried by vehicles which are smaller than the load, then it is expected that the energetic consumption is reduced. In this paper, the transportation support system using multi-vehicle's formation is considered. The aim of our research is to carry the load using four vehicles keeping the formation with information exchange among vehicles. The control law based on a Lyapunov function is proposed based on the virtual structure.

First, the model of vehicles is indicated where the kinematics model and dynamics model are considered. The virtual vehicle is introduced for each vehicle, the control problem is converted into the convergence of each virtual vehicle. A Lyapunov function by using the graph Laplacian is proposed. It is shown that the control law is derived from the Lyapunov function.

In the simulation, the performance of the proposed transportation support system is verified using the four wheeled mobile vehicles. It is shown that the vehicles can reach the reference position under keeping the initial formation. In the experiments, the same performance as the simulation is verified by using four vehicles with micro computer and wireless LAN network. The information of the position and rotation is exchanged among the neighbors through the camera. It is indicated that four vehicles can reach the reference position.

2. PROBLEM FORMULATION

Consider four wheeled mobile vehicles. The coordinate of the i th wheeled mobile vehicle consists of the position $(x_i,\ y_i)$ and the rotation θ_i as shown in Fig. 1. The control inputs are given by the velocity v_i and steering angle $\dot{\theta}_i$. The kinematics model of the mobile robot is derived as follows

$$\begin{bmatrix}\dot{x}_i\\ \dot{y}_i\\ \dot{\theta}_i\end{bmatrix} = \begin{bmatrix}\cos\theta_i & 0\\ \sin\theta_i & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v_i\\ \dot{\theta}_i\end{bmatrix}.\ (i = 1 \sim 4) \tag{1}$$

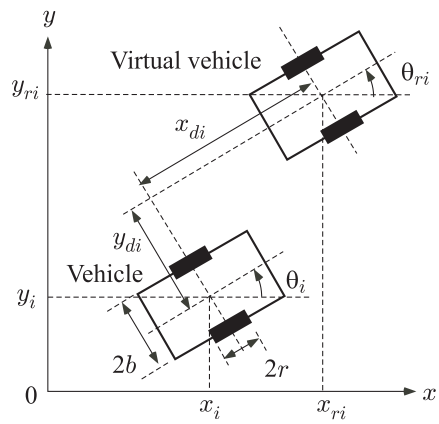

Fig. 1 Vehicle and virtual vehicle (VV)

The velocity v_i and steering angle $\dot{\theta}_i$ are related to the angle velocities of the right and left wheels $\omega_{ri},\ \omega_{li}$

$$\begin{bmatrix}v_i\\ \dot{\theta}_i\end{bmatrix} = \begin{bmatrix}\frac{r}{2} & \frac{r}{2}\\ \frac{r}{2b} & \frac{r}{2b}\end{bmatrix}\begin{bmatrix}\omega_{ri}\\ \omega_{li}\end{bmatrix}. \tag{2}$$

The symbol r, b is the radius of the wheels and the half-width of the wheeled mobile vehicle, respectively. From the equations (1), (2), the kinematics model of the vehicle is derived as follows

$$\begin{bmatrix}\dot{x}_i\\ \dot{y}_i\\ \dot{\theta}_i\end{bmatrix} = \frac{r}{2b}\begin{bmatrix}b(\omega_{ri}+\omega_{li})\cos\theta_i\\ b(\omega_{ri}+\omega_{li})\sin\theta_i\\ \omega_{ri}-\omega_{li}\end{bmatrix}. \quad (i = 1 \sim 4) \tag{3}$$

By the positional relationship between the vehicle and virtual vehicle(VV) in Fig. 1, the kinematics model of i th VV is described as

$$\begin{bmatrix} x_{ri} \\ y_{ri} \\ \theta_{ri} \end{bmatrix} = \begin{bmatrix} x_i + x_{di}\cos\theta_i - y_{di}\sin\theta_i \\ y_i + x_{di}\sin\theta_i + y_{di}\cos\theta_i \\ \theta_i \end{bmatrix} \tag{4}$$

where $(x_{ri},\, y_{ri})$ are positions of the center of gravity of i th VV, θ_{ri} is heading angle of i th VV and x_{di}, y_{di} are distance between the VV and vehicle. The derivative of (4) is given by

$$\begin{bmatrix} \dot{x}_{ri} \\ \dot{y}_{ri} \\ \dot{\theta}_{ri} \end{bmatrix} = \begin{bmatrix} \dot{x}_i - x_{di}\dot{\theta}_i\sin\theta_i - y_{di}\dot{\theta}_i\cos\theta_i \\ \dot{y}_i + x_{di}\dot{\theta}_i\cos\theta_i - y_{di}\dot{\theta}_i\sin\theta_i \\ \dot{\theta}_i \end{bmatrix}. \tag{5}$$

Using the equations (1), (2), the equation (5) becomes as follows

$$\begin{aligned} \begin{bmatrix} \dot{x}_{ri} \\ \dot{y}_{ri} \\ \dot{\theta}_{ri} \end{bmatrix} &= \begin{bmatrix} \cos\theta_i & -x_{di}\sin\theta_i - y_{di}\cos\theta_i \\ \sin\theta_i & x_{di}\cos\theta_i - y_{di}\sin\theta_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \dot{\theta}_i \end{bmatrix} \\ &= \begin{bmatrix} B_i \\ B_\theta \end{bmatrix} \begin{bmatrix} \omega_{ri} \\ \omega_{li} \end{bmatrix} \end{aligned} \tag{6}$$

where

$$B_i = \frac{r}{2b}\begin{bmatrix} b\cos\theta_i - x_{di}\sin\theta_i - y_{di}\cos\theta_i & b\cos\theta_i + x_{di}\sin\theta_i + y_{di}\cos\theta_i \\ b\sin\theta_i + x_{di}\cos\theta_i - y_{di}\sin\theta_i & b\sin\theta_i - x_{di}\cos\theta_i + y_{di}\sin\theta_i \end{bmatrix} \tag{7}$$

$$B_\theta = \frac{r}{2b}\begin{bmatrix} 1 & -1 \end{bmatrix}. \tag{8}$$

Define the vector q_{ri}, S_i, ω_i

$$q_{ri} = \begin{bmatrix} x_{ri} \\ y_{ri} \\ \theta_{ri} \end{bmatrix}, \quad S_i = \begin{bmatrix} B_i \\ B_\theta \end{bmatrix}, \quad \omega_i = \begin{bmatrix} \omega_{ri} \\ \omega_{li} \end{bmatrix} \tag{9}$$

the kinematics model of the i th VV is derived as

$$\dot{q}_{ri} = S_i\omega_i. \tag{10}$$

It is assumed that the dynamics of the i th vehicle is linear model

$$J_i\dot{\omega}_i + C_i\omega_i = \tau_i \tag{11}$$

where

$$J_i = \begin{bmatrix} J_{ri} & 0 \\ 0 & J_{li} \end{bmatrix}, \quad C_i = \begin{bmatrix} C_{ri} & 0 \\ 0 & C_{li} \end{bmatrix}, \quad \tau_i = \begin{bmatrix} \tau_{ri} \\ \tau_{li} \end{bmatrix} \tag{12}$$

$J_{ri} > 0$, $J_{li} > 0$ are the right and left moment of the inertia, $C_{ri} > 0$, $C_{li} > 0$ are the right and left viscous friction, τ_{ri}, τ_{li} are the right and left torque, respectively.

For the above equations (10) and (11), we consider the transportation support system using multi-vehicle's formation. The objective is to transport the products using four vehicles keeping the formation based on information exchange as shown in Fig. 2. In order to achieve the multi-vehicle's formation, the virtual vehicles VV1-VV4 of each vehicle have to keep the same position. The vehicle 1 is able to exchange the information of the position and rotation for vehicle 2. As the same as vehicle 1, each vehicle can obtain the information as the arrow shown in Fig. 2.

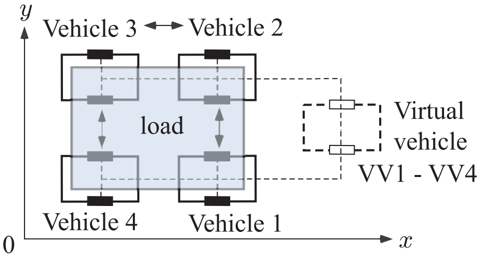

Fig. 2 Four vehicles and information exchange

3. STABILITY OF TRANSPORTATION SUPPORT SYSTEM

The graph Laplacian L represents the network structures. The graph Laplacian is defined as

$$L = D - A \tag{13}$$

The diagonal element of D shows the number that i th vehicle communicate with neighbors and the off-diagonal element is zero. The A is adjacency matrix, if j th vehicle communicate with i th vehicle, then $a_{ij} = 1$, other elements are zero. In Fig. 2, matrix D and A are defined as

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{14}$$

Therefore, graph Laplacian L is given as follows

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}. \tag{15}$$

A positive definite function for the system is proposed using the graph Laplacian L

$$\begin{aligned} V &= \frac{1}{2}\big(\omega^T J\omega + q_r^T \mathrm{diag}K_{G1}L_{.3}q_r \\ &\quad + (q_r - q_g)^T \mathrm{diag}K_{G2}(q_r - q_g)\big) \end{aligned} \tag{16}$$

where

$$\omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix}, \quad J = \begin{bmatrix} J_1 & 0 & 0 & 0 \\ 0 & J_2 & 0 & 0 \\ 0 & 0 & J_3 & 0 \\ 0 & 0 & 0 & J_4 \end{bmatrix}, \quad q_r = \begin{bmatrix} q_{r1} \\ q_{r2} \\ q_{r3} \\ q_{r4} \end{bmatrix} \tag{17}$$

q_{gi} is the reference of i th virtual vehicle,

$$q_g = \begin{bmatrix} q_{g1} \\ q_{g2} \\ q_{g3} \\ q_{g4} \end{bmatrix}, \quad q_{gi} = \begin{bmatrix} x_g \\ y_g \\ \theta_g \end{bmatrix}. \tag{18}$$

The symbol $L_{.3}$ represents $L \otimes I_3$, $\otimes$ denotes Kronecker product, I_3 indicates 3×3 unit matrix. The matrix

$$\mathrm{diag}K_{G1} = \mathrm{diag}(K_{G1}, K_{G1}, K_{G1}, K_{G1}) \tag{19}$$

is weighting matrix of the error among vehicles,

$$\mathrm{diag}K_{G2} = \mathrm{diag}(K_{G2}, K_{G2}, K_{G2}, K_{G2}) \tag{20}$$

is weighting matrix of the error between references and states of VVs, where

$$K_{G1} = \mathrm{diag}(K_{x1}, K_{y1}, K_{\theta 1}) \quad (21)$$

$$K_{G2} = \mathrm{diag}(K_{x2}, K_{y2}, K_{\theta 2}). \quad (22)$$

The second term of positive definite function (16) is calculated as follows

$$\begin{aligned}
& q_r^T \mathrm{diag} K_{G1} L_{.3} q_r \\
&= \begin{bmatrix} q_{r1} \\ q_{r2} \\ q_{r3} \\ q_{r4} \end{bmatrix}^T \begin{bmatrix} K_{G1} & 0 & 0 & 0 \\ 0 & K_{G1} & 0 & 0 \\ 0 & 0 & K_{G1} & 0 \\ 0 & 0 & 0 & K_{G1} \end{bmatrix} \\
& \qquad \times \begin{bmatrix} I_3 & -I_3 & 0 & 0 \\ -I_3 & 2I_3 & -I_3 & 0 \\ 0 & -I_3 & 2I_3 & -I_3 \\ 0 & 0 & -I_3 & I_3 \end{bmatrix} \begin{bmatrix} q_{r1} \\ q_{r2} \\ q_{r3} \\ q_{r4} \end{bmatrix} \\
&= \frac{1}{2} K_{x1} \Big[(x_{r1} - x_{r2})^2 + (x_{r2} - x_{r1})^2 + (x_{r2} - x_{r3})^2 \\
& \quad + (x_{r3} - x_{r2})^2 + (x_{r3} - x_{r4})^2 + (x_{r4} - x_{r3})^2 \Big] \\
& \quad + \frac{1}{2} K_{y1} \Big[(y_{r1} - y_{r2})^2 + (y_{r2} - y_{r1})^2 + (y_{r2} - y_{r3})^2 \\
& \quad + (y_{r3} - y_{r2})^2 + (y_{r3} - y_{r4})^2 + (y_{r4} - y_{r3})^2 \Big] \\
& \quad + \frac{1}{2} K_{\theta 1} \Big[(\theta_{r1} - \theta_{r2})^2 + (\theta_{r2} - \theta_{r1})^2 + (\theta_{r2} - \theta_{r3})^2 \\
& \quad + (\theta_{r3} - \theta_{r2})^2 + (\theta_{r3} - \theta_{r4})^2 + (\theta_{r4} - \theta_{r3})^2 \Big] \\
& > 0 \qquad (23)
\end{aligned}$$

The second term $q_r^T \mathrm{diag} K_{G1} L_{.3} q_r$ indicates the errors among vehicles which can exchange the information.

The derivative of equation (16) along the trajectories of the system (10), (11) is given by

$$\begin{aligned}
\dot{V} &= \frac{1}{2} \big[\dot{\omega}^T J \omega + \omega^T J \dot{\omega} + \dot{q_r}^T \mathrm{diag} K_{G1} L_{.3} q_r \\
& \quad + q_r^T \mathrm{diag} K_{G1} L_{.3} \dot{q}_r \big] + \frac{1}{2} \big[\dot{q}_r^T \mathrm{diag} K_{G2} (q_r - q_g) \\
& \quad + (q_r - q_g)^T \mathrm{diag} K_{G2} \dot{q}_r) \big] \\
&= \omega^T J \dot{\omega} + \dot{q}_r^T \mathrm{diag} K_{G1} L_{.3} q_r \\
& \quad + \dot{q}_r^T \mathrm{diag} K_{G2} (q_r - q_g) \qquad (24)
\end{aligned}$$

From equation (10), the kinematics of four vehicles is shown as follows

$$\dot{q}_r = (\mathrm{diag} S_i)\, \omega \quad (25)$$

where

$$\mathrm{diag} S_i = \begin{bmatrix} S_1 & 0 & 0 & 0 \\ 0 & S_2 & 0 & 0 \\ 0 & 0 & S_3 & 0 \\ 0 & 0 & 0 & S_4 \end{bmatrix}. \quad (26)$$

The dynamics of four vehicles is derived

$$J\dot{\omega} + C\omega = \tau. \quad (27)$$

The equations (25) and (27) are substituted into equation (24)

$$\begin{aligned}
\dot{V} &= \omega^T (\tau - C\omega) + \omega^T (\mathrm{diag} S_i)^T \mathrm{diag} K_{G1} L_{.3} q_r \\
& \quad + \omega^T (\mathrm{diag} S_i)^T \mathrm{diag} K_{G2} (q_r - q_g) \\
&= -\omega^T C \omega + \omega^T \left(\tau + (\mathrm{diag} S_i)^T (\mathrm{diag} K_{G1} L_{.3} q_r \right. \\
& \quad \left. + \mathrm{diag} K_{G2} (q_r - q_g)) \right). \qquad (28)
\end{aligned}$$

If second term of equation (28) equals 0, $\dot{V} \leq 0$ is derived. Therefore, the torque τ is given by

$$\begin{aligned}
\tau &= -(\mathrm{diag} S_i)^T (\mathrm{diag} K_{G1} L_{.3} q_r \\
& \quad + \mathrm{diag} K_{G2} (q_r - q_g)). \qquad (29)
\end{aligned}$$

Then, equation (28) becomes

$$\dot{V} = -\omega^T C \omega. \quad (30)$$

By using the Lyapunov's stability theorem, the stability of transportation support system is proved. The four vehicles can transport the load keeping the multi-vehicle's formation. However, equation (30) shows that the Lyapunov function (16) is equal or less than 0. Though angle velocity ω converge to zero, it cannot be guaranteed that errors among VVs and errors between the VVs and reference converge to zero.

4. SIMULATION

In this section, the performance of the transportation system proposed in section 3 is verified. The model parameters are assumed as

$$J_{ri} = J_{li} = 1 \ [\mathrm{kg \cdot m^2}] \quad (31)$$

$$C_{ri} = C_{li} = 1 \ [\mathrm{N \cdot s/m}] \quad (32)$$

$$r = 0.1 \ [\mathrm{m}], \ \ b = 0.1 \ [\mathrm{m}]. \quad (33)$$

The control parameters of K_{G1}, K_{G2} are designed

$$K_{G1} = \begin{bmatrix} K_{x1} & 0 & 0 \\ 0 & K_{y1} & 0 \\ 0 & 0 & K_{\theta 1} \end{bmatrix} = \begin{bmatrix} 200 & 0 & 0 \\ 0 & 200 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (34)$$

$$K_{G2} = \begin{bmatrix} K_{x2} & 0 & 0 \\ 0 & K_{y2} & 0 \\ 0 & 0 & K_{\theta 2} \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (35)$$

The initial conditions are given as follows and Table 1,

$$\begin{bmatrix} \dot{x}_i(0) \\ \dot{y}_i(0) \\ \dot{\theta}_i(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} \dot{\omega}_{ri}(0) \\ \dot{\omega}_{li}(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (36)$$

Table 1 Initial condition

Vehicle i	Vehicle's position (x_i, y_i, θ_i)	Virtual vehicle's position (x_{di}, y_{di})
1	(1, 0, 0)	(1, 0.5)
2	(1, 1, 0)	(1, -0.5)
3	(-1, 1, 0)	(3, -0.5)
4	(-1, 0, 0)	(3, 0.5)

The references of VVs are given as

$$q_{gi} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}. \ (i = 1 \sim 4) \quad (37)$$

Fig. 3 shows the position of vehicles, VVs, and the references GV.

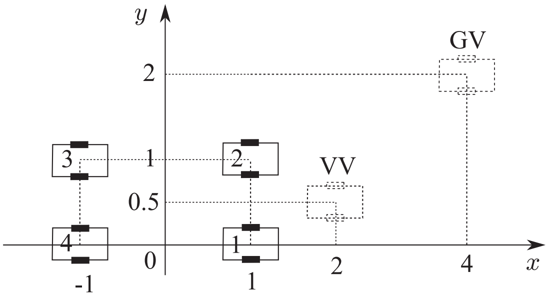

Fig. 3 Simulation setup

The simulation results are obtained by using MATLAB. Figs. 4-6 indicates the position and rotation of VVs, where the solid line, dashed line, thick line, dot-dashed line represents the VV1, VV2, VV3, VV4, respectively. The positions of VVs converge to the reference positions. However, the rotations of VVs are different from the references. Fig. 7 shows the trajectories VVs. It is shown that the vehicles can reach the references keeping the distances among vehicles. Figs. 8-9 represents the convergence the angle velocities of vehicles. The convergence of ω proves the (30) using control law (29).

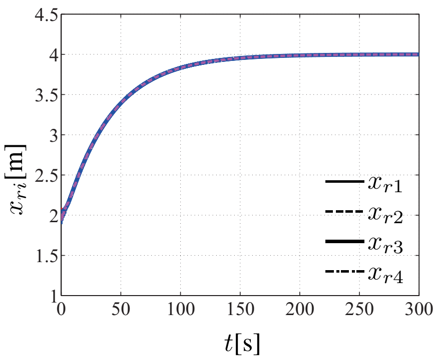

Fig. 4 Time responses of x_{ri}

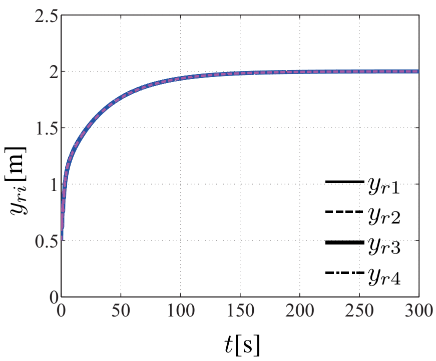

Fig. 5 Time responses of x_{ri}

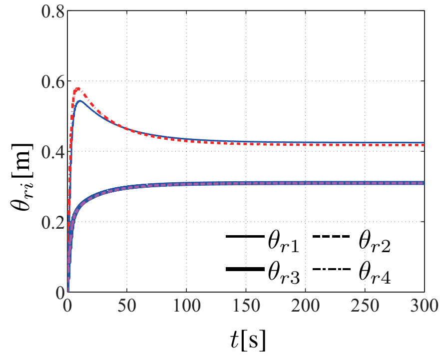

Fig. 6 Time responses of θ_{ri}

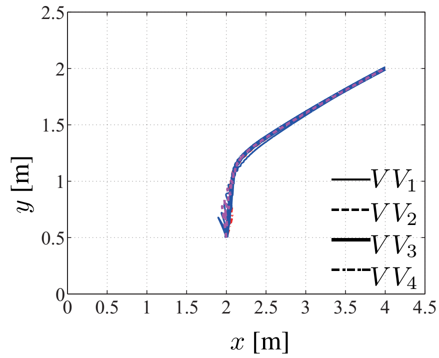

Fig. 7 Trajectories of VV1-VV4

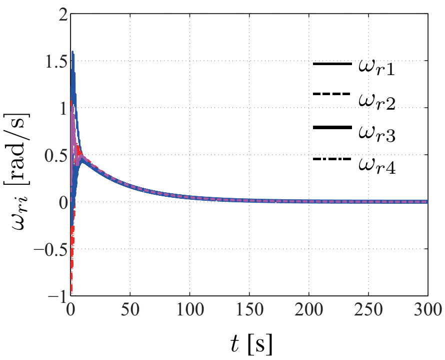

Fig. 8 Time responses of ω_{ri}

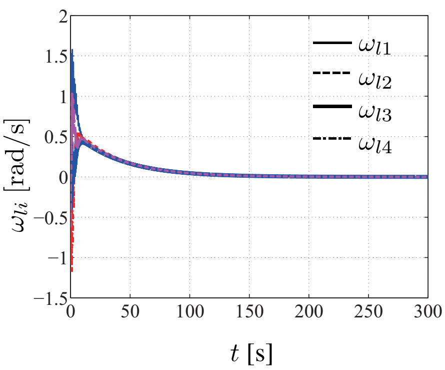

Fig. 9 Time responses of ω_{li}

5. EXPERIMENTS

The experimental setup consists of four vehicles with micro computers, wireless LAN, camera, access point and computer such as Figs. 10. The each vehicle is composed of DC motors, amplifier, micro computer (H8/3052), wireless LAN converter and battery as shown in Fig. 11. The image captured by the camera is sent to a computer. The image is processed by using OpenCV software and the position and rotation of vehicles are computed. The position and the rotation of vehicles are sent to each micro computer on vehicles through the wireless LAN. The control law (29) is implemented on the micro computer. The control input is converted into the PWM signal, the PWM signal drive the right and left motors of the vehicles.

The initial conditions of vehicles and VVs, the references are shown in Fig. 10 and Table 2. The references of VVs are given as

$$q_{gi} = \begin{bmatrix} 400 \\ 300 \\ 0 \end{bmatrix} . \ (i = 1 \sim 4) \tag{38}$$

Table 2 Initial condition of experiment

Vehicle i	Vehicle's position (x_i, y_i, θ_i)	Virtual vehicle's position $(x_{ri}, y_{ri}, \theta_{ri})$
1	(293, 185, 0)	(329, 213, 0)
2	(288, 254, 0)	(329, 213, 0)
3	(222, 256, 0)	(329, 213, 0)
4	(223, 179, 0)	(329, 213, 0)

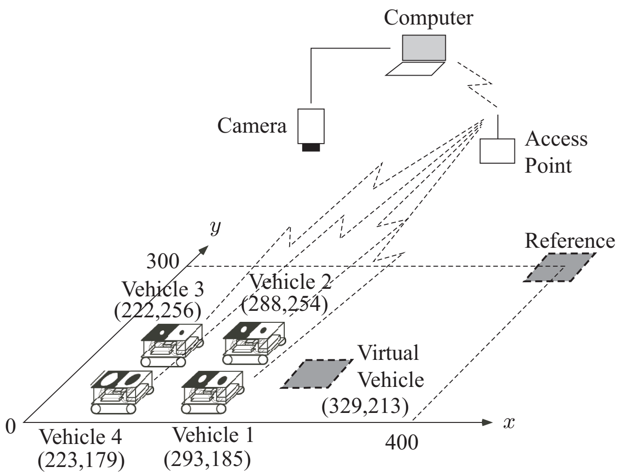

Fig. 10 Experimental setup

Figs. 12-15 indicates the position and rotation of VVs. Figs. 16-18 represent the position and rotation of vehicles. Figs. 19-20 shows the control inputs. The unit pixel is used, because the camera data is directly illustrated.

It is shown that the position of VVs can reach the reference position under keeping the initial formation. However, the rotation is not satisfied as the same as simulations.

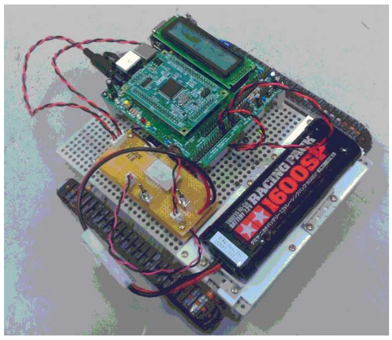

Fig. 11 Vehicle

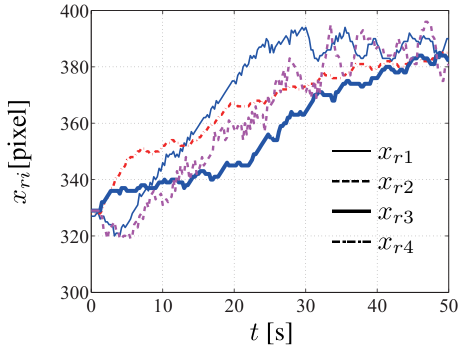

Fig. 12 Time responses of position x_{ri}

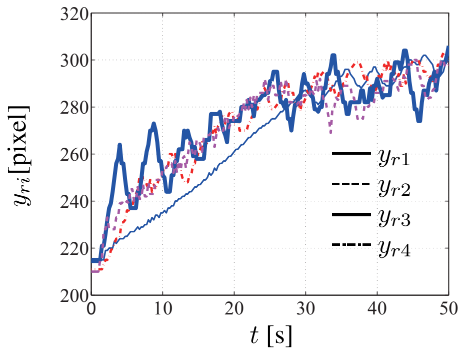

Fig. 13 Time responses of position y_{ri}

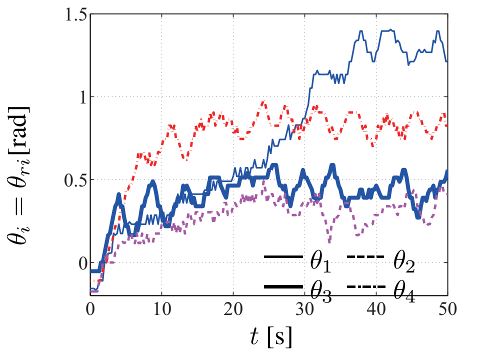

Fig. 14 Time responses of rotation θ_i, θ_{ri}

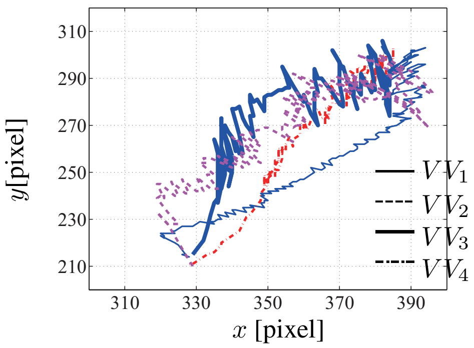

Fig. 15 Trajectories of VVs

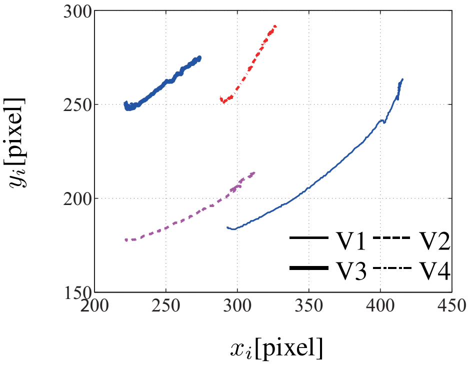

Fig. 18 Trajectories of vehicles

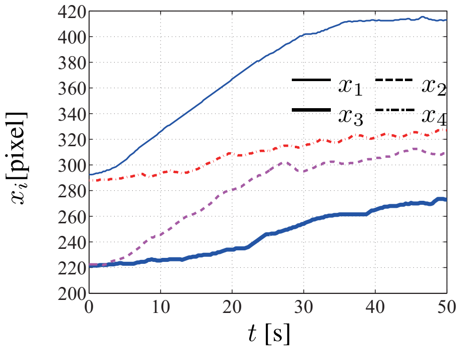

Fig. 16 Time responses of position x_i

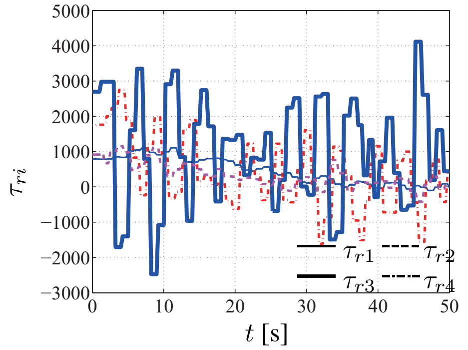

Fig. 19 Time responses of τ_{ri}

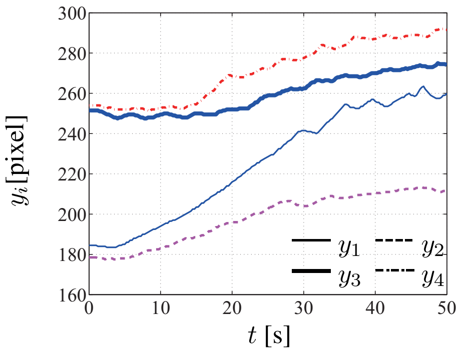

Fig. 17 Time responses of position y_i

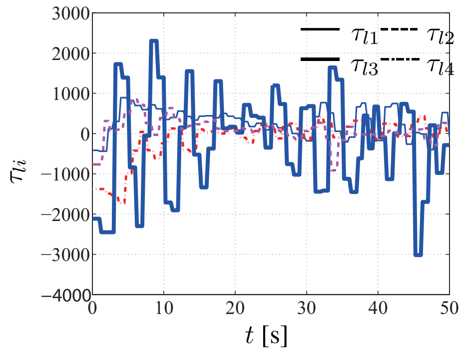

Fig. 20 Time responses of τ_{li}

6. CONCLUSIONS

This paper considered the transportation support system using multi-vehicle's formation. The control law was proposed using virtual structure, graph Laplacian, Lyapunov function. In the simulation and experiment, it was shown that four vehicles can reach the reference position by using four wheeled mobile vehicles.

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