The 8th France-Japan and
6th Europe-Asia Congress on Mechatronics
November 22-24, 2010, Yokohama, Japan

Four-channel Force Reflecting Teleoperation System Based on ISS Small Gain Theorem

Nam Duc Do
Division of Electrical Engineering and Computer Science
Kanazawa University
Kanazawa, Ishikawa, 920-1192, Japan
Email: namdd@scl.ec.t.kanazawa-u.ac.jp

Toru Namerikawa
Department of System Design Engineering
Keio University
3-14-1 Hiyoshi, Yokohama, 223-8522, Japan
Email: namerikawa@sd.keio.ac.jp

***Abstract*—This paper focuses on a new force-reflection (FR) algorithm based on input-to-state stability (ISS) small gain theorem in cooperative work of Single Master-Multiple Slave (SMMS) teleoperation system with time varying communication delay. In this work, we propose a new position tracking control for object grasping and a new FR algorithm based on a PD control to transfer the positions, velocities and force information between both sides of teleoperation. The goal of these methods are to achieve the secure grasping by multiple slave robots and improve the tracking performance of the cooperation SMMS teleoperation system. To analyze stability of the system, the ISS small gain approach is used to show the overall force-reflecting teleoperation to be input-to-state stable. Several experimental results show the effectiveness of our proposed method.**

I. Introduction

Teleoperation systems allow person to extend their sense and manipulation capabilities to remote place. In general, slave robot can perform some real tasks at the remote place by controller signals that send from the master side. In bilateral control, contact information will feedback to the master side, this information is necessary to improve the manipulation capability [1]. One absolutely unsolved problem of the control of teleoperation system is time delay in communication line. The delay may destabilize and deteriorate the transparency of the teleoperation system. Therefore, it is necessary to design a control law to guarantee the stability of the system under communication delays.

Up to now, many successful control schemes have been proposed for the teleoperation system with single master single slave (SMSS). However, the teleoperation systems with multirobot are relative rare. In [2], [3], [4], [5] some control methods were proposed for the system with multiple master multiple slave (MMMS). In this system, one human can control one slave robot to perform separate operation in a cooperative task, thus the system may demand a large of number of human operators if the task requires many slave robots. In [6], [7], [8] the single master multiple slave (SMMS) systems were considered, but these control methods were only proposed for the motion coordination.

Many surveys concern the motion and force control problems of SMSS system, however it is relative rare with SMMS system, especially in case of contact between the grasping object and the environment. When contact occurs, the arising forces will be dictated by the dynamic balancing of two coupled systems, the cooperative slave robots and the environment. In addition, to improve the transparency of SMSS bilateral teleoperation with communication delays, a force-reflection (FR) scheme was addressed by Polushin et al. [11], a stabilization scheme for force reflecting teleoperation was introduced.

In this paper, we propose a novel cooperative control method of the SMMS system with four-channel force reflection based on ISS small gain theorem. This method is developed from one of our previous results [12], [13]. In this work, the position tracking control in a cooperative task between a master and multiple slave robots was proposed, however the real force-reflection has not been treat. In this paper, we propose using two forward and backward forces to transfer the force information from both sides of teleoperation, it makes a four-channel FR algorithm under time varying delay in the communication lines. The goal of our control method is to guarantee the overall stability, the master and slave spacing zero errors achievement and the stability of reflecting force when the interaction occurs. In addition, we also assume using an individual gain for a different structure of the master and the slave robots. In the independent design, a scaling power can be set to both sides of teleoperation. To improve the stability analysis of our previous work [13], the ISS small gain approach is used to show the overall FR teleoperation system to be input-to-state stable, we also can see in [14]. In the experiment, two slave robots hold and carry one object to one desired position following the control signals that send from the master side. The results of experiment show the effectiveness of our proposed control technique.

II. Problem formulation

A. Dynamics of Teleoperation System

In this section, the dynamics of the SMMS system that composed one master and N slave robots can be shown by a motion equation of a general robot arm. The dynamic of the master with m-DOF and the dynamics of the i slave with n_i-DOF are shown as follows:

$$\begin{cases} M_m(q_m)\ddot{q}_m + C_m(q_m,\dot{q}_m)\dot{q}_m = \tau_m + J_m^T(q_m)F_{op} \\ M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i = \tau_i + J_i^T(q_i)F_i \end{cases} \tag{1}$$

where the subscript "m" denotes the master and the subscript "i" denotes the order indexes of the slave; $q_m \in R^{m\times 1}$, $q_i \in R^{n_i\times 1}$ are the joint angle vectors; $\tau_m \in R^{m\times 1}$, $\tau_i \in R^{n_i\times 1}$ are the input torque vectors; $F_{op} \in R^{m\times 1}$ is the operational force vector; $F_i \in R^{n_i\times 1}$ are the grasping force vectors; $M_m \in R^{m\times m}$, $M_i \in R^{n_i\times n_i}$ are the symmetric and positive definite inertia matrices; $C_m(q_m,\dot{q}_m)\dot{q}_m \in R^m$, $C_i(q_i,\dot{q}_i)\dot{q}_i \in R^{n_i}$ are the centripetal and Coriolis torque vectors; $J_m(q_m) \in R^{m\times m}$, $J_i(q_i) \in R^{n_i\times n_i}$ are Jacobian matrices.

In this paper, we propose a control law for different structural teleoperation. This control law of the system maybe not possible with some parameters in the joint space, therefore it is useful to rewrite the master and slave robot dynamics directly in the task space, we have:

$$\dot{x}_k(t) = J_k(q_k)\dot{q}_k(t),\ k = m,\ i. \quad (2)$$

by further differentiation of (2) as:

$$\ddot{x}_k(t) = J_k(q_k)\ddot{q}_k(t) + \dot{J}_k(q_k)\dot{q}_k^2(t),\ k = m,\ i. \quad (3)$$

where $\dot{x}_m,\ \ddot{x}_m \in R^{m\times 1}$ and $\dot{x}_i,\ \ddot{x}_i \in R^{n_i\times 1}$ are the end-effector velocities and acceleration vectors, respectively. Substituting (2) and (3) into (1), we can get the master and multiple slave robots dynamics in the task space as follows:

$$\widetilde{M}_m(q_m)\ddot{x}_m + \widetilde{C}_m(q_m,\dot{q}_m)\dot{x}_m = J_m^{-T}\tau_m + F_{op} \quad (4)$$

$$\widetilde{M}_i(q_i)\ddot{x}_i + \widetilde{C}_i(q_i,\dot{q}_i)\dot{x}_i = J_i^{-T}\tau_i + F_i \quad (5)$$

where: $\widetilde{M}_k = J_k^{-T}M_kJ_k^{-1}$; $\widetilde{C}_k = J_k^{-T}\{C_k - M_kJ_k^{-1}\dot{J}_k\}J_k^{-1}$, $(k = m,\ i)$, x_i is end-effector of each slave robot in Cartesian coordinate system of multiple slaves. Let us denote the total degree of freedom of the N slave robots by: $n = \sum_i^N n_i$, hence the group dynamics of the N slave robots can be rewritten as:

$$\widetilde{M}(q)\ddot{x} + \widetilde{C}(q,\dot{q})\dot{x} = \tau + F \quad (6)$$

where $x = [x_1^T,\ldots,x_N^T]^T \in R^n$, $\tau = [\tau_1^T J_1^{-T},\ldots,\tau_N^T J_N^{-T}]^T \in R^n$, $F = [F_1^T,\ldots,F_N^T]^T \in R^n$, and $\widetilde{M}(q) = diag[\widetilde{M}_1(q_1),\ldots,\widetilde{M}_N(q_N)] \in R^{n\times n}$, $\widetilde{C}(q,\dot{q}) = diag[\widetilde{C}_1(q_1,\dot{q}_1),\ldots,\widetilde{C}_N(q_N,\dot{q}_N)] \in R^{n\times n}$ are the inertia matrices and Coriolis matrices, respectively.

In teleoperation, the signals are transferred between both sides of master and slave. Communication delay is assumed as follows:

Assumption 1: Both time varying delay $T_m(t)$ and $T_s(t)$ are continuously differentiable functions and possibly bounded as:

$$0 \le T_h(t) \le T_h^+ < \infty,\ |\dot{T}_h(t)| < 1,\ h = m,s \quad (7)$$

where $T_h^+ \in R$ is upper bounds of the communication delays. In this paper, the remote environment is assumed to be a simple spring-damper system with constant parameter. This system is as a perturbed system described by the equations below in the form of input-to-state stability properties:

$$\begin{cases} \dot{x}_e = F_{env}(t,x_e,x_L,\dot{x}_L) + g_e(t,x_e,x_L,\dot{x}_L) \\ F_L = \Gamma_{env}(t,x_e,x_L,\dot{x}_L) \end{cases} \quad (8)$$

where $x_e \in R^n$ is a position of the environment, x_L and $\dot{x}_L \in R^{n\times 1}$ are the position and velocity vectors of the cooperative-slave robots in Locked-System (this system will be presented in Section III); F_L is the environment force. We assume that $F_{env}(t,x_e,x_L,\dot{x}_L)$, $\Gamma_{env}(t,x_e,x_L,\dot{x}_L)$ are piecewise continuous in t and locally Lipschitz in $x_e,\ x_L,\ \dot{x}_L$. The input $(x_L(t),\dot{x}_L(t))$ is a piecewise continuous and essentially bounded function of t for all $t \ge 0$; $g_e(t,x_e,x_L,\dot{x}_L)$ is the perturbation term. The environment satisfies following assumptions:

Assumption 2: The cooperative-slave contacts with following spring-damper environment with constant parameter

$$\Gamma_{env}(t,x_e,x_L,\dot{x}_L) \le |x_e| + a|\dot{x}_L| + b|x_L| \quad (9)$$

holds for all $t \ge 0$; $a,\ b$ are constant parameters $(a,\ b > 0)$.

Assumption 3: Let $x_e = 0$ be a uniformly asymptotically stable equilibrium point of the nominal system (8). There exists a Lyapunov function of the nominal system such that $\alpha_{1e}(|x_e|) \le V_e(x_e) \le \alpha_{2e}(|x_e|)$ holds for all $x_e \in R^n$, and $V_e = 0$ while $x_e = 0$. The time derivative of V_e along trajectories of (8) satisfies:

$$\dot{V}_e(t) \le -\alpha_{3e}|x_e|^2 + \left|\frac{\partial V_e}{\partial x}\right| g(t,x_e,x_L,\dot{x}_L) \quad (10)$$

where $\alpha_{1e}(|x_e|)$, $\alpha_{2e}(|x_e|)$ are class $\mathscr{K}$ functions and $\alpha_{3e} > 0$. The perturbation $g_e(t,x_e,x_L,\dot{x}_L)$ in (10) satisfies the uniform bound:

$$\left|\frac{\partial V_e}{\partial x}\right| |g_e(t,x_e,x_L,\dot{x}_L)| \le \delta\alpha_{4e}|x_e| \le F_L^T s_e(t) \quad (11)$$

for almost all $t \ge 0$ and $\in R^n$, $\alpha_{4e} > 0$, and δ is a perturbation gain.

Let us define:

$$s_e(t) = \dot{x}_L(t) + \Lambda_{env}x_L(t) \quad (12)$$

where $\Lambda_{env} \in R^{n\times n}$ is a positive diagonal gain matrix.

Note the first bound of the perturbation in (11), we have:

$$\begin{aligned} \dot{V}_e &\le -\alpha_{3e}|x_e|^2 + \theta_e\alpha_{3e}|x_e|^2 - \theta_e\alpha_{3e}|x_e|^2 + \delta\alpha_{4e}|x_e| \\ &= -(1-\theta_e)\alpha_{3e}|x_e|^2 - |x_e|(\theta_e\alpha_{3e}|x_e| - \delta\alpha_{4e}) \\ &\le -(1-\theta_e)\alpha_{3e}|x_e|^2;\ \forall |x_e| \ge \frac{\delta\alpha_{4e}}{\theta_e\alpha_{3e}} \end{aligned} \quad (13)$$

where θ_e is some positive constant, $\theta_e < 1$.

Therefore, the upper bound of perturbation in (11) satisfies the time derivative of V_e as follows:

$$\dot{V}_e(t) \le -\alpha_{3e}|x_e|^2 + F_L^T s_e(t) \quad (14)$$

B. Control Objectives

In this paper, the SMMS system is constructed with one master and two slave robots. We would like to design a control law for SMMS system to satisfy following Control Objectives.

Control Objective 1: (Autonomously grasping by multiple slave robots) In this work, the grasping achievement following condition:

$$x_S = x_S^d \quad (15)$$

where $x_S \in R^{n-m}$ is the relative position of the end-effectors of the slaves, $x_S^d \in R^{n-m}$ is a desired position of x_S.

Control Objective 2: (Movement of grasped object) When the grasping is achieved, the movement of the grasped object is achieved as:

$$x_L = x_m \quad (16)$$

where $x_L = \alpha x_{L0} - C$, $x_{L0} \in R^m$ and x_m are the center position of the slave end-effectors and the position of the master, respectively; $\alpha \in R$ is the position scale, $C \in R^m$ is shown as a translation value.

Control Objective 3: (Static force reflection) The teleoperation with static force reflection is achieved as $\dot{x}_j = \ddot{x}_j = 0(j = m,L)$ such that:

$$F_{op} = -\beta F_L \quad (17)$$

where F_L is the contact force of cooperative-slave, $\beta > 0 \in R$ is a positive scalar and it expresses a force scaling factor.

III. CONTROL DESIGN

In this section, we propose a novel control law for the SMMS system to achieve above Control Objectives.

A. Passive-Decomposition

First, base on Passive-Decomposition [9] that was introduced by D. Lee, the dynamic of multiple slave robots is decomposed into two decoupled systems: the Shape-System describing "movement of the multiple slaves with grasping object" and the Locked-System describing "movement of the multiple slaves according to the instruction from the master".

Utilizing the Passive-Decomposition, the velocity of multiple slave robots is rewritten with each system as follows:

$$\dot{x} = S^{-1}\begin{bmatrix}\dot{x}_S \\ \dot{x}_L\end{bmatrix} \quad (18)$$

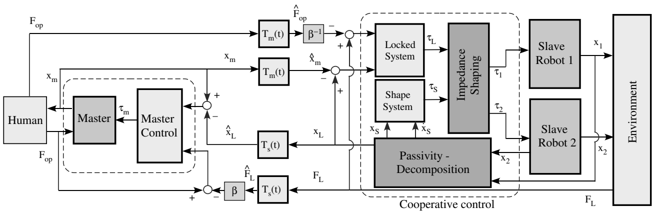


Fig. 1. Four-channel FR Teleoperation of SMMS system

where $\dot{x}_S \in R^{n-m}$ is relative velocity of the end-effectors of the slaves in the Shape-System and $\dot{x}_L \in R^m$ is the velocity the Locked-System. S is the non-singular decomposition matrix. In the following formula of $S^{-T}\widetilde{M}S^{-1}$, the non-diagonal terms become zeros as:

$$S^{-T}\widetilde{M}S^{-1} = \begin{bmatrix} M_S & 0 \\ 0 & M_L \end{bmatrix} \tag{19}$$

where $M_S \in R^{(n-m)\times(n-m)}$, $M_L \in R^{m\times m}$ are inertia matrices of the Shape-System and the Locked-System, respectively. In addition, a local compensation of impedance shaping is necessary. We have relationships of the forces as follows:

$$\begin{bmatrix} F_S \\ F_L \end{bmatrix} = S^{-T}\begin{bmatrix} F_1 \\ F_2 \end{bmatrix}, \quad \begin{bmatrix} \tau_S \\ \tau_L \end{bmatrix} = S^{-T}\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \tag{20}$$

where F_1 and F_2 are forces in each end-effector of slave robot that exert on grasping object. We define:

$$\begin{bmatrix} C_S & C_{SL} \\ C_{LS} & C_L \end{bmatrix} = S^{-T}\widetilde{M}\frac{d}{dt}(S^{-1}) + S^{-T}\widetilde{C}S^{-1} \tag{21}$$

note (6), the Passive-Decomposition form is written as:

$$M_S(q)\ddot{x}_S + C_S(q,\dot{q})\dot{x}_S + C_{SL}(q,\dot{q})\dot{x}_L = \tau_S + F_S \tag{22}$$

$$M_L(q)\ddot{x}_L + C_L(q,\dot{q})\dot{x}_L + C_{LS}(q,\dot{q})\dot{x}_S = \tau_L + F_L \tag{23}$$

Above dynamic equations include friction terms $C_{SL}(q,\dot{q})\dot{x}_L$ and $C_{LS}(q,\dot{q})\dot{x}_S$. However, ignore the remote control by the human, decoupling of the Shape-System and the Locked-System is desired for the slave that maybe autonomous grasping. Therefore, the decoupling control inputs are given as follows:

$$\tau_S = C_{SL}(q,\dot{q})\dot{x}_L + \tau_S' \tag{24}$$

$$\tau_L = C_{LS}(q,\dot{q})\dot{x}_S + \tau_L' \tag{25}$$

where τ_S', τ_L' are new control inputs. Substituting (24), (25) into (22), (23), we get:

$$M_S(q)\ddot{x}_S + C_S(q,\dot{q})\dot{x}_S = \tau_S' + F_S \tag{26}$$

$$M_L(q)\ddot{x}_L + C_L(q,\dot{q})\dot{x}_L = \tau_L' + F_L \tag{27}$$

hence, two above dynamics become a decoupling.

Some properties of this SMMS system are given as follows:

Property 1: $M_i(q) (i = S, L)$ is a positive symmetric matrix, and there exist some constant parameters with below relationship as :

$$0 < m_{i1} \leq \| M_i \| \leq m_{i2}$$
$$\| C_i \| \leq c_i \| \dot{x}_i \| \tag{28}$$

Property 2: $\dot{M}_i(q) - 2C_i(q,\dot{q})$ $(i = S, L)$ are skew-symmetric matrices.

Property 3: $\dot{x}_i, \ddot{x}_i$ $(i = S, L)$ are bounded and $\dot{M}_i, \dot{C}_i$ are also bounded.

Properties 1∼3 denote the feature of motion equation of normal robots. The following assumption is from (1), (26), (27) as follows:

Assumption 4: The operator and the environment can be modeled as passive systems, where the velocities $\dot{x}_m$, $\dot{x}_L$ are system inputs and bounded, the force F_{op}, F_L are system outputs, respectively. Moreover, these forces are bounded by the functions of the velocities of the master and the Locked-System. The velocities $\dot{x}_m$, $\dot{x}_L$ also equal zero for $t < 0$.

B. Proposal Control Law

Concerning the control law of the Shape-System (26), the Control Objective of this system is: $x_S = x_S^d$, then the position tracking with this control law is proposed as follows:

$$\tau_S' = M_S\{\ddot{x}_S^d(t) - K_d^S(\dot{x}_S - \dot{x}_S^d(t)) - K_p^S(x_S - x_S^d(t))\} + C_S\dot{x}_S - F_S \tag{29}$$

Substituting (29) into (26) we obtain the following closed-loop systems:

$$\ddot{e} + K_d^S\dot{e} + K_p^S e = 0, \; e = x_S - x_S^d \tag{30}$$

where K_d^S, K_p^S are positive definite diagonal gain matrices.

Considering the coupling control of the master and the Locked-System. Note the Control Objectives 2 and 3, the SMMS teleoperation system with four-channel FR architecture of the communication lines is concerned. The cooperative control law is defined as follows:

$$\tau_m = J_m^T\{-K_d^m\dot{x}_e^m - K_p^m x_e^m - (1 - K_F^m)F_{op} - K_F^m\beta F_L(t - T_s(t))\} \tag{31}$$

$$\tau_L' = -K_d^L\dot{x}_e^L - K_p^L x_e^L + K_F^L\beta^{-1}F_{op}(t - T_m(t)) - (1 + K_F^L)F_L \tag{32}$$

where $\dot{x}_e^{m/L}$, $x_e^{m/L}$ are the position and velocity errors of master and cooperative slave robots; β is force scaling factor; $T_m(t)$ and $T_s(t)$ are assumed to be time varying delays. If the positions and velocities of the master and cooperative slaves are transmitted to each side with communication delays $T_{m/s}(\cdot)$, and similar to the exerted and contact forces, the following signals

$$\hat{x}_{m/L}(t) = x_{m/L}(t - T_{m/s}(t)), \; \dot{\hat{x}}_{m/L}(t) = \dot{x}_{m/L}(t - T_{m/s}(t))\dot{T}_{m/s}(t);$$
$$\hat{F}_{op} = F_{op}(t - T_m(t)), \; \hat{F}_L = F_L(t - T_s(t)) \tag{33}$$

are available for the controller on both sides of teleoperation. We define:

$$x_e^m = x_m - \hat{x}_L, \; \dot{x}_e^m = \dot{x}_m - \dot{\hat{x}}_L;$$
$$x_e^L = x_L - \hat{x}_m, \; \dot{x}_e^L = \dot{x}_L - \dot{\hat{x}}_m \tag{34}$$

Substituting above control law (31), (32) into the Locked-System (27) and dynamic equation of the master (4), we obtain a closed-loop system as follows:

$$\widetilde{M}_m(q_m)\ddot{x}_m + \widetilde{C}_m(q_m,\dot{q}_m)\dot{x}_m = -K_d^m\dot{x}_e^m - K_p^m x_e^m + K_F^m\left(F_{op} - \beta\hat{F}_L\right) \tag{35}$$

$$M_L(q)\ddot{x}_L + C_L(q,\dot{q})\dot{x}_L = -K_d^L\dot{x}_e^L - K_p^L x_e^L + K_F^L\left(\beta^{-1}\hat{F}_{op} - F_L\right) \tag{36}$$

where K_p^j, K_d^j and K_F^j $(j = m, L)$ are gains and defined as follows: $K_{(p,d,F)}^m = k_m K_{(p,d,F)}$; $K_{(p,d,F)}^L = k_L K_{(p,d,F)}$; where $K_{(p,d,F)} \in R^{n\times n}$ are positive definite diagonal control gain matrices; $k_m > 0, k_L > 0$ are constant gains of scalar that designed separately on the master and the slave side. We can see in the Fig. 1, it shows a block diagram of the cooperative control system with four-channel force reflecting teleoperation.

IV. Stability Analysis

A. Stability of Shape-System

The below theorem concerns the Shape-System.

Theorem 1: Consider the closed-loop Shape-System (30), desired value of relative position of spaces between the slave robots is conversed as follows:

$$e = x_S - x_S^d \to 0 \quad as \; t \to \infty \tag{37}$$

Proof: The equation (30) can be rewritten as follows:

$$\begin{bmatrix} \dot{e} \\ \ddot{e} \end{bmatrix} = \phi \begin{bmatrix} e \\ \dot{e} \end{bmatrix}, \quad \phi = \begin{bmatrix} 0 & I \\ -K_P^S & -K_d^S \end{bmatrix} \tag{38}$$

where K_P^S, K_d^S are positive diagonal matrices, eigenvalues of ϕ become negative, therefore following errors of position and velocity are achieved:

$$e = x_S - x_S^d \to 0 \quad as \quad t \to \infty \tag{39}$$

$$\dot{e} = \dot{x}_S - \dot{x}_S^d \to 0 \quad as \quad t \to \infty \tag{40}$$

it means the Control Objective 1 is achieved and the autonomous grasping of multiple slaves is also achieved. ■

B. Stability of Locked-System

This section deals with the stability of the overall teleoperation system that includes the master and the cooperative slave subsystems of Locked-System.

Lemma 1: Consider the closed-loop master subsystem to be a piecewise continuous in t and locally Lipschitz in the state $x_M = (x_m^T, \dot{x}_m^T)^T$; the input $u_M = (\hat{x}_L^T, \dot{\hat{x}}_L^T, \hat{F}_L^T)^T$. There exists a continuous differentiable, positive definite, radially unbounded Lyapunov function V_m:$R^r \to R$ of the subsystem that satisfies the inequalities:

$$\alpha_{1m}(|x_M|) \leq V_m \leq \alpha_{2m}(|x_M|) \tag{41}$$

$$\frac{\partial V_m}{\partial t} + \frac{\partial V_m}{\partial x} f(t, x_M, u_M) \leq -\alpha_{3m}(|x_M|), \tag{42}$$
$$\forall |x_M| \geq \rho_m(|u_M|) > 0$$

$\forall t \geq 0$, $D = \{x_M \in R^n;\ |x_M| < r_m\}$, $D_u = \{u_M \in R^m;\ |u_M| < r_{mu}\}$, where $\alpha_{1m}(|x_M|)$, $\alpha_{2m}(|x_M|)$, $\alpha_{3m}(|x_M|)$ and ρ_m are class $\mathscr{K}$ functions, then the subsystem is locally input-to-state stable.

Proof: First, consider an ISS-Lyapunov function candidate as follows:

$$V_m = k_m^{-1} \dot{x}_m^T \widetilde{M}_m \dot{x}_m + 2 x_m^T K_p x_m - 2K_F \int_0^t F_{op}^T(\xi) \dot{x}_m(\xi) d\xi \tag{43}$$

where $\widetilde{M}_m, K_p, K_F$ are positive definite matrices; $k_m, \beta > 0$. Following the Assumption 4, the environment and the manipulator are passive, then V_m is the positive function. We also easily check that this function satisfies (41), and $V_m = 0$ while $\dot{x}_m = 0$, $x_m = 0$. Since $\alpha_{1m}(x_M)$ and $\alpha_{2m}(x_M)$ are radially unbounded, hence V_m is said to be radially unbounded.

The derivative of above function along trajectories of the system (35) with concerning Property 2 as:

$$\begin{aligned} \dot{V}_m =& -2\dot{x}_m^T K_d \dot{x}_m + 2\dot{x}_m^T K_d \dot{\hat{x}}_L + 2\dot{x}_m^T K_p \hat{x}_L - 2K_F \beta \dot{x}_m^T \hat{F}_L \\ & + 2\dot{x}_m^T \theta_m \dot{x}_m - 2\dot{x}_m^T \theta_m \dot{x}_m \\ =& -2\dot{x}_m^T (K_d - \theta_m)\dot{x}_m - 2\dot{x}_m^T(\theta_m \dot{x}_m - K_d \dot{\hat{x}}_L - K_p \hat{x}_L + K_F \beta \hat{F}_L) \\ \leq& -2\dot{x}_m^T (K_d - \theta_m) \dot{x}_m \end{aligned} \tag{44}$$
$$\forall |\dot{x}_m| \geq \frac{K_d |\dot{\hat{x}}_L| + K_p |\hat{x}_L| + K_F \beta |\hat{F}_L|}{\theta_m} (= \rho_m(|u_M|))$$

where θ_m is some positive constant. We can choose θ_m to satisfy the derivative of V_m to be negative as follows:

$$\theta_m < K_d \tag{45}$$

Remark 1: Note the Assumptions 1, 4 and the expression (33), there exists at least one value of $|\dot{x}_m|$ to guarantee above condition of (44), since $\dot{T}_s(t)$ is bounded, the delay parameters $\hat{x}_L$ and $\dot{\hat{x}}_L$ are also bounded.

Using the fact that, the signal $\dot{x}_m$ is bounded, the feedback force from slave side is also bounded or the input u_M is bounded. Following the Theorem 5.2 in [15], we can choose a class $\mathscr{K}$ function $\gamma_m = \alpha_{1m}^{-1} \circ \alpha_{2m} \circ \rho_m$, positive constant $k_{1m} = \alpha_{2m}^{-1}(\alpha_{1m}(r_m))$ and $k_{2m} = \rho_m^{-1}(\min\{k_{1m}, \rho_m(r_{mu})\})$ for any initial state $x_M(t_0)$ and any bounded input $u_M(t)$, and we can choose r_m and r_{mu} large enough that satisfies the inequalities given below:

$$\begin{aligned} & |x_M(t_0)| < \alpha_{2m}^{-1}(\alpha_{1m}(r_m)); \\ & \rho_m\big(\sup_{t \geq t_0} |u_M|\big) < \min\{\alpha_{2m}^{-1}(\alpha_{1m}(r_m)), \rho_m(r_{mu})\} \end{aligned} \tag{46}$$

Using the Definition 5.2 in [15] we have the solution $x_M(t)$ exists and satisfies:

$$|x_M(t)| \leq \mu(|x_M(t_0)|, t - t_0) + \rho_m\big(\sup_{t_0 \leq \tau \leq t} |u_M(\tau)|\big),$$
$$\forall 0 \leq t_0 \leq t \tag{47}$$

where μ is a class $\mathscr{KL}$ function. Then the solution $x_M(t)$ only depends on $u_M(\tau)$ for $t_0 \leq \tau \leq t$, and the master subsystem is locally input-to-state stable. ■

Now, we consider the slave-environment interconnection with the cooperative-slave subsystem.

Lemma 2: State of the closed-loop cooperative-slave subsystem is assumed as: $x_S = (x_L^T, \dot{x}_L^T, x_e^T)^T$, and input: $u_S = (\hat{x}_m^T, \dot{\hat{x}}_m^T, \hat{F}_{op}^T)^T$. We suppose the environment dynamics (8) satisfy Assumptions 2 and 3. There exits a continuous differentiable, positive definite, radially unbounded Lyapunov function V_s of the subsystem that satisfies the below inequalities:

$$\alpha_{1s}(|x_S|) \leq V_s \leq \alpha_{2s}(|x_S|) \tag{48}$$

$$\frac{\partial V_s}{\partial t} + \frac{\partial V_s}{\partial x} f(t, x_S, u_S) \leq -\alpha_{3s}(|x_S|) \tag{49}$$
$$\forall |x_S| \geq \rho_s(|u_S|) > 0$$

$\forall t \geq 0$, $D = \{x_S \in R^n; |x_S| < r_s\}$, $D_u = \{u_S \in R^s; |u_S| < r_{su}\}$ where $\alpha_{1s}(|x_S|)$, $\alpha_{2s}(|x_S|)$, $\alpha_{3s}(|x_S|)$ and ρ_s are class $\mathscr{K}$ functions, then the subsystem is locally input-to-state stable.

Proof: We consider the ISS-Lyapunov function candidate as follows:

$$V_s = k_L^{-1} \dot{x}_L M_L \dot{x}_L + x_L K_p x_L + 2K_F \int_0^t F_L(\xi) \dot{x}_L(\xi) d\xi + V_e \tag{50}$$

where V_e was introduced in the Assumption 3, M_L, K_p, K_F are the positive definite matrices. Similar to the master subsystem, the first and the second term of the right-side in (50) are radially unbounded; note that in the Assumption 3, V_e satisfies the inequality (9) with any radially unbounded α_{1e} and α_{2e}, then V_s is also said to be radially unbounded and satisfies the inequality (48). We also can easily check that $V_s(0) = 0$ while $x_S = 0$ ($\dot{x}_L = 0$, $x_L = 0$, $x_e = 0$).

The derivative of V_s along the trajectories of the system (36) with concerning Property 2 as follows:

$$\dot{V}_s = -2\dot{x}_L^T K_d \dot{x}_L + 2\dot{x}_L^T K_d \dot{\hat{x}}_m + 2\dot{x}_L^T K_p \hat{x}_m + 2K_F \beta^{-1} \hat{F}_{op}^T \dot{x}_L + \dot{V}_e \tag{51}$$

Note the derivative of V_e in (14) and the expressions of F_L and s_e in Assumption 3, we have:

$$\begin{aligned} \dot{V}_e \leq & -\alpha_{3e} x_e^2 + |x_e^T| \dot{x}_L - |x_e^T| \Lambda_{env} x_L + a \dot{x}_L^2 \\ & - b |x_L^T| \Lambda_{env} x_L + |x_L^T| (b - a\Lambda_{env}) \dot{x}_L \end{aligned} \tag{52}$$

Applying Young's quadratic inequality with $|A^T B| \leq (\varepsilon/2)|A|^2 + (1/2\varepsilon)|B|^2$ that holds for all $\varepsilon > 0$, therefore we can obtain the following bound of the second and third terms in (52) as:

$$|x_e|^T \dot{x}_L \leq \frac{\lambda}{4} |x_e|^2 + \frac{1}{\lambda} \dot{x}_L^2 \tag{53}$$

$$|x_e|^T \Lambda_{env} x_L \leq \frac{\lambda}{4} |x_e|^2 + \frac{\Lambda_{env}}{\lambda} x_L^2 \tag{54}$$

where λ is a small positive constant. If we choose $a = b = 1/\lambda$ and $\Lambda_{env} = I$, we can rewrite the derivative of the Lyapunov function as follows:

$$\begin{aligned}\dot{V}_s \leq & -\dot{x}_L^T 2K_d\dot{x}_L - \left(\alpha_{3e} - \frac{\lambda}{2}\right)|x_e|^2 + 2K_F\beta^{-1}\dot{x}_L^T\hat{F}_{op} \\ & + 2\dot{x}_L^T K_d\dot{\hat{x}}_m + \dot{x}_L^T K_d\hat{x}_m + 2\dot{x}_L^T\theta_L\dot{x}_L - 2\dot{x}_L^T\theta_L\dot{x}_L \\ \leq & -2\dot{x}_L^T\left(K_d - \theta_L\right)\dot{x}_L - \left(\alpha_{3e} - \frac{\lambda}{2}\right)|x_e|^2 \\ & -2\dot{x}_L^T\left(\theta_L\dot{x}_L - K_d\dot{\hat{x}}_m - K_d\hat{x}_m - K_F\beta^{-1}\hat{F}_{op}\right)\end{aligned} \tag{55}$$

where θ_L is some positive constant. We can choose the values of λ and θ_L to satisfy first two terms of (55) to be negative. We have:

$$\begin{cases}\theta_L < K_d \\ \lambda < 2\alpha_{3e}\end{cases} \tag{56}$$

from (55), we receive:

$$\dot{V}_s \leq -2\dot{x}_L^T\left(K_d - \theta_L\right)\dot{x}_L - \left(\alpha_{3e} - \frac{\lambda}{2}\right)|x_e|^2 \tag{57}$$

$$\forall|\dot{x}_L| \geq \frac{K_d|\dot{\hat{x}}_m| + K_d|\hat{x}_m| + K_F\beta^{-1}|\hat{F}_{op}|}{\theta_L}\left(= \rho(|u_S|)\right)$$

Similar to the master subsystem case, note the Assumptions 1, 4 and the expression (33), we can conclude that the slave+environment subsystem is also locally input-to-state stable. ■

Based on the Lemma 1 and Lemma 2, the following theorem concerning stability properties of the closed-loop system is obtained.

Theorem 2: Consider the cooperative teleoperation system (1), the FR algorithm in (31) and (32). Suppose the environment dynamic satisfies Assumption 3, there exists $\gamma_\Lambda(\cdot) \in \mathscr{K}$ such that $\gamma_\Lambda = \gamma_m \circ \gamma_s$ implies that: for the four-channel FR teleoperation, the overall system is input-to-state stable.

Proof: We choose the state of the overall FR teleoperation as: $x_T = (x_m^T, \dot{x}_m^T, x_L^T, \dot{x}_L^T, x_e^T)^T$ and the output as: $u_T = \left(\hat{x}_m^T, \dot{\hat{x}}_m^T, \hat{x}_L^T, \dot{\hat{x}}_L^T, \hat{F}_{op}^T, \hat{F}_L^T\right)^T$. Now we can combine above presented results and the consecutive application of the ISS theorem. Indeed, denote by the ISS gain $\gamma_{m[u_M \to x_M]}(\cdot) \in \mathscr{K}$ of the closed-loop master subsystem, whole existence is guaranteed by Lemma 1. And also, we let $\gamma_{s[u_S \to x_S]}(\cdot) \in \mathscr{K}$ be the ISS gain of the closed-loop slave+environment subsystem (8). Choose γ_Λ such that the satisfying:

$$\gamma_\Lambda = \gamma_{[u_M \to x_M]}(\cdot) \circ \gamma_{[u_S \to x_S]}(\cdot) \tag{58}$$

Applying the Definition 5.2 [15], we can conclude the overall FR teleoperation system is input-to-state stable. The proof is completed. ■

V. Evaluation by Control Experiments

A. Impedance Shaping

In this paper, the SMMS system was constructed with two of 2-DOF serial-link arm of slave robots. Some parameters x_S, x_S^d, x_L are defined as follows:

$$x_S = \bar{x}_1 - \bar{x}_2 = \begin{bmatrix} x_1 - x_2 \\ y_1 - y_2 \end{bmatrix};\ x_S^d = \begin{bmatrix} d \\ 0 \end{bmatrix} \tag{59}$$

$$x_L = \alpha\frac{\bar{x}_1 + \bar{x}_2 - C}{2} = \frac{\alpha}{2}\begin{bmatrix} x_1 + x_2 - c \\ y_1 + y_2 \end{bmatrix} \tag{60}$$

where $C = [c\ \ 0]^T$ is the transport value of the coordinates at master and slave robot, α is the position scale; $\bar{x}_1 = [x_1\ y_1]^T$, $\bar{x}_2 = [x_2\ y_2]^T$ are position of the end-effector of slave robots, respectively. From (59) and (60) we get:

$$\begin{bmatrix} \dot{x}_S \\ \dot{x}_L \end{bmatrix} = \begin{bmatrix} \dot{\bar{x}}_1 - \dot{\bar{x}}_2 \\ \frac{\alpha}{2}(\dot{\bar{x}}_1 + \dot{\bar{x}}_2) \end{bmatrix} = \begin{bmatrix} I & -I \\ \frac{\alpha}{2}I & \frac{\alpha}{2}I \end{bmatrix}\begin{bmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \end{bmatrix} \tag{61}$$

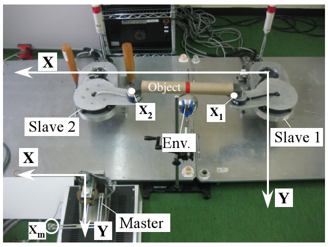


Fig. 2. Experimental setup.

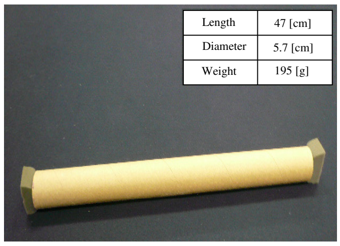


Fig. 3. Grasping object.

We define the decomposition matrix S as follows:

$$S = \begin{bmatrix} I & -I \\ \frac{\alpha}{2}I & \frac{\alpha}{2}I \end{bmatrix} \tag{62}$$

However, the non-diagonal and coupling terms between the Shape-System and the Locked System still exist even by using this decomposition matrix S. Thus, a linearization technique with the impedance shaping is then introduced as:

$$\tau_i = J_i^T\{M_iH^{-1}(\tau_i' + F_i) - F_i + C_i\dot{x}_i\} \quad (i = 1, 2) \tag{63}$$

where τ_i' is a new control input, H is inertia matrix of device. Therefore, the Shape-System and the Locked-System to be decoupling, we get:

$$\begin{bmatrix} M_S & 0 \\ 0 & M_L \end{bmatrix}\begin{bmatrix} \ddot{x}_S \\ \ddot{x}_L \end{bmatrix} = \begin{bmatrix} \tau_S' \\ \tau_L' \end{bmatrix} + \begin{bmatrix} F_S \\ F_L \end{bmatrix} \tag{64}$$

Therefore, by the definition of x_S, x_L mentioned above, the Shape- System and the Locked-System are decoupling by the impedance shaping only.

B. Experimental Setup and Results

In the experiment, the SMMS system is constructed by one master with two DOFs parallel link type arm and two slaves with two-two DOFs series link type arms. The experimental setup is shown in Fig. 2. The remote environment on the slave side is a hard iron global as also shown in the Fig. 2. The cylindrical grasping object is used and shown in Fig. 3. For implementation of the controllers and communication lines, we utilise a dSPACE digital control system (dSPACE Inc.). All experiments have been done with the artificial time varying communication delays and the sampling time is $1[ms]$:

$$\begin{cases} T_m(t) = 0.1\sin t + 0.15\ [s] \\ T_s(t) = 0.1\sin t + 0.15\ [s] \end{cases} \tag{65}$$

From above equation, maximum round-trip delay is $0.39[s]$. The controller gains are chosen as: $K_p^m = diag(120, 120)$, $K_d^m = diag(24, 24)$; $K_p^L = diag(450, 450)$, $K_d^L = diag(90, 90)$; $K_F^m = diag(0.15, 0.15)$, $K_F^L = diag(0.55, 0.55)$, $k_m = 1$, $k_L = 3.75$, $K_p^S = diag(300, 300)$, $K_d^S = diag(50, 50)$; power scale $\beta = 1/4$; position scale $\alpha = 1.5625$.

Two kind of experimental conditions are given as follows:

Case 1: Control the grasping object without any contact with remote environment

Case 2: Control the grasping object in contact with remote environment

However, in actual experiments, it is difficult for entirety time synchronization on master and slave side in the system configuration. The data that received from master and the data of slave that measured from slave side need be compared, especially the position data on the slave side.

The experimental results are shown in Figs. 4-6. The Fig. 4 shows the position of the master mini-robot and the Locked-System with cooperative robots in case two slaves robot move

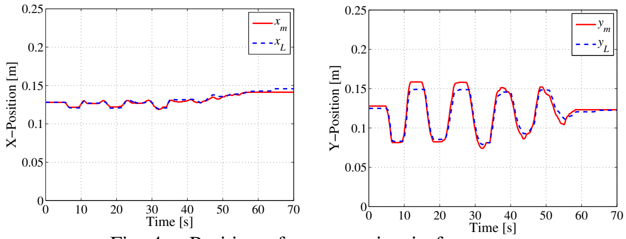
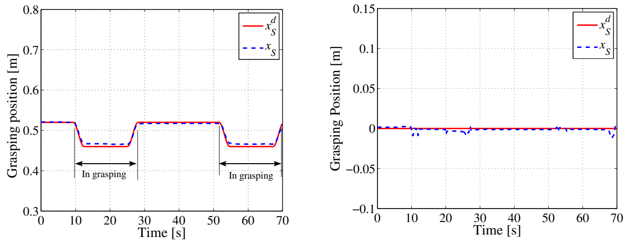


Fig. 4. Position of non-grasping in free space.



(a) X-Position (b) Y-Position

Fig. 5. Position of Shape-System and x_S^d.

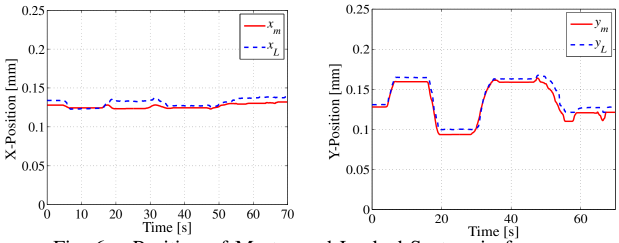


Fig. 6. Position of Master and Locked-System in free space.

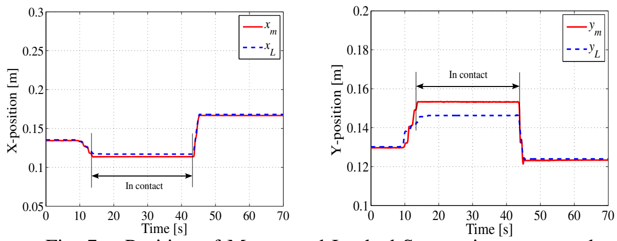


Fig. 7. Position of Master and Locked-System in contact task.

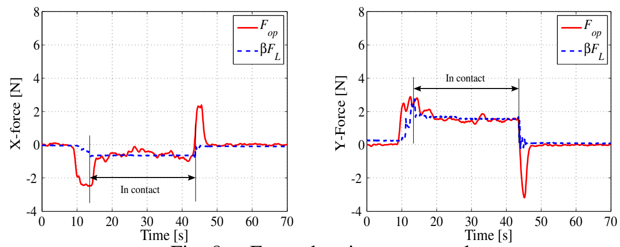


Fig. 8. Force data in contact task.

in the free space. We can see the positions of both side are achieved. The Fig. 5 shows the time responses of the end-effector position of slave of the Shape-System, In this figure, we can conclude that the relative position between two slaves following the target trajectory with grasping object is achieved. In the Fig. 6, the grasping object at the center position between two end-effectors of the slaves is able to transported following the end-effector of the master robot. The object is presumed to mix with closed links of the slaves. When grasping, the distance between the slaves is narrowed.

The experimental results in case of contact with the environment is shown in Figs. 7, 8. The objected is grasped and come to contact with the environment following vertical Y axis. Fig. 7 shows the time responses of the end-effector position of the Locked-System with the master. The Fig. 8 shows the reflection forces when the object contacts with the environment. We can see that the reflecting force from the environment and the scaling force of the human are same values.

VI. Conclusions

In this paper, we proposed a new control law with four-channel force-flection (FR) algorithm for a Single Master-Multiple Slave (SMMS) teleoperation system based on ISS small gain theorem. This proposal resolves the dynamics of multiple slaves system such as the Shape-System dynamic and the Locked-System dynamic of the control law. Moreover, the proposal control law can be used to achieve an autonomous grasping object by multiple slave and the transportation of the object by the control experiment. In this work, the slaves are possible to hold even if unknown objects or the width extendable of object if it can be held by the force control. The force information on the grasping object is necessary for the position control law to keep the object to be held. To analyze stability, the input-to-state stability (ISS) small gain approach was used to show the overall FR teleoperation system to be input-to-state stable. Finally, several experimental results show the effectiveness of our proposal control method.

References

[1] P. F. Hokayem and M. W. Spong,“Bilateral teleoperation: An historical survey”, *Automatica*, vol. (42)-12, pp. 2035-2057, 2006.
[2] I. H. Elhaji, A. Goradia, N. Xi, C. M. Kit, Y.H. Liu, T. Fukuda,“Design and Analysis of Internet-Based Tele-Coordinated Multirobot systems”, *Autonomous Robots*, vol. (15)-3, pp. 237-254, 2003.
[3] N. Y. Chong, S. Kawabata, K. Ohbaabd, T. Kotoku, K. Komoriya, K. Takase, and K. Tanie, “Multioperator teleoperation of multirobot systems with time delay: part 1-aids for collision-free control”, *Presence*, vol. (11)-3, pp. 277-291, 2002.
[4] S. Sirouspour, “Modeling and Control of Cooperative Teleoperation Systems”, *IEEE Trans. on Robotics*, vol. (21)-6, pp. 1220-1226, 2005.
[5] S. Sirouspour and P. Setoodeh, “Adative Nonlinear Teleoperation Control in Multi-master/Multi-slave Environments”, *Proc. IEEE Conference on Control Applications*, pp. 1263-1268, 2005.
[6] T. Suzuki, T. Sekine, T. Fujii, H. Asama, and I. Endo: “Cooperative formation among multiple mobile robot teleoperation in inspection task”, *Proc. 39th IEEE Conference on Decision and Control*, pp. 358-363, 2000.
[7] O. Khatib, K. Yokoi, K. Chang, D. Ruspini, R. Holmberg, A. Casal, and A. Baader: Force strategies for cooperative tasks in multiple mobile manipulation systems, *International Symposium of Robotics Research*, 1995.
[8] C. Preeda, G.G. Hwang and H. Hashimoto, “VR Simulation for Nano SMMS Teleoperation over the Delayed Networks”, *Proc. SICE-ICASE International Joint Conference*, pp. 4826-4831, 2006.
[9] D. Lee and M.W. Spong, “Bilateral Teleoperation of Multiple Cooperative Robots over Delayed Communication Networks: Theory”, *Proc. IEEE International Conference on Robotics and Automation*, pp. 362-367, Barcelona, Spain, April 2005.
[10] C. C. de Wit, B. Siciliano and G. Bastin (Eds): *Theory of Robot Control*, Springer 1996.
[11] I. G. Polushin, P. X. Liu and C. H. Lung, “A force-reflection algorithm for improved transparency in bilateral teleoperation with communication delay”, *IEEE/ASME Transactions on Mechatronics*, vol. (12)-3, pp. 361-374, 2007.
[12] D. D. Nam, Y. Yamashina and T. Namerikawa,“Bilateral Teleoperation of Multiple Cooperative Robots with Time-Varying Delay”, *IEEE Multi-conference on Systems and Control*, Yokohama, Japan, September 8th-10th, 2010 (to be published)
[13] D. D. Nam and T. Namerikawa, “Impendance Control for Force-Reflecting Teleoperation with Commnunication Delays Based on IOS Small Gain Theorem”, *ICCAS-SICE International Joint Conference*, pp. 4079-4086, August 18th-21th, 2009.
[14] E. D. Sontag and Y. Wang, “On characterizations of the input-to-state stability property”, *Sys. Control lett.*, vol. (24), pp. 351-359, 1995.
[15] H. Khalil, *Nonlinear systems*. Prentice Hall, Upper Saddle River, NJ07458, 1996.