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H_∞ Filter-based Robotic Localization and Mapping with Intermittent Measurements

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***Abstract*—In this paper, a mobile robot localization and mapping with intermittent measurements is considered. A mobile robot is arbitrarily placed in an unknown environment and then must constructs a map and localize itself in the builded map whenever observations are missing by using H_∞ Filter(HF). We show that, even if a measurement data is missing or there are some uncertainties exist during observations, part of information still available for the robot to estimate its location and landmarks effectively. One solution for the problem is by using measurement innovation to sufficiently provides information. We guarantee that, even if measurements are sometimes missing, the measurement innovations contributes information for estimation purposes.**

I. Introduction

Mobile robot localization and mapping plays an important role in realizing autonomous robot behavior. It states a problem that a mobile robot is placed in an unknown environment and makes observations about its surroundings. Information are then used to build a map while at the same time, robot attempts to localize itself in the constructed map(See Fig.1). The problem which is also known as SLAM(Simultaneous Localization and Mapping) problem dominate most of the robot problem based on the fact that the robot must continuously confirmed its location to accomplish their given task. The research of SLAM has been proceeded to either theoretical and practical approach in various kinds of ways e.g[1], [2], [3]. Up to date, E.Asadi et al.[1] proposed a method that considered information fusion by two different channel of sensory data to enhance the estimation in SLAM problem. They claimed that this is a way to improve estimation and lead to better results. SLAM depends on the efficiency of robot localization and sensory data to fulfill it given task. There are also different types of SLAM approaches available such as EKF(Extended Kalman Filter), HF, Particle Filtering(PF)[4], [5], etc., which already been proposed and analyzed in some desirable conditions to achieve various kind of tasks. However EKF suffer in non-gaussian noise while PF has high computational cost and is complex. We seek for a robust filter that can overcome such problems for SLAM and therefore propose the study of HF-based SLAM.

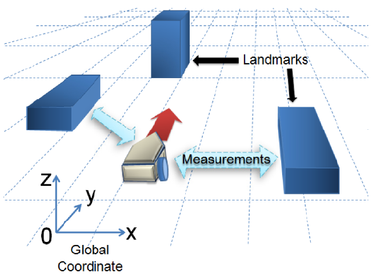

Fig. 1. Mobile robot localization

In this paper, we investigate the HF-based SLAM convergence and its behavior under intermittent measurements. Hamzah et.al[6] investigated the convergence properties of towards HF SLAM and they inspires us to determine its potential in intermittent measurement. Intermittent measurements states a situation where measurement data is not arrived at a specific time due to system failure or sensor defects. Mobile robot equipped with various kinds of sensors fall as one of the possible candidates in this condition. This is due to some data may partially lost during its observations.

The research of intermittent measurement have been focused mainly on Kalman Filter[7][8][9] for network system and surprisingly very limited studies on mobile robot are found. Intermittent measurement studies originated in 1969 when Nahi[9] examined two different cases of data loss and then derived an optimal estimator as a solution to the problem. His results are then being analyzed further by some researchers constitutes more on network packet drops. Sinopoli[7] stated that a critical value is existed for missing measurement data in the sensor network and proposed some bounded value to the expected error. However, their results does not show explicitly the error bound of the system. Plare and Bullo[8] demonstrated that there is an exact critical value whenever observations are missing for a detectable system by analyzing cones of the positive semidefinite matrix to the system. They claimed that the critical value is bounded to some exact value which supports Sinopoli results.

On behalf of robotics research, Payeur[10] studied on the propagation of uncertainties by using Jacobian to build a scene of environment in the sense of intermittent measurements. The information is then being combined into the occupancy grid to visualize the system characteristics. Other than this approach, we did not find any analysis of intermittent measurement in robotics application.

Instead of formulating the problem under the covariance characteristics which is mostly focussed in this research, the measurement innovation is applied to contribute information about the system. We prove that the measurement innovation defines the system uncertainties whenever measurement data are missing. To investigate the problem, we present HF-SLAM as a method to explain the measurement innovation behavior during intermittent measurement.

This paper is organized as follows. *Section II* introduces the problem statement and describes the whole dynamical system of the robot under HF with an underlying theory on intermittent measurements. *Section III* discuss and analyzes the convergence analysis for intermittent measurements and propose our main results. *Section IV* determine the experimental results and discussion. Finally, *Section V* concludes our paper.

II. Problem Statements

The HF-SLAM is discussed to understand its estimation whenever measurements are missing during mobile robot observations. The mobile robot that does not know its initial conditions, has to estimate its location and landmarks in a an unknown environment. The problem describes that, for a given $\gamma > 0$, an H_∞ Filter attempts to find a solution for estimated state $\hat{X}_k$, that satisfies

$$\sup_{X_0,v,w} \frac{\sum_{k=0}^{N} ||X_k - \hat{X}_k||}{\left\{ ||X_0 - \bar{X}_0||^2_{P_0^{-1}} + \sum_{k=0}^{N} ||v_k||^2_{R_k^{-1}} + \sum_{k=0}^{N} ||w_k||^2_{Q_k^{-1}} \right\}} < \gamma^2$$

where $X_0, X_k \in \mathbb{R}^{3+2m}$ is the robot($\in \mathbb{R}^3$) and landmarks($\in \mathbb{R}^{2m}, m = 1,2,...,N$) states. w, v, are the process and measurement noises with covariance of $Q_k \geq 0$, $D_k > 0$ respectively and $P_0 > 0$ is the initial state covariance. The above equation alternatively means that the estimation error to the noise ratio is less than a certain level of γ.

To ensure that the robot has a level of confidence about its position, the measurement innovation is guaranteed to be available. We made some assumptions as follows.

Assumption 1: Mobile robot is equipped with sufficient proprioceptive and exteroceptive sensors.

Assumption 2: Landmarks are stationary and robot cannot sense any occluded landmarks.

An initial study of intermittent measurements using HF has been proposed by Liu et.al[11]. They designed the filter such that to achieve better performance in two dimensional system estimation whenever there is a packet loss during observations. The suggested that a parameter-dependent technique is applicable in the filter to ensure stability throughout estimation.

In this paper, we analyze the behavior of measurement innovation towards partially loss measurements. The analysis is mainly cover a case where both measurement data from proprioceptive and exteroceptive sensors are not arrived. *Finite escape time*[12], [13] is not considered in this paper.

A. H_∞ Filter-Based SLAM

We consider a nonlinear discrete-time dynamical system as follows.

$$\theta_{k+1} = \theta_k + f_\theta(\omega_k, v_k, \delta\omega, \delta v) \quad (1)$$
$$x_{k+1} = x_k + (v_k + \delta v)T\cos[\theta_k] \quad (2)$$
$$y_{k+1} = y_k + (v_k + \delta v)T\sin[\theta_k] \quad (3)$$
$$L_{k+1} = L_k \quad (4)$$

where θ_k is the mobile robot pose angle, and ω_k, v_k are mobile robot turning rate and its velocity. While, x_k, y_k are the x,y cartesian coordinate of the mobile robot and $L_k \in \mathbb{R}^{2m}, m = 1,2,...N$ is each respective landmark location. v_k is the mobile robot velocity while ω_k is the mobile robot angular acceleration. Both control inputs have noises shown by $\delta\omega, \delta v$ which defines noises on angular acceleration and velocity respectively. We denote the robot and landmarks states by $X_k \in \mathbb{R}^{(3+2m)}$ from this point. T is the sampling rate. The process model for the landmarks is unchanged as the landmarks are assumed to be stationary and are given. Under HF algorithm, the prediction and update processes which starts from an initial state is shown by

$$\hat{X}_k = F_x(\hat{X}_0, \omega_0, v_0, 0, 0) \quad (5)$$

where F_x is the transition matrix, and $\hat{X}_k$ is the predicted robot and landmarks states. The associated covariance P_k is shown by

$$P_k = \nabla f_r P_0[I - \gamma^{-2}P_0 + \nabla H_k^{-1} R_k^{-1} \nabla H_k P_0]^{-1} \nabla f_r^T + \nabla G_{\omega v} \Sigma_0 \nabla G_{\omega v}^T \quad (6)$$

Here ∇f_r is the Jacobian evaluated from the mobile robot motion in (1)-(4), Σ is the control noise covariance and $\nabla G_{\omega v}$ is the Jacobian transformation for the process noise. When $T = 1$,

$$\nabla f_r = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -v\sin\theta & 1 & 0 & 0 \\ v\cos\theta & 0 & 1 & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \quad (7)$$

where I is an identity matrix with an appropriate dimension. The above expression can also be written as

$$P_k = \nabla f_r P_0 \psi_k^{-1} \nabla f_r^T + \nabla G_{\omega v} \Sigma_k \nabla G_{\omega v}^T \quad (8)$$

where $\psi_k = I + (\nabla H_k^T R_k^{-1} \nabla H_k - \gamma^{-2} I)P_0$. ∇H_k explains the Jacobian transformation regarding the measurement process for any landmark i and this is shown later.

The measurement process has the following equation.

$$\begin{aligned} z_{i_{k+1}} &= \eta_{k+1} \begin{bmatrix} r_i \\ \theta_i \end{bmatrix} \\ &= \eta_{k+1} \begin{bmatrix} \sqrt{(x_i - x_{k+1})^2 + (y_i - y_{k+1})^2} + \varepsilon_{r_i} \\ \arctan \frac{y_i - y_{k+1}}{x_i - x_{k+1}} - \theta_{k+1} + \varepsilon_{\theta_i} \end{bmatrix} \\ &= \eta_{k+1} H_i(X_{k+1}) + \varepsilon_{r_i \theta_i} \end{aligned} \quad (9)$$

where r_i and θ_i is the relative distance and measurements between robot and landmark i respectively. This equation show that the mobile robot measures relative distance and angle from a specific i^{th} landmark with some associated noises of ε_{r_i}, ε_{θ_i}.

η is a scalar quantity independent of observation time, k, either take values of 1 or 0. For a measurement between mobile robot and any landmark i, we have the following results.

$$\nabla H_k = \begin{bmatrix} 0 & -\frac{dx_k}{r} & -\frac{dy_k}{r} & \frac{dx_k}{r} & \frac{dy_k}{r} \\ -1 & \frac{dy_k}{r^2} & -\frac{dx_k}{r^2} & -\frac{dy_k}{r^2} & \frac{dx_k}{r^2} \end{bmatrix} \quad (10)$$

where $r = \sqrt{(x_i - x_k)^2 + (y_i - y_k)^2}$, $dx_k = x_i - x_k$ and $dy_k = y_i - y_k$. Notice that in (9), unlike the standard HF algorithm, η is added to the system. $\eta = 1$ means that the measurement data are available and $\eta = 0$ show the opposite means.

$$Pr\{\eta = 1\} = p \quad (11)$$

$$Pr\{\eta = 0\} = 1 - p \quad (12)$$

$$E[\eta] = E[\eta^2] = p \quad (13)$$

The corrected state update is describe by

$$\hat{X}_{k+1} = \hat{X}_k + K_{k+1}(H_k(X_k) - H_k(\hat{X}_k)) \quad (14)$$

where $K_k = P_k(I - \gamma^{-2}P_k + \nabla H_k^T R_k^{-1} \nabla H_k P_k)^{-1}$. The difference between EKF and HF exist in its gain and state covariance. γ is not included in K_k and ψ_k, then the equation show the EKF algorithm. Notice that in (14) onward, H_i is replaced by H_k to indicate the Jacobian is evaluated at time k.

III. Convergence Analysis for Intermittent Measurements

Before further analyzing the convergence of intermittent measurements, same notations by Huang et.al[3] are used. For a mobile robot observing a landmark at point A, the Jacobian matrix is given by

$$\nabla H_A = \begin{bmatrix} -e & -A & A \end{bmatrix} \quad (15)$$

where

$$e = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad A = \begin{bmatrix} \frac{dx_A}{r_A} & \frac{dy_A}{r_A} \\ -\frac{dy_A}{r_A^2} & \frac{dx_A}{r_A^2} \end{bmatrix} \quad (16)$$

with

$$dx_A = \begin{bmatrix} x_i - x_A \end{bmatrix} \quad (17)$$

$$dy_A = \begin{bmatrix} y_i - y_A \end{bmatrix} \quad (18)$$

$$r_A = \sqrt{dx_A^2 + dy_A^2} \quad (19)$$

Measurement innovation can be a method to determine whether estimations are successfully achieved. In HF, the updated state is given by

$$\hat{x}_{k+1} = \hat{x}_k + K_{k+1}H_{k+1}(x_k - \hat{x}_k) \quad (20)$$

$$= \hat{x}_k + K_{k+1}d_k \quad (21)$$

From above, we easily understood that the measurement innovation gives some bounded error to the estimation. In the case of intermittent observations, if there is a lost of observation data at a specific time $k(1 < k < \infty)$, then the estimation at k probably acquire the previous value at $k-1$ as $\nabla H_{k+1}(x_k - \hat{x}_k) = 0$. However we show in this paper, the measurement innovation has some distinct value, which specifically contributes the uncertainties information to the system. The measurement innovation is found by subtracting (14) from the mobile robot kinematics.

$$\begin{aligned} d_{k+1} &= F_x(X_k) - \left[F_x(\hat{X}_k) + K_{k+1}(H_k(X_k) - H_k(\hat{X}_k))\right] \\ &= (F_x - K_{k+1}H_k)(X_k - \hat{X}_k) \end{aligned} \quad (22)$$

After one-step prediction, it then further show that

$$\begin{aligned} d_{k+2} &= (F_x - K_{k+1}H_i)(X_k - \hat{X}_k) \\ &\quad + K_{k+2}(H_k X_{k+1} - H_k \hat{X}_{k+1}) \end{aligned} \quad (23)$$

The term $F_x - K_{k+1}H_i$ is very important as it determines the system stability. For a linearized system especially in a case of a one dimensional monobot(a robot with single coordinate system), it has been shown that this term is marginally stable and bounded[9]. In normal HF, it is known that the convergence properties are similar to Kalman Filter if γ is set to be large enough[6].

The investigation now proceeds to examine the SLAM convergence in intermittent measurements.

Proposition 1: Let $P_0 = P_0^T > 0$. For $n-$times successive observations, a stationary mobile robot state covariance is monotonically decreasing.

Proof: Given that the initial state covariance, P_0 as

$$P_0 = \begin{bmatrix} P_{vv} & 0 \\ 0 & P_{mm} \end{bmatrix}$$

where $P_{vv} \in \mathbb{R}^{3\times3}$ is the initial robot covariance and $P_{mm} \in \mathbb{R}^{2m\times2m}, m = 1, 2, ...N$ as the initial landmark covariance. If measurement data is not missing, then the analysis can be derived using the normal EKF algorithm. The covariance updates is given by (6) and as the robot is non-moving, then

$$\begin{aligned} P_k &= P_0 \psi^{-1} \\ &= [P_0^{-1} + (\nabla H_k^T R_k^{-1} \nabla H_k - \gamma^{-2} I_k)]^{-1} \end{aligned} \quad (24)$$

If $P_0 = P_0^T \geq 0$, and utilizing (15) we found that as $\nabla H_A = [-H_A \quad A]$, we have the following equation. For $n-$times observations, the state covariance exhibit the following.

$$P_{k+1}^n = \left(P_0^{-1} + n\nabla H_{A_k}{}^T R_{A_k}^{-1} \nabla H_{A_k} - \gamma^{-2} I_k)\right)^{-1} \quad (25)$$

$$= \left(P_0^{-1} + n\begin{bmatrix} -H_{A_k}^T \\ A_k^T \end{bmatrix} R_{A_k}^{-1} \begin{bmatrix} -H_{A_k} & A_k \end{bmatrix} - \gamma^{-2} I_k)\right)^{-1} \quad (26)$$

Calculate further, we arrive in below equation.

$$P_{k+1}^n = \begin{bmatrix} P_{vv}^{-1} + n(H_{A_k}^T R_{A_k}^{-1} H_{A_k} - \gamma^{-2} I_k) & -nH_{A_k}^T R_{A_k}^{-1} A \\ -nA_k{}^T R_{A_k} - 1H_{A_k} & P_{mm}^{-1} + n(A_k{}^T R_{A_k}^{-1} A_k - \gamma^{-2} I_k) \end{bmatrix}^{-1}$$

By *Matrix Inversion Lemma*, we understand that especially whenever γ is choose to has bigger value, the state error covariance is monotonically decreasing. This characteristics shown the same characteristics as Kalman Filter. ■

A. Intermittent Measurements Analysis

This paper attempts to prove that even if there are some missing measurements data, the measurement innovation is not exactly unavailable but bound to some exact value. These information are important as it provides some variance whenever the observations data are missing as stated by (21), which then can be used to infer the measurement innovation covariance for the system.

Definition 1: Measurement data lost is defined whenever measurement data is not arrived after one sample time and occurred randomly in mobile robot observations.

The above definition describes that if a measurement is unavailable at time k, then $\eta_k = 0$. We now demonstrates how HF behaves if this is partially happened during mobile robot observations. The expectation of state covariance can be illustrated by e.g Sinopoli et.al[5] as

$$\begin{aligned} & \mathbb{E}[(z_{k+1} - \hat{z}_{k+1})(z_{k+1} - \hat{z}_{k+1})^T] \\ = \; & p\nabla H_{k+1} P_{k+1} \nabla H_{k+1}^T \end{aligned}$$

Therefore, if the measurement innovation at some specific times is missing, then state covariance updates is unavailable and thus it seems that the state covariance take the previous state covariance data. In this respect, we show that the information when the measurements are missing being bounded to some respective value.

Lemma 1: If the measurement data is not available intermittently at $1 < k < \infty$ time, for a robot observing a landmark and constantly move, the measurement innovation takes the following expression.

$$d_k = \eta_{k-1} A_{k-1}(L_{m_{k-1}} - R_{k-1}) \tag{27}$$

where A is defined in (16) and d is the measurement innovation. L_m and R shows the landmark and robot x, y location respectively.

Proof: We begin the proof by analyzing the measurement innovation for a specific time step[3].

$$\begin{aligned} d &= z - H_i\hat{X} && (28) \\ &= \eta H_i X + \omega_{r1\theta1} - \eta H_i \hat{X} && (29) \\ &= \eta H_i (X - \hat{X}) + \omega_{r1\theta1} && (30) \\ &= \eta \begin{bmatrix} -e & -A & A \end{bmatrix} \begin{bmatrix} \theta - \hat{\theta} \\ R - \hat{R} \\ L_m - \hat{L}_m \end{bmatrix} + \omega_{r1\theta1} && (31) \\ &= \eta \left[-e(\theta - \hat{\theta}) - A(R - \hat{R}) + A(L_m - \hat{L}_m) \right] \\ &\quad + \omega_{r1\theta1} && (32) \end{aligned}$$

where A is the Jacobian evaluated at the robot position and landmarks respectively. Rearranging above and assuming no noises yield

$$\eta\left[e\theta + AR - AL_m\right] = -d + \eta\left[e\hat{\theta} + A\hat{R} - A\hat{L}_m\right] \tag{33}$$

For better visualization, the analysis moves to next time update, that expressing the result in Jacobian evaluation in different time based on the estimated position. This step is also important as one time observation is insufficient to improve the position estimation.

$$\eta_1\left[e\theta + A_1 R - A_1 L_m\right] = -d_1 + \eta_1\left[e\hat{\theta}_1 + A_1\hat{R}_1 - A_1\hat{L}_{m1}\right] \tag{34}$$

$$\eta_2\left[e\theta + A_2 R - A_2 L_m\right] = -d_2 + \eta_2\left[e\hat{\theta}_2 + A_2\hat{R}_2 - A_2\hat{L}_{m2}\right] \tag{35}$$

After an arrangement and subtracting above equations, we arrive in

$$\begin{aligned} \eta_1 A_1^{-1} e\theta - \eta_2 A_2^{-1} e\theta &= -A_1^{-1} d_1 + A_2^{-1} d_2 + \eta_1 A_1^{-1} e\hat{\theta}_1 \\ &\quad - \eta_2 A_2^{-1} e\hat{\theta}_2 + \eta_1 \hat{R}_1 - \eta_2 \hat{R}_2 \\ &\quad - \eta_1 \hat{L}_{m1} + \eta_2 \hat{L}_{m2} && (36) \end{aligned}$$

When the observation is not available at the 2^{-nd} measurement time, we found that

$$\begin{aligned} \eta_1 A_1^{-1} e\theta &= -A_1^{-1} d_1 + A_2^{-1} d_2 \\ &\quad + \eta_1 A_1^{-1} e\hat{\theta}_1 + \eta_1 \hat{R}_1 - \eta_1 \hat{L}_{m1} && (37) \\ A_2^{-1} d_2 &= A_1^{-1} d_1 + \eta_1 (A_1^{-1} e(\theta - \hat{\theta}_1) \\ &\quad - \hat{R}_{r1} + \hat{L}_{m1}) && (38) \\ \therefore d_2 &= \eta_1 A_1 (L_{m1} - R_1) && (39) \end{aligned}$$

Therefore, even if the measurements data are intermittently missing, we now understand that the estimation is still possible, especially for a stationary robot case. Measurement innovation provides sufficient information regarding the missing measurement to the system and is employed to update the estimation. ■

Recognize that η is representing the previous observation which must be available in order to ensure that (39) is applicable for update purposes. In other words, we distinctly show that the measurement innovation when measurement data is unavailable is defined by (39) without the notion of η.

Corollary 1: If a measurement data is missing intermittently at $1 \le k \le \infty$, the measurement innovation is bounded to the following equation.

$$d_k = \eta_{k-1} A_{k-1}(L_{m_{k-1}} - R_{k-1}) \tag{40}$$

Proof: The proof is easily obtained by the above lemma. It is observable that the measurement innovation is bounded to $d_{k+1} = \gamma_{k-1} A_{k+1}(L_{m_k} - X_{r_k})$ if measurement data is not available at $1 \le k \le \infty$. Note that the measurement innovation takes either positive or negative values, which also significantly means its bounded limit. ■

Lemma 2: The measurement innovation is bounded to (41) even if there are multiple loss of measurement data during $1 \le k \le \infty$, observations if and only if $A_k = A_{k+1}$.

Proof: The proof is easily verified by the above *Lemma 1* and therefore, is omitted. ■

Above *Lemma 1, Lemma 2* and *Corollary 1* may sufficiently describe that, even if measurements data are missing at $0 \le k \le n$, the mobile robot can estimate its location. And with respect to the special matrix A, the solution to localization is guaranteed to be available. Analyzing further the above *Lemma 1* and *Corrollary 1*, we can determine the relationship between d_{k+1} and d_k.

From (40), it has been demonstrated that the measurement innovation is useful to infer the state error covariance. We define the *Measurement Innovation Error* as $d_k - d_{k+1}$. The *measurement innovation error* is used to evaluate the state covariance behavior whenever measurement data is missing.

Theorem 1: If a measurement data is partially having lost at $1 \le k \le \infty$, then the measurement innovation error are shown by the following.

$$e(\theta_{k-1} - \hat{\theta}_{k-1}) + A_{k-1}(\hat{R}_{k-1} - \hat{L}_{m_{k-1}}) \tag{41}$$

The uncertainties is decreasing if below equations are satisfied.

$$(\hat{L}_{m^i_x} - \hat{R}_x) \leq \frac{dy_A}{dx_A}(\hat{R}_y - \hat{L}_{m^i_y}) \quad (42)$$

$$\frac{dx_A}{r^2}(\hat{L}_{m^i_y} - \hat{R}_y) \leq \hat{\theta}_{k-1} - \theta_{k-1} + \frac{dy_A}{r_A}(\hat{L}_{m^i_x} - \hat{R}_x) \quad (43)$$

This is when a robot is observing at point A and dx_A, dy_A and r_A are shown in (18),(19) and (20) respectively. $L_{m^i_x}$, $L_{m^i_y}$ state the landmark number in $x-y$ location being observed by the robot and $\hat{R}_x$ is the measurement noise at time $k-1$. Else, the uncertainties is increasing.

Proof: Lemma 1 has proven that the measurement innovation when the measurement data is not available is equal to (40). Furthermore, (40) also equivalently means that if $d_k \geq d_{k+1}$, then the state covariance is generally decreasing. This fact is applied to analyze the measurement innovation in a case where measurement data is not available. To analyze the situation, it can be easily viewed that if the *Measurement Innovation Error* $|d_k| - |d_{k-1}| \geq 0$, then the measurement innovation error gives information about the robot confidence in position inference.

$$\begin{aligned} |d_k| - |d_{k-1}| &= A_k(L_{m_{k-1}} - X_{r_{k-1}}) \\ &\quad -[-e(\theta_{k-1} - \hat{\theta}_{k-1}) \\ &\quad -A_{k-1}(R - \hat{R}_k) \\ &\quad +A_{k-1}(L_{m_{k-1}} - \hat{L}_{m_{k-1}})] \end{aligned} \quad (44)$$

If $A_k = A_{k-1}$, then to ensure the uncertainties is decreasing, it must satisfy

$$\begin{aligned} |d_k| - |d_{k-1}| &= e(\theta_{k-1} - \hat{\theta}_{k-1}) - \\ &\quad A_{k-1}(\hat{L}_{m_{k-1}} - \hat{R}_{k-1}) \geq 0 \end{aligned} \quad (45)$$

This condition is achieved such as if below equations are fulfilled.

$$\frac{dx_A}{r_A}(\hat{L}_{m^i_x} - \hat{R}_x) \leq \frac{dy_A}{r_A}(\hat{R}_y - \hat{L}_{m^i_y}) \quad (46)$$

$$\frac{dx_A}{r_A^2}(\hat{L}_{m^i_y} - \hat{R}_y) \leq \hat{\theta}_{k-1} - \theta_{k-1} + \frac{dy_A}{r_A^2}(\hat{L}_{m^i_x} - \hat{R}_x) \quad (47)$$

when a robot is observing at point A and dx_A, dy_A and r_A are shown in (17),(18) and (19) respectively. $L_{m^i_x}$, $L_{m^i_y}$ state the landmark number in $x-y$ location being observed by the robot and $\hat{R}_x$ is the measurement noise at time $k-1$. Eq. (45) under conditions of (46), (47) equivalently states the error becomes bigger and not converges but bounded by d_k. Remark that (46), (47) are the estimation at $k-1$ when the measurement data is available. Moreover, from (45) it can be deduced that as it have two opposite values, if (46), (47) are not satisfied, then the measurement innovation error is decreasing. ■

Based on these results, it we expect that Kalman Filter based SLAM has the same characteristics whenever the measurement data is missing during mobile robot observations. However, this study is left for future development.

IV. Experimental Results and Discussion

Experiments are conducted for above proposed theorems. An E-puck robot is used to observe an uknown environment. We observed the results when measurement data are missing at $50[s], 80[s]$ and $120[s]$ for 20[s], 2[s] and 1[s] observations respectively. The experimental results are evaluated under an environment that has a non-Gaussian noise as provided in experimental parameters in Table 1. Note that the initial state covariance and both process and measurement noises have the appropriate dimensions to describes the whole system behavior.

TABLE I
Experiment Parameters

Sampling Time, T	$0.1s$
Process noise,Q distribution	1×10^{-7}
Observation noise,R distribution	min = [-0.4 -0.05] max = [1 0.5]
γ	9
Robot Initial Covariance *Pvv*	1×10^{1}
Landmarks Initial Covariance *Pmm*	1×10^{2}

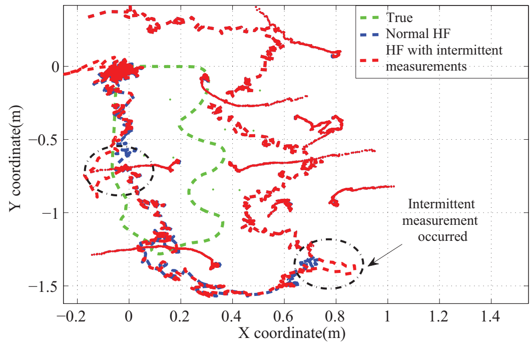

Fig. 2. Map construction between normal HF and HF with partially missing measurement data

Fig.2 illustrates map construction between HF and HF with intermittent measurements. It is viewable that at certain times, the estimations are diverging when the measurement data are not arrived as specified above. Even though the estimation is diverging for some time, we observe that the estimation still fairly shows good results for both robot and landmarks. Besides, RMSE(Root Mean Square Error) are also evaluated to support this result(Fig.3). There are not much differences to the normal HF estimations.

Fig.4 explains about the state error covariance update regarding both mobile robot and landmarks states. The results consistently describes the same behavior as presented on previous figure. The covariance update shows the same characteristics to normal HF estimation if no measurement data is arrived. The measurement innovation behavior sufficiently defines the information when measurement data is unavailable as presented in Fig.5. Some of information refer

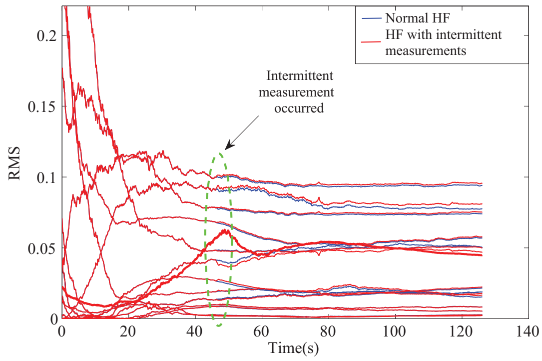

Fig. 3. Some of the estimation errors characteristics when measurements data are missing

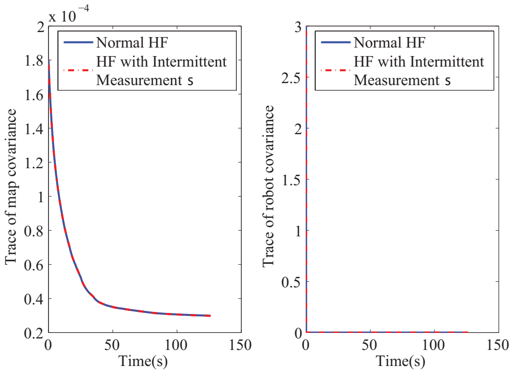

Fig. 4. State error covariance update characteristics

to the previous estimation while others increase or decrease with respect to the previous estimation. Even though we observed some difference between these two filter conditions especially after 50[s], the estimations are still almost result in the same outcome. Furthermore, we understand that, the filter is not guaranteed to acquire to the previous information but interestingly depend on the characteristics described by (40).

A. Discussion

Above experimental results have shown that even though a measurement data is not arrived randomly at some time, the estimation is still possible. Besides, these results also significantly proved that HF has achieved a desired performance if the γ are properly selected such that satisfying the condition described in Section II. Furthermore, the finite escape time, which is a problem in HF, is not occurred during the whole observations.

V. Conclusion

This paper presented the result of intermittent measurements of some missing measurement data when the robot observing its environment. It has been demonstrated that even though the data are missing, the robot is still able to estimate its location in HF. The results are also consistent with the findings which

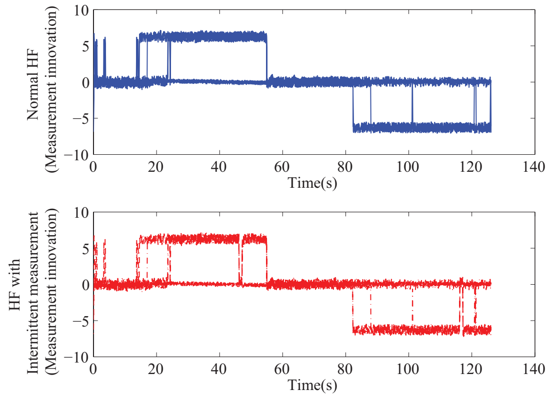

Fig. 5. The measurement innovation characteristics during measurement data loss at the specified time

is described in the literature. Furthermore, the measurement innovation of HF finally arrived in the same as the normal EKF without measurement data lost if measurements are available after the data lost. The measurement innovation is also bounded and thus enabling estimation to fairly be done by the robot.

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