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A Passivity-based Approach to Wide Area Stabilization of Magnetic Suspension Systems

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Abstract— **This paper deals with a passivity-based approach to wide area stabilization of magnetic suspension systems. It is well known that the magnetic suspension systems have strong nonlinearity. In order to realize wide area stabilization of magnetic suspension systems, we design the passivity-based feedback controller considering a nonlinear dynamics of the magnetic suspension systems and a leakage inductance of the electromagnet especially. The systems can be decomposed two subsystems; an electrical subsystem and a mechanical subsystem. We design nonlinear passivity-based controllers for both two subsystems. Steady-state and transient time responses results for some operating points show the effectiveness of the proposed controller experimentally.**

I. INTRODUCTION

Magnetic suspension systems support objects via magnetic force without any physical contacts. These systems are used for magnetic bearings, transport devices in clean rooms and the maglev trains [1]. The most serious problem of these systems are instability, so they need feedback control for their stabilization. Another problem is a strong nonlinearity of an electromagnetic force of the magnetic suspension system. The most frequent approach taken here is a linear robust control which includes a linearization of the electromagnetic force around the equilibrium points and a stabilization by a linear robust controller[2].

Hence it is difficult to take a wide area operating point for the system. For this problem, Ortega[3] and Shimizu[4] focus on the control based on the passivity which is used in the field of the robotics and so on, they derived the controller using the passivity of the system. In the reference[3][4], the system of voltage control type magnetic suspension system was divided into two subsystems: an electrical subsystem and a mechanical subsystem. The authors realized a wide area stabilization by designed controllers to each subsystem.

In the references[3][4], the authors derived a controller based on passivity using a model which a utilized simplified characteristic of the magnetic suspension system. Therefore, in this research we derive a passivity-based nonlinear controller for the strict and complex model of the magnetic suspension system and we stabilize it in the wide area of the state space. We apply the derived controller to the actual magnetic suspension experimental system and verify the effectiveness of the controller based on proposed model compared with the controller based on conventional model[3][4].

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II. PASSIVE SYSTEM [5][6]

We consider nonlinear systems described by the following equation

$$\left\{ \begin{array}{l} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{array} \right. \tag{1}$$

with state of the system $x \in R^n$, control inputs $u \in R^m$ and outputs $y \in R^m$.

Definition The system (1) is said to be passive if there exists a semi-definite function $H(x)$, such that $\forall u(t)$

$$H(x(t)) - H(x(0)) \leq \int_0^t y^T(\tau)u(\tau)d\tau, \quad \forall t \geq 0. \tag{2}$$

Here $H(x)$ is called a storage function. The above inequality is called the dissipation inequality. This equation (2) means “stored energy” is less than “supplied energy”.

In addition, if a positive definite function $Q(x)$ and a following inequality exists, this system is called strictly passive.

$$\begin{aligned} &H(x(t)) - H(x(0)) \\ &\leq \int_0^t y^T(\tau)u(\tau)d\tau - \int_0^t Q(x(\tau))d\tau, \quad \forall u \end{aligned} \tag{3}$$

III. MODEL OF MAGNETIC SUSPENSION SYSTEM AND PASSIVITY[3][4]

A. Mathematical Model of Magnetic Suspension System

In this research, we deal with the voltage control type magnetic suspension system.

We consider the magnetic suspension system of the rigid body ball as shown in Fig. 1. Here shows the steady-state suspension position of the iron ball X. $x(t)$ shows the displacement of the iron ball from X and its direction is defined downward. Also, M means the mass of the suspension body. $u(t)$ is the voltage which is inputed to the electromagnet, $i(t)$ is the electric current which flows through the electromagnet, $f(t)$ is the electromagnetic force derived by $i(t)$.

Here, it is possible to show the inductance L of the electromagnet in the following form as the function of the position $x(t)$ of the iron ball.

$$L(x(t)) = \frac{2k}{x_0 + X + x(t)} + L_0 \tag{4}$$

k is an attractive force coefficient, x_0 is an offset term, L_0 is a leakage inductanceCand a displacement $x(t)$ satisfies this inequality.

$$x(t) > -(X + x_0) \tag{5}$$

A characteristic of this inductance is shown in Fig. 2.

The curve (a) is an ideal property that means the magnetic permeability of the iron core is infinity and there is not leakage magnetic flux of the electromagnet. Curve (b) is the characteristic of an actual inductance with finite magnetic permeability with the existing leakage magnetic flux. In (b) case, we can expresses $L(x)$ by using x_0, L_0 as in (4). In reference[3][4], the authors assumed that the leakage inductance L_0 is small enough, and derived a controller by the inductance $L(x)$ for $L_0 = 0$. In this research, we deal with the model which explicitly considers L_0.

The mathematical model of the system is given by an electric circuit equation in (6) and a motion equation in (7), where R is an internal resistance of electromagnet, and the electromagnet force $f(t)$ is defined in (8)D

$$u(t) = L(x(t))\dot{i}(t) - \frac{2k}{(x_0 + X + x(t))^2}\dot{x}(t)i(t) + Ri(t) \tag{6}$$

$$M\ddot{x}(t) = Mg - f(t) \tag{7}$$

$$f(t) = \frac{k}{(x_0 + X + x(t))^2} i^2(t) \tag{8}$$

Here the magnetic flux λ is given by

$$\lambda(t) = L(x(t))i(t) = \frac{2k + L_0(x_0 + X + x(t))}{x_0 + X + x(t)} i(t). \tag{9}$$

The electric circuit equation (6) with magnetic suspension system using this λ, (6) is rewritten as follows:

$$\dot{\lambda}(t) = -\frac{R(x_0 + X + x(t))}{2k + L_0(x_0 + X + x(t))}\lambda(t) + u(t). \tag{10}$$

Moreover, using this λ, the electromagnetic force $f(t)$ (8) which the electromagnet be generated in (6) is rewritten as follows:

$$f(t) = \frac{k}{(2k + (x_0 + X + x(t))L_0)^2}\lambda^2(t). \tag{11}$$

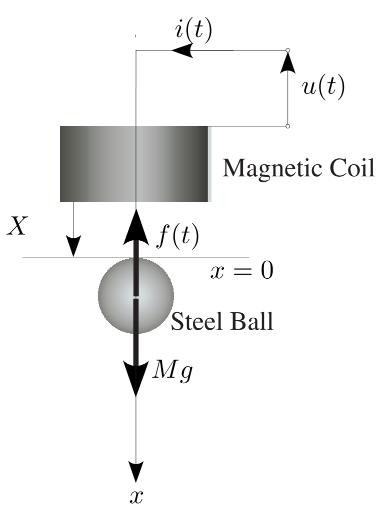

Fig. 1. Magnetic Suspension System

Using this equality, equation of motion (7) becomes the following.

$$\Sigma \left\{ \begin{array}{l} \Sigma_1 : \left\{ \begin{aligned} \dot{\lambda}(t) &= -\frac{R(x_0 + X + x(t))}{2k + L_0(x_0 + X + x(t))}\lambda(t) \\ &\quad + u(t) \\ f(t) &= \frac{k}{(2k + (x_0 + X + x(t))L_0)^2}\lambda^2(t) \end{aligned} \right. \\ \Sigma_2 : \left\{ \; M\ddot{x} = Mg - f(t) \right. \end{array} \right. \tag{12}$$

The conventional model[3] is assumed that $L_0 = 0$ in (12). The magnetic suspension system shown in (12) can be divided into the electrical subsystem Σ_1 and the mechanical subsystem Σ_2, and the total system Σ is combined by the feedback connection as shown in Fig. 3 [3][4]D

B. Verification of Passivity

We verify the passivity of the electrical subsystem Σ_1 and the mechanical subsystem Σ_2 of the Magnetic suspension system which are composed of the feedback connection of Fig. 3.

First, we verify the passivity of electrical subsystem Σ_1. The input of Σ_1 is u and the output is λD The candidate of the storage function of Σ_1 is shown by the following function.

$$H_\lambda = \frac{1}{2}\lambda^2 \tag{13}$$

The time derivative of the storage function is given by

$$\dot{H}_\lambda = \lambda\dot{\lambda} = -\frac{R(x_0 + X + x(t))}{2k + L_0(x_0 + X + x(t))}\lambda^2 + \lambda u. \tag{14}$$

Here, using the following positive α from $x(t) > -(X + x_0)$

$$0 < \alpha \le \frac{R(x_0 + X + x(t))}{2k + L_0(x_0 + X + x(t))} \tag{15}$$

(14) is rewritten as follow.

$$\dot{H}_\lambda \le -\alpha\lambda^2 + \lambda u \tag{16}$$

The time integral of both sides of the formula above from 0 to T become

$$\int_0^T \lambda u dt \ge \alpha \int_0^T \lambda^2 dt + H_\lambda(T) - H_\lambda(0), \tag{17}$$

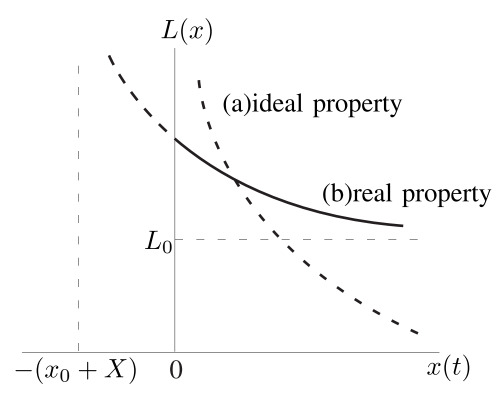

Fig. 2. Property of Inductance

because (17) fills (3), the electrical subsystem Σ_1 is the strictly passivity system which input is u and output is y.

Next, we verify the passivity of the mechanical subsystem Σ_2. The input of Σ_2 is $Mg-f$, the output is $\dot{x}$. The candidate of the storage function of Σ_2 is shown by the following.

$$H_m = \frac{1}{2}M\dot{x}^2 \tag{18}$$

The time derivative of the storage function is given by

$$\dot{H}_m = M\dot{x}\ddot{x} = \dot{x}(Mg - f). \tag{19}$$

The time integral of both sides of the formula above from 0 to T become

$$\int_0^T \dot{H}_m dt = H_m(T) - H_m(0) \tag{20}$$

$$= \int_0^T \dot{x}(Mg - f)dt. \tag{21}$$

Because (21) fills (2), the mechanical subsystem Σ_2 is passiveD

For these reason, we have shown that the magnetic suspension system is composed of the electrical subsystem Σ_1 which is strictly passive and the mechanical subsystem Σ_2 which is passive.

IV. CONTROLLER DESIGN[3][4]

We design a nonlinear passivity-based controller for each two subsystems. The controller for two subsystems is designed by following way:

1) The controller for the mechanical subsystem calculates the desired electromagnetic force f_d that makes the iron ball converges to the desired position x_*.
2) The controller for the electrical subsystem calculates the desired magnetic flux λ_d that makes the electromagnetic force f follows to a desired electromagnetic force f_d. Then determine the applied voltage u in order to the electrical subsystem generates the desired magnetic flux f_d.

A. Determine the Input Voltage u

It is assumed to that the desired electromagnetic force f_d and the desired magnetic flux λ_d are known, we can define the control input u that makes $\lambda \to \lambda_d$. The passivity property above suggests to assign the desired storage function of the closed-loop system

$$H_d = \frac{1}{2}\tilde{\lambda}^2 \tag{22}$$

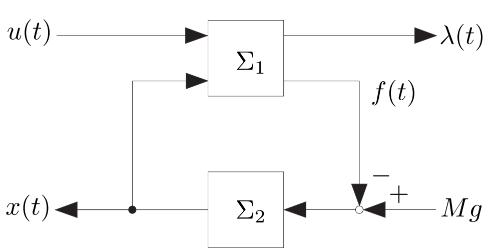

Fig. 3. Feedback Decomposition

where $\tilde{\lambda} = \lambda - \lambda_d$, and with λ_d is the desired magnetic flux that is determined by the desired position x_*. If the applied voltage u is selected as

$$u = \dot{\lambda}_d + \frac{R(x_0 + X + x(t))}{2k + L_0(x_0 + X + x(t))}\lambda_d + v \tag{23}$$

where $v = -R_{DI}\tilde{\lambda}$. Then we get the error dynamics from (12) and (23).

$$\dot{\tilde{\lambda}} = -\frac{R(x_0 + X + x(t))}{2k + L_0(x_0 + X + x(t))}\tilde{\lambda} + v \tag{24}$$

The error dynamics (24) is strictly passive systems with the storage function (22), the input v, and the output $\tilde{\lambda}$.

Therefore, from the equation (15), the dynamics of $\tilde{\lambda}$ in (24) is exponential stable if and only if $v \equiv 0$, this means $\lambda \to \lambda_d$.

B. Determine the Desired Magnetic Flux λ_d

The electromagnetic force f can be written in terms of the magnetic flux error $\tilde{\lambda}$ and the desired magnetic flux λ_d as

$$f = \frac{k}{(2k + (x_0 + X + x(t))L_0)^2}\left\{\lambda_d^2 + \tilde{\lambda}\big(\tilde{\lambda} + 2\lambda_d\big)\right\} \tag{25}$$

From the discussion in the previous section, $\tilde{\lambda}$ is guaranteed to go to 0 if and only if $v \equiv 0$. Hence, the desired electromagnetic force is chosen the solution of the following equation that is (25) with $\tilde{\lambda} = 0$. Then f_d is given as

$$f_d = \frac{k}{(2k + (x_0 + X + x(t))L_0)^2}\lambda_d^2 \tag{26}$$

Solving above equation, the desire flux λ_d and its derivative $\dot{\lambda}_d$ with respect to time t are given by

$$\lambda_d = (2k + (x_0 + X + x(t))L_0)\sqrt{\frac{f_d}{k}} \tag{27}$$

$$\dot{\lambda}_d = L_0\sqrt{\frac{f_d}{k}}\dot{x} + \frac{(2k + (x_0 + X + x(t))L_0)}{2}\sqrt{\frac{1}{kf_d}}\dot{f}_d \tag{28}$$

where $f_d > 0$ and $k > 0$. A derivative $\dot{f}_d$ is assumed to be known. We can write the control input (23) in terms of f_d and $\dot{f}_d$ as

$$u = L_0\sqrt{\frac{f_d}{k}}\dot{x} + \frac{(2k + (x_0 + X + x(t))L_0)}{2}\sqrt{\frac{1}{kf_d}}\dot{f}_d + R(x_0 + X + x(t))\sqrt{\frac{f_d}{k}} + v \tag{29}$$

C. Determine the Desired Magnetic Force f_d

In this final step we define the desired electromagnetic force f_d that makes x follow to x_*, where x_* is a desired position of the iron ball. The mechanical subsystem is the 2nd order system, so f_d is defined as the following equation by using a PID controller,

$$f_d = -M\{\ddot{x}_* - k_d\dot{\tilde{x}} - k_p\tilde{x} - k_i\int_0^t \tilde{x}(\tau)d\tau\} \tag{30}$$

where, $\tilde{x} = x - x_*$ is the position error.

The block diagram of the designed controller is shown in Fig. 4. In Fig. 4, C_1 a calculation given by (29), C_2 also shows the equation (30).

V. CONTROL EXPERIMENTS

In order to evaluate the effectiveness of the controller based on the proposed model, several steady state responses and transient responses are measured.

A. Model Parameters

Model parameters of the plant are shown in Table I. These model parameters are used in the design of the controller.

B. Experimental Method

An available range of the steady-state X[mm] of this experiment is 0[mm] $< X \leq$ 8[mm]. In order to remove the effect of the noise, saturated derivatives $\dot{f}_d$ and $\dot{\tilde{x}}_d$ are used for a controller calculation. The controller is designed by the adjustment of the PID gain. The parameters of designed controller are shown in Table II.

Steady-state responses and transient responses are done at $X = 2, 5, 7$[mm] respectively. For an evaluation of transient response, a step reference signal is added to the system around 0.1(s), where the magnitude of the step signal is 1.0[mm].

C. Experimental Results

Experimental results are shown in Figs. 5-16.

Figs. 5-7 are steady-state responses at 2, 5, 7[mm] by the controller based on conventional model. Fig. 8-10 are steady-state responses at 2, 5, 7[mm] by the controller based on proposed model.

The iron ball can be stably levitated by both controllers. The results of controller based on the conventional model, there is a small fluctuation in the levitation at 2[mm].

Figs. 11-13 are step responses at 2,5,7[mm] by the controller based on conventional model. Figs. 14-16 are step responses at 2,5,7[mm] by the controller based on proposed model. Step responses of 2 and 7[mm] are worse than step responses 5[mm] by both controllers. Settling times of 2 and 7[mm] are slower than settling times of 5[mm] by both controllers. At the transient from 0(s) to 0.5(s) , the responses are vibrating. The vibration of the results of the controller based on proposed model is smaller than the results of the controller based on conventional model.

We can see that the controller based on proposed model achieves better control performance because its model represents the real behavior of the system.

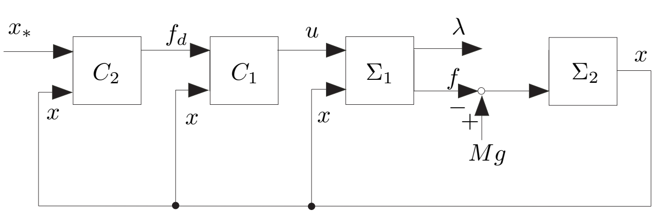

Fig. 4. Block Diagram of the Control System

TABLE I
PHYSICAL MODEL PARAMETERS

Parameter	Symbol	Value	Unit
M	Mass of Steel Ball	0.286	[kg]
X	Steady Gap	5.00×10^{-3}	[m]
k	Coefficient of $f(t)$	2.14×10^{-4}	[Nm^2/A^2]
x_0	Offset	4.36×10^{-3}	[m]
L_0	Leakage Inductance	0.248	[H]
R	Resistance	9.49	[Ω]

VI. CONCLUSIONS

In this research, we considered the new model which considered a leakage inductance L_0 of the magnetic suspension system and derived a passivity-based controller for the model. We divide the model into the electrical subsystems and the mechanical subsystems, and passivity of these two subsystems are shown. We also derived controllers for both subsystems and carried out steady-state response experiments step responses experiments for some steady state positions $X = 2,\ 5,\ 7$[mm]. In the steady-state response at 2[mm], the vibration of the proposed method is smaller than the controller based on conventional model. In the transient time from 0(s) to 0.5(s), the vibration of the controller based on proposed model is smaller than the results of the controller based on conventional model. The controller based on proposed model achieves better control performance because its model represents the real behavior of the system.

The future work is to consider robustness and a design of the controller satisfies robust stability and L_2 disturbance attenuation performance[7].

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TABLE II
PARAMETERS OF THE CONTROLLER

Parameter	Conventional Method	Proposed Method
Saturated Derivative of $\dot{f}_d$	$\frac{s}{\frac{1}{50}s+1}$	$\frac{s}{\frac{1}{50}s+1}$
Saturated Derivative of $\dot{x}$	$\frac{s}{\frac{1}{3000}s+1}$	$\frac{s}{\frac{1}{3000}s+1}$
k_p	4000	4000
k_i	4500	4500
k_d	420	420

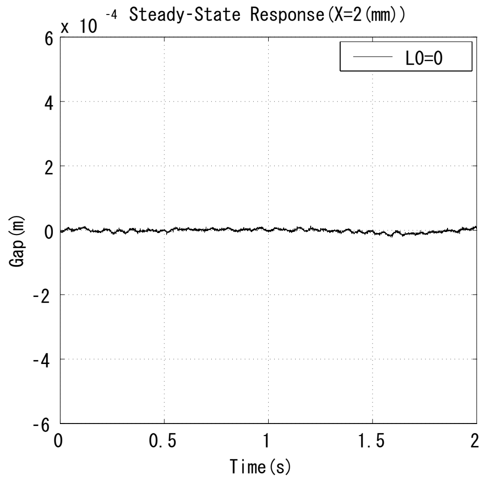

Fig. 5. Steady-State Response of $X = 2$[mm]: conventional method

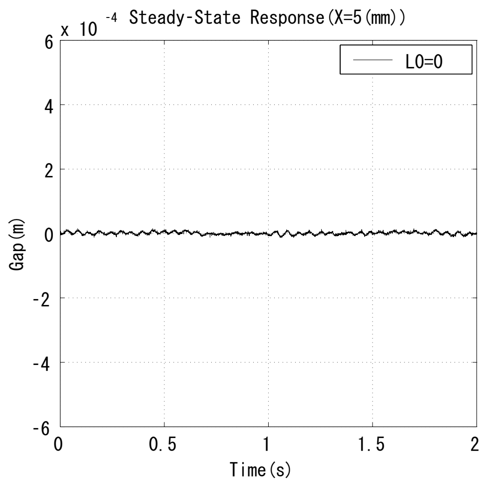

Fig. 6. Steady-State Response of $X = 5$[mm]: conventional method

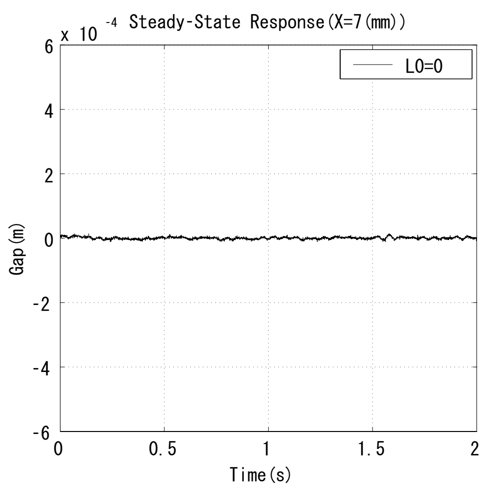

Fig. 7. Steady-State Response of $X = 7$[mm]: conventional method

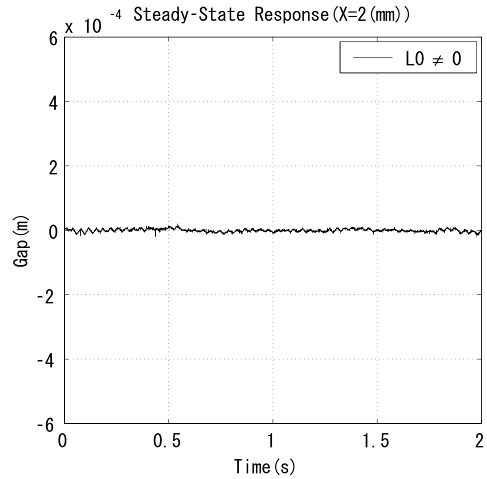

Fig. 8. Steady-State Response of $X = 2$[mm]: proposed method

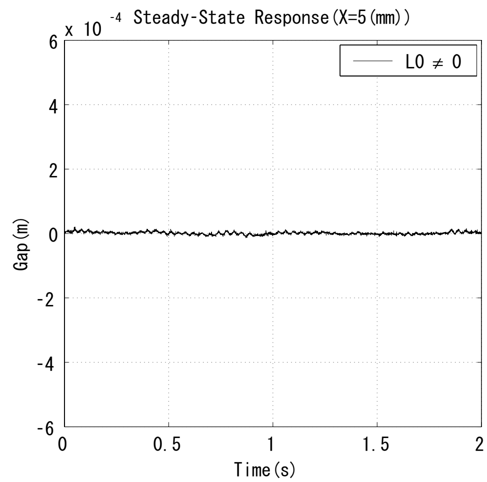

Fig. 9. Steady-State Response of $X = 5$[mm]: proposed method

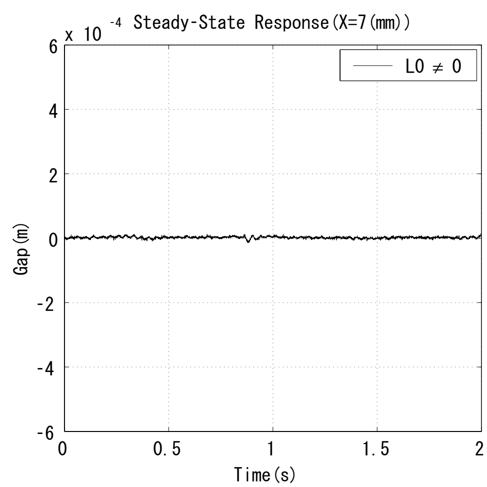

Fig. 10. Steady-State Response of $X = 7$[mm]: proposed method

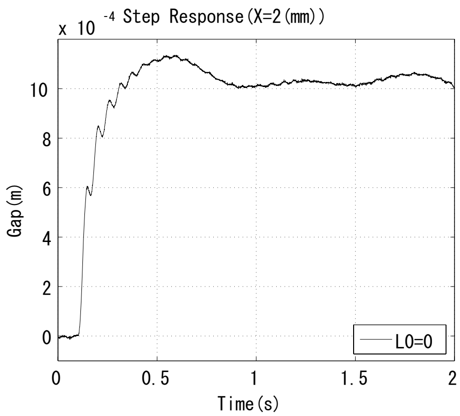

Fig. 11. Step Response of $X = 2$[mm]: conventional method

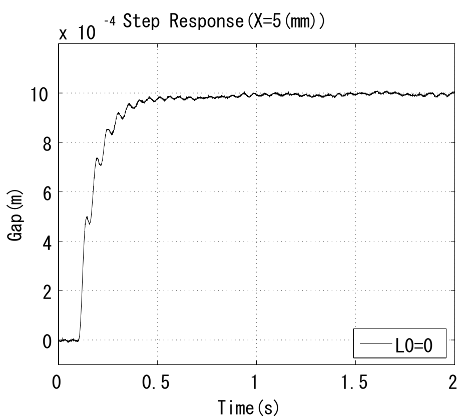

Fig. 12. Step Response of $X = 5$[mm]: conventional method

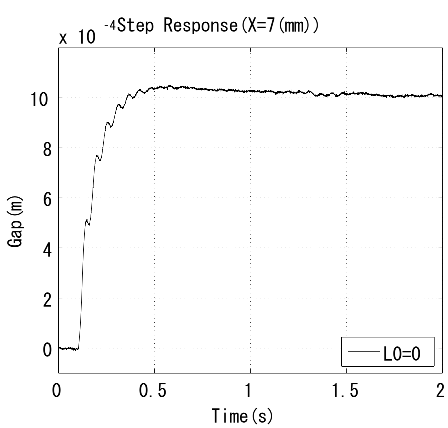

Fig. 13. Step Response of $X = 7$[mm]: conventional method

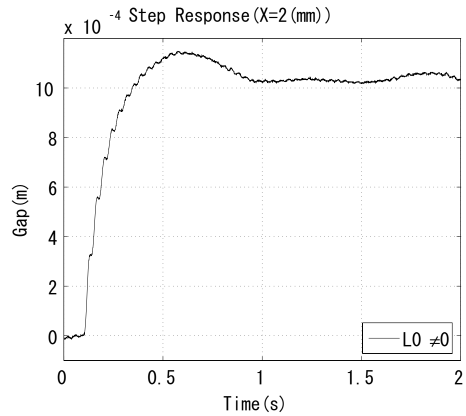

Fig. 14. Step Response of $X = 2$[mm]: proposed method

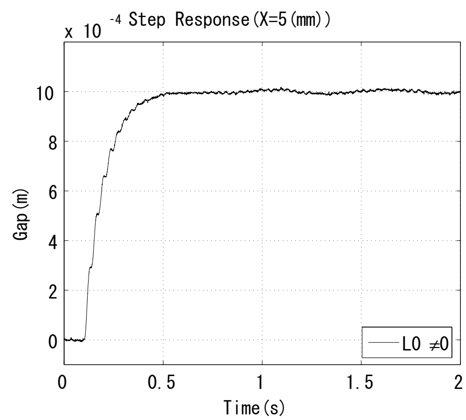

Fig. 15. Step Response of $X = 5$[mm]: proposed method

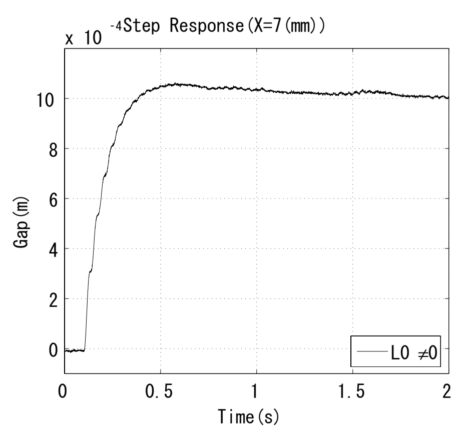

Fig. 16. Step Response of $X = 7$[mm]: proposed method