

High Performance Robust Control of Magnetic Suspension Systems Using GIMC Structure

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Abstract—This paper deals with a high performance and robust control scheme based on Generalized Internal Model Control(GIMC) Structure. We apply the GIMC structure to the unstable magnetic suspension system and construct a high performance and robust control system. GIMC structure can switch two controllers which have high performance and high robustness respectively. The two controllers using in GIMC structure are designed via \mathcal{H}_∞ mixed sensitivity problem. The experimental results show the effectiveness of GIMC structure.

I. INTRODUCTION

Conventional robust control design techniques such as \mathcal{H}_∞ control, μ -synthesis, etc, had shown good results. But almost all robust control design techniques cannot satisfy nominal performance in nominal plant, because they are based on the worst possible scenarios which may occur in a only particular situation [1], [2]. Nevertheless, the ability to keep the stability of system under the worst-case scenario is also very important. From these observations, a desired property for control architecture has a high performance for the nominal plant and a high robustness in order to keep stability for perturbed plants. In other words, it is expected to achieves both performance and robustness.

General control architecture cannot achieve both performance and robustness because there are tradeoff in these specifications. Then multiple control architectures should be used on plant conditions, e.g, the nominal controller should be applied for the nominal plant model, and the robust controller should be employed for the perturbed plant model.

“Generalized Internal Model Control(GIMC) structure” was proposed for this problem[3], [4]. GIMC structure is IMC generalized by introducing outer feedback controller. This structure can switching controllers according to whether free parameter is used or it doesn’t use it, using parameterization of stabilizing controller based on left coprime factorization.

This GIMC structure was applied to gyroscope and motor control so far, and experimentally it achieves to keep the stability of perturbed plant such as sensor failure [5], [6]. But this structure has not been applied to unstable plant which have poles in right-half plain yet.

Our goal is to apply, GIMC structure to Magnetic Suspension Systems which are unstable, and evaluate its effectiveness via experiments [7]. GIMC structure can achieve both

high performance for nominal plant and high robustness for perturbed plant. We design two controllers using \mathcal{H}_∞ mixed sensitivity problem. To check performance and robustness of the GIMC structure, we show step response experiments in nominal and perturbed plant respectively. We show a designed \mathcal{H}_∞ controller cannot achieve both performance and robustness for comparison. On the other hand, the GIMC structure keeps stability for perturbation of plant by using switching controllers experimentally.

II. GIMC STRUCTURE

Let $G(s)$ be a nominal plant model of plant $\tilde{G}(s)$ and $K_0(s)$ be a stabilizing controller for $G(s)$. Suppose that K_0 and G have the left coprime factorizations expressed by (1).

$$G(s) = \tilde{M}(s)^{-1}\tilde{N}(s), \quad K_0(s) = \tilde{V}(s)^{-1}\tilde{U}(s) \quad (1)$$

It is well known that every stabilizing controller $K(s)$ for $G(s)$ can be written in (2) and (3) by using free-parameter $Q(s) \in RH_\infty$,

$$K(s) = (\tilde{V}(s) - Q(s)\tilde{N}(s))^{-1}(\tilde{U}(s) + Q(s)\tilde{M}(s)), \quad (2)$$

$$\det(\tilde{V}(\infty) - Q(\infty)\tilde{N}(\infty)) \neq 0. \quad (3)$$

GIMC structure is shown in Fig.1. This has an outer feedback loop($K_0(s) = \tilde{V}(s)^{-1}\tilde{U}(s)$) and an internal feedback loop.

Note that the reference signal $ref(t)$ in Fig.1 enters into the stabilizing controller K in the GIMC structure, but stability of system does not change from $K(s)$ because a transfer function from $y(t)$ to $u(t)$ is same with $K(s) = (\tilde{V}(s) - Q(s)\tilde{N}(s))^{-1}(\tilde{U}(s) + Q(s)\tilde{M}(s))$. The free-parameter $Q(s) \in RH_\infty$ can be chosen within (3) and $K(s)$ is a set of the stabilizing controllers. In the following, $K(s)$ is fixed by some specified $Q(s)$. We assume that $Q(s)$ is fixed in the following.

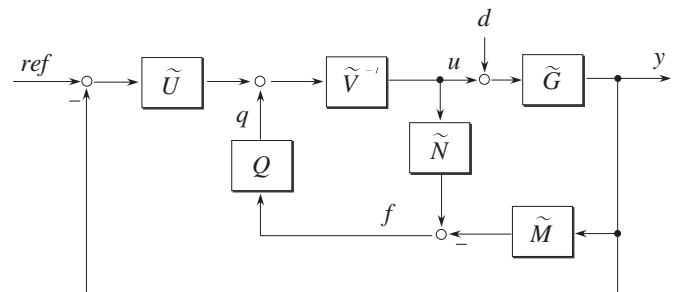


Fig. 1. GIMC structure

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GIMC structure can achieve both high performance and high robustness because it can utilize both controllers $K_0(s)$ and $K(s)$ by switching them, it depends on an internal signal $f(s)$. The internal signal $f(s)$ can be expressed in (4) [4].

$$f(s) = \tilde{N}(s)u(s) - \tilde{M}(s)y(s) \quad (4)$$

This signal $f(s)$ is an error of an estimated signal and an actual signal. Consider two cases which are $\tilde{G}(s) = G(s)$ and $\tilde{G}(s) \neq G(s)$.

$\tilde{G}(s) = G(s)$:

$f(s) = 0$ if there are no model uncertainties, disturbance or faults, then $q(s) = 0$. The control system is controlled by $K_0(s) = \tilde{V}(s)^{-1}\tilde{U}(s)$.

$\tilde{G}(s) \neq G(s)$:

$f(s) \neq 0$ if there are either model uncertainties or disturbance or faults, then the inner loop is active because $q(s) \neq 0$. The feedback system is controlled by $K(s) = (\tilde{V}(s) - Q(s)\tilde{N}(s))^{-1}(\tilde{U}(s) + Q(s)\tilde{M}(s))$.

GIMC structure can switch two controllers which are $K_0(s)$ and $K(s)$ using the internal signal $f(s)$ in the above way. This switching characteristic gives a desired control property to the GIMC structure. The high performance controller $K_0(s)$ is applied to the nominal model ($f(s) = 0$) and the high robustness controller $K(s)$ is applied to the perturbed plant ($f(s) \neq 0$).

The design procedure of GIMC structure is given by the following three steps.

Controller Design Step[3]

Step 1. Design a high performance controller $K_0(s)$ for the nominal model $G(s)$.

Step 2. Design a high robust controller $K(s)$ for the perturbed model $\tilde{G}(s)$.

Step 3. Construct an internal controller $Q(s)$ based on the following equation.

$$Q(s) = \tilde{V}(s)(K(s) - K_0(s))(\tilde{N}(s)K(s) + \tilde{M}(s))^{-1} \quad (5)$$

The internal controller $Q(s)$ is not used in the nominal model then GIMC structure is controlled by only $K_0(s)$, and the internal controller $Q(s)$ is activated for the perturbed plant. This means the GIMC structure is controlled by $K(s)$.

Implementation of GIMC-based Switching Controller

Actually it is impossible to construct a completely accurate plant model such as $\tilde{G}(s) = G(s)$, then $K(s)$ is applied even for the nominal plant because $\tilde{G}(s) \simeq G(s)$ in nominal mode.

Consider a new GIMC structure with a detector and a switch in the internal loop as shown in Fig.2. This structure makes the high performance controller $K_0(s)$ work even if there exists a small perturbation $\tilde{G}(s) \simeq G(s)$. That means the high performance controller K_0 can be applied to a slightly perturbed nominal model.

In this new GIMC structure for implementation, a switching timing and its decision is judged by a signal $r(s)$ which is an output of a function $H(s)$. The signal $r(s)$ is expressed

in eq. (6) and the function $H(s)$ is a filter of the signal $f(s)$ to judge a current mode of the plant.

$$r(s) = H(s)(\tilde{N}(s)u(s) - \tilde{M}(s)y(s)) \quad (6)$$

A judgment index J_{th} of the nominal and the robust modal is a magnitude of the signal $r(s)$ in (7).

The index J_{th} is utilized to decide a model among the multiple candidates of the plant models. If $r(s) < J_{th}$ then switch is OFF which means the candidate of the perturbed plant is selected and if $r(s) > J_{th}$ then the switch is ON.

$$J_{th} = \max_{\Delta=0,u,d} |r(s)|, \quad \tilde{G} = G(1 + \Delta) \quad (7)$$

III. SYSTEM CONFIGURATION AND MODELING

The controlled plant in this research is a magnetic suspension system shown in Fig.3 where m : mass of iron ball, $f_{mag}(t)$: electromagnetic force, $x(t)$: displacement, $v(t)$: input voltage, $i(t)$: current, respectively.

The equation of motion is expressed by (8) and an electromagnetic force is given by (9).

$$m \frac{d^2 x(t)}{dt^2} = mg - f_{mag}(t) \quad (8)$$

$$f_{mag}(t) = k \left(\frac{i(t)}{x(t) + x_0} \right)^2 \quad (9)$$

The coefficients k and x_0 in (9) are determined by identification experiments. Equation (9) is transformed into (11) by using Taylor series expansion of (10) around the equilibrium point. The variables in (10) are defined as, X : steady gap between the electromagnet and the iron ball, $\delta x(t)$: displacement from the steady gap, I : steady current of the electromagnet, $\delta i(t)$: current from steady current.

$$x(t) = X + \delta x(t), i(t) = I + \delta i(t) \quad (10)$$

$$f_{mag}(t) \simeq k \left(\frac{I}{X + x_0} \right)^2 + k_i \delta i(t) - k_x \delta x(t) \quad (11)$$

$$k_x = \frac{2kI^2}{(X + x_0)^3}, k_i = \frac{2kI}{(X + x_0)^2}$$

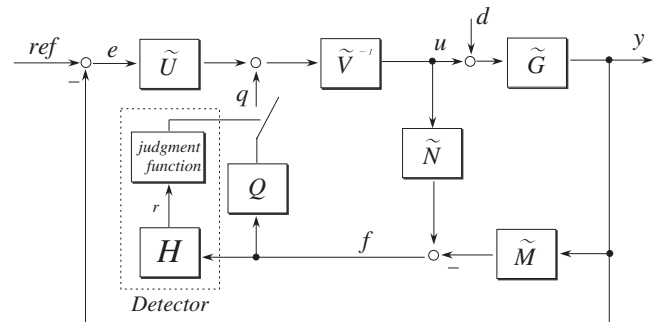


Fig. 2. GIMC structure with Detector and Switch

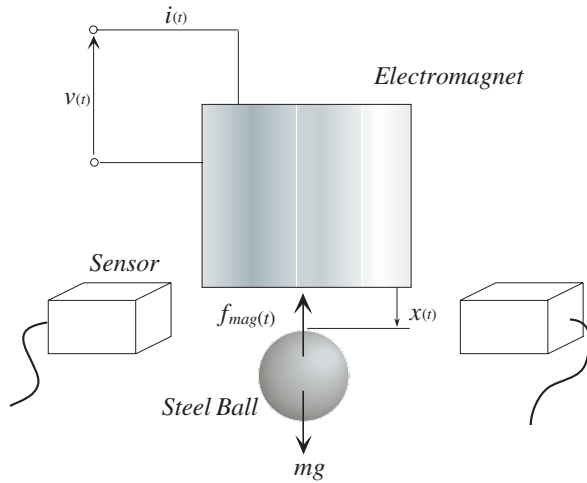


Fig. 3. Magnetic Suspension System

Redefine $x(t) = \delta x(t)$ and $i(t) = \delta i(t)$, then state-space equation is given as (12). The model parameters are shown in Table I.

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ \frac{k_x}{m} & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ -\frac{k_i}{m} \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{aligned} \quad (12)$$

$$x = \begin{bmatrix} x & \dot{x} \end{bmatrix}^T, y = x, u = i$$

IV. CONTROLLER DESIGN FOR GIMC STRUCTURE

The controller design step is already mentioned in the Section II. At first, we design two controllers which are a nominal controller $K_0(s)$ and a robust controller $K(s)$ using \mathcal{H}_∞ mixed sensitivity problem, respectively on Step 1, 2.

The \mathcal{H}_∞ mixed problem is a design problem to find a controller which satisfies the condition (13), where $S(s)$:sensitivity function, $T(s)$:complementarity sensitivity function, $W_S(s)$:weighting function for sensitivity function, $W_T(s)$:weighting function for complementarity sensitivity function.

$$\left\| \begin{bmatrix} W_S S \\ W_T T \end{bmatrix} \right\|_\infty < 1 \quad (13)$$

The generalized plant for the \mathcal{H}_∞ mixed sensitivity problem is shown in Fig.4. The weighting functions chosen for

TABLE I
NOMINAL PARAMETERS

m	0.357[kg]
k	$11.641 \times 10^{-4} [\text{Nm}^2/\text{A}^2]$
x_0	$4.737 \times 10^{-3} [\text{m}]$
X	$5 \times 10^{-3} [\text{m}]$
I	0.54[A]

$K_0(s)$ and $K(s)$ are written in (14), (15), respectively. W_{SP} and W_{TP} in (14) is used to design $K_0(s)$, and W_{SR} and W_{TR} in (15) is used to design $K(s)$, respectively. It is well-known that there exists a constraint $S(s) + T(s) = I$. Then $W_S(s)$ should be selected to have high gain if the designed controller should have high performance, on the other hand, $W_T(s)$ should be selected to have high gain if the controller should have high robustness.

$$W_{SP}(s) = \frac{400}{s+0.01},$$

$$W_{TP}(s) = 1 \times 10^{-6} \times (s+0.02)(s+0.1) \quad (14)$$

$$W_{SR}(s) = \frac{10}{s+0.01},$$

$$W_{TR}(s) = 1 \times 10^{-5} \times (s+0.02)(s+800) \quad (15)$$

In these steps, $K_0(s)$ and $K(s)$ are designed to let them have a high performance and a high robustness respectively. The frequency responses of two controllers are shown in Fig.5, where a solid line shows $K_0(s)$, a dashed line shows $K(s)$.

Finally, we construct the internal controller $Q(s)$ by using $K_0(s)$ and $K(s)$ based on Step 3. In order to construct $Q(s)$ by using (5), coprime factorizations of plant $G(s)$ and $K_0(s)$ are necessary. Suppose that state-spaces of $K_0(s)$ and $G(s)$ are given as

$$G = \begin{bmatrix} A & B \\ C & D \end{bmatrix}, K_0 = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} \quad (16)$$

and (A, B) is controllable, (C, A) is observable, (A_k, B_k) is controllable and (C_k, A_k) is observable.

The coprime factorizations of $G(s)$ and $K_0(s)$ are given by (17) and (18) respectively. Note that L and L_k stabilize $A + LC$ and $A_k + L_k C_k$, respectively.

$$\begin{bmatrix} \tilde{N} & \tilde{M} \end{bmatrix} = \begin{bmatrix} A+LC & B+LD & L \\ C & D & I \end{bmatrix} \quad (17)$$

$$\begin{bmatrix} \tilde{V} & \tilde{U} \end{bmatrix} = \begin{bmatrix} A_k+L_k C_k & L_k & B_k+L_k D_k \\ C_k & I & D_k \end{bmatrix} \quad (18)$$

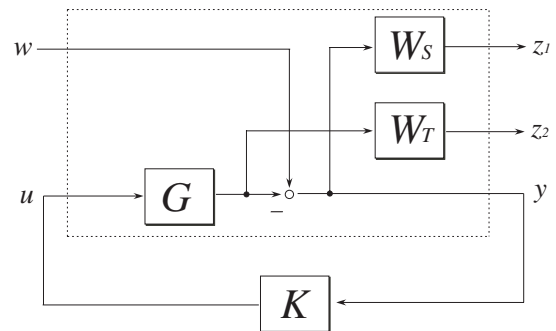


Fig. 4. Generalized plant

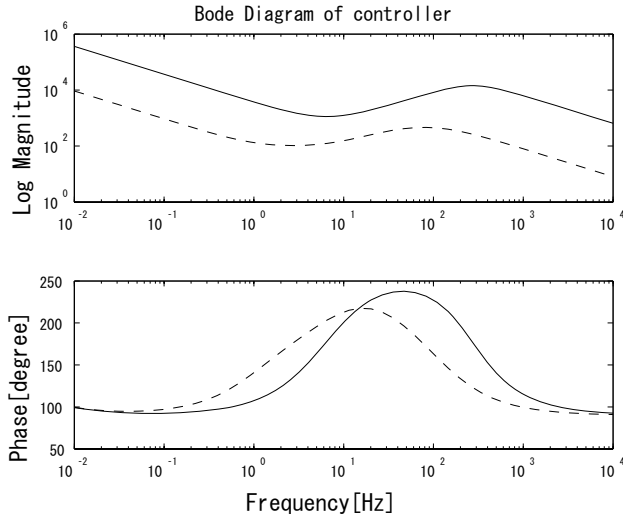


Fig. 5. Bode Diagram of Controllers

V. EXPERIMENTAL EVALUATION

A. Evaluation of Control Performance

To compare GIMC structure with a conventional robust controller, we show some time responses of GIMC structure, $K_0(s)$ and $K(s)$, respectively. In order to evaluate controller characteristics, we consider the perturbation of mass of the iron ball and a communication delay of the plant, and perturbations of the parameters are shown in Table II.

1) *GIMC Structure*: At first, the case of GIMC structure in Fig.1 is evaluated. Transient responses for 1[mm] step reference signal are measured where parameters in Table I are used for the nominal plant and parameters in Table II are used in the perturbed plant. Nominal responses are shown in Fig.6. The responses with the perturbed plant are shown in Fig.7. The solid lines show responses of GIMC structure and the dashed lines and dash-dot lines show responses of $K_0(s)$ and $K(s)$ in Figs.6 and 7. The internal signals $f(s)$ are shown in Fig.8 where the lower line shows the nominal response and the upper line shows the perturbed response.

From Fig.6, $K_0(s)$ shows best responses but GIMC structure is not so good. From Fig.7, the response of K_0 is deteriorated with the perturbed plant, but the response of GIMC structure does not change so much. These results are caused by the internal signal $f(s)$ in Fig.8. The plant was not controlled by the high performance controller $K_0(s)$ because $f(s)$ is not zero even in the nominal mode.

TABLE II
PERTURBATION OF PARAMETERS

	Nominal Parameters	Perturbed Parameters
m[kg]	0.357	0.197
delay[s]	0.000	0.001

2) *GIMC Structure with Detector and Switch*: The reason why responses of GIMC are not corresponding to responses $K_0(s)$ is that there is error between $\tilde{G}(s)$ and $G(s)$ then $f(s) \neq 0$ even for the nominal plant. Hence we apply the GIMC structure with a detector and a switch in Fig.2. Resulting time responses are shown in Fig.9 and we can see that this structure can achieve nominal performance. Here the filter $H(s)$ in the detector is given by (19).

$$H(s) = \frac{1}{1 + \frac{s}{2\pi f_s}}, \quad f_s = 2 \quad (19)$$

Time responses of the structure in Fig.2 for the perturbed plant are same with the responses in Fig.7. The residual signal $r(s)$ from detector is shown in Fig.10. The solid line shows a response of the nominal plant and the dashed line shows a response of the perturbed plant.

The threshold value for the judgment for both nominal and perturbed plant is decided as $J_{th} = 4.5 \times 10^{-5}$ based on Fig.10. If the signal $r(s)$ is less than J_{th} , the high performance controller $K_0(s)$ is applicable. If $r(s)$ is larger than J_{th} , the feedback control system is controlled by the high robust controller $K(s)$. An excellent switching of two controllers can be done by using a detector and a switch. From these results, we have confirmed that the GIMC structure can achieve a high performance robust control.

B. Evaluation of Stability

The time responses of two controller $K(s)$ and $K_0(s)$ are measured when the parameters of the plant are changed in the real-time feedback control. A 1.5(ms) communication delay as a model perturbation occurs in real time. Resulting time responses of controller $K(s)$, $K_0(s)$ and GIMC structure in Fig.2 are shown in Figs.11, 12 and 13, respectively. Here the communication delay as a model perturbation is added to the plant at 0.5[s].

Time response of $K_0(s)$ in Fig.11 shows very big vibration after 0.5[s]. Time response of $K(s)$ in Fig.12 does not change after the perturbation supplement. In the time response of GIMC in Fig.13, a vibration shows after 0.5[s] but the controller is switched around 1.0[s] then the vibration is getting small and the position signal converges on 0.

The control input is shown in Fig.14 in the same period. The signal becomes to be vibrate after 0.5[s], and after around 1.0[s] the vibration begin to go to zero because the controller is switched.

From these results, GIMC structure can keep the stability if the parameter of the plant is changed in the real time control. But, the response vibrates in the transient of controller switching.

VI. CONCLUSION

We have shown that the GIMC structure can achieve both a high performance and a high robustness. GIMC structure can switch a high performance controller and a high robustness controller based on a residual signal $f(s)$.

We applied GIMC to magnetic suspension system then showed that GIMC could achieve a high performance and a

high robustness control compared with a conventional \mathcal{H}_∞ control.

In the experimental evaluations for stability, GIMC structure can keep stability even if a parameter of the plant is changed in the real time control. In addition, we confirmed that GIMC structure can keep stability for model perturbations which destabilizes the nominal controller K_0 .

REFERENCES

- [1] Kemin Zhou and John C.Dolye, *Essential of Robust Control*, Prentice Hall (1998)
- [2] Gier E.Dullerud and Fernando Paganini, *A Course in Robust Control Theory : A Convex Approach*, Springer (1999)
- [3] Kemin Zhou, A Natural Approach to High Performance Robust Control: Another Look at Youla Parameterization, *Proceedings of SICE Annual Conference 2004*, pp. 869-874, 2004
- [4] Kemin Zhou and Zhang Ren, A New Controller Architecture for High Performance, Robust, and Fault-Tolerant Control, *IEEE Transaction on Automatic Control*, Vol.46, No.10, pp. 1613-1618, 2001
- [5] Daniel U, Campos-Delgado and Kemin Zhou, Reconfigurable Fault-Tolerant Control Using GIMC Structure, *IEEE Transactions on Automatic Control*, Vol.48, No.5, pp. 1613-1618, 2003
- [6] D.U.Campos-Delgado, S.Martinez Martinez and K.Zhou, Integrated Fault Tolerant Scheme with Disturbance Feed-forward, *Proceeding of the 2004 American Control Conference*, pp. 1799-1804, 2004
- [7] Masayuki Fujita, Toru Namerikawa, Fumio Matsumura and Kenko Uchida, μ -Synthesis of an Electromagnetic Suspension System, *IEEE Transactions on Automatic Control*, Vol.40, No.3, pp. 530-536, 1995

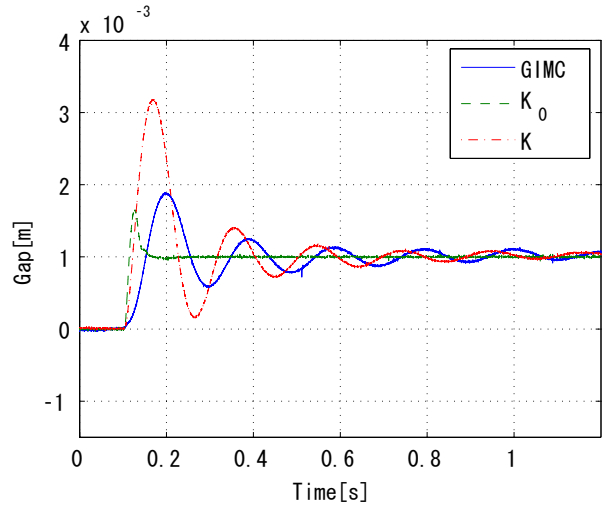


Fig. 6. Step responses of nominal plant

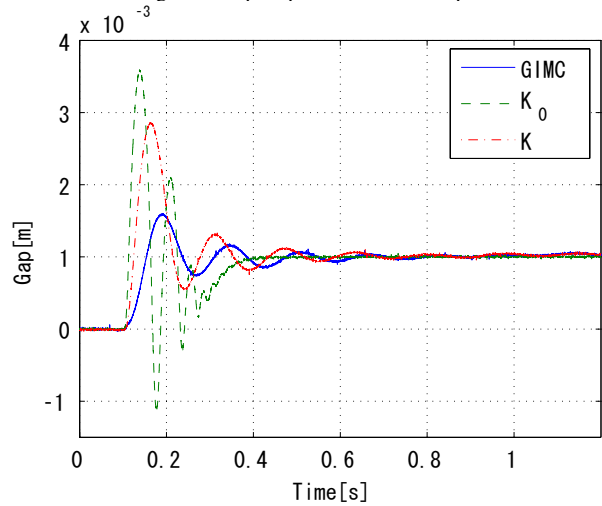


Fig. 7. Step responses of Perturbed plant

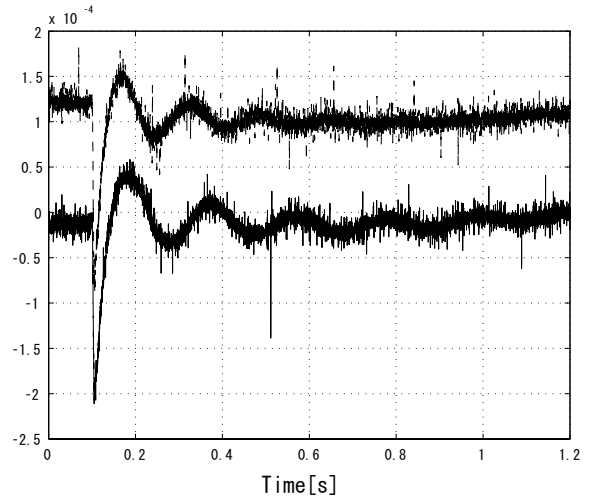


Fig. 8. Internal signals $f(s)$ of GIMC structure

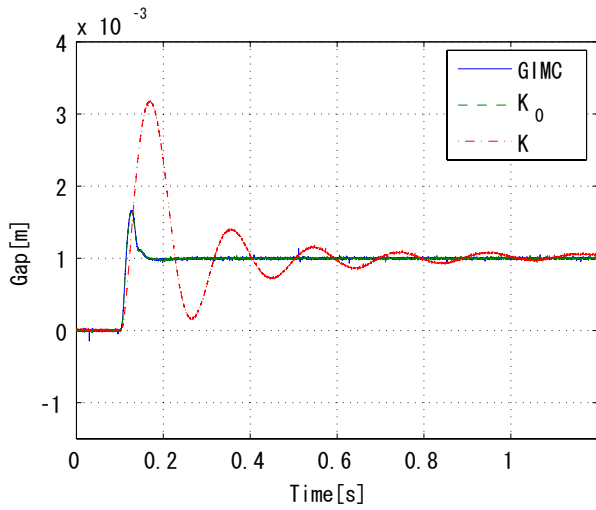


Fig. 9. Step Responses of Nominal Plant with Detector and Switch

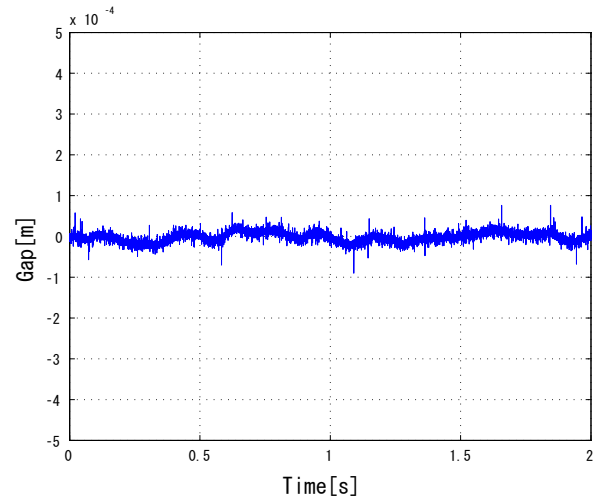


Fig. 12. Time Response of $K(s)$

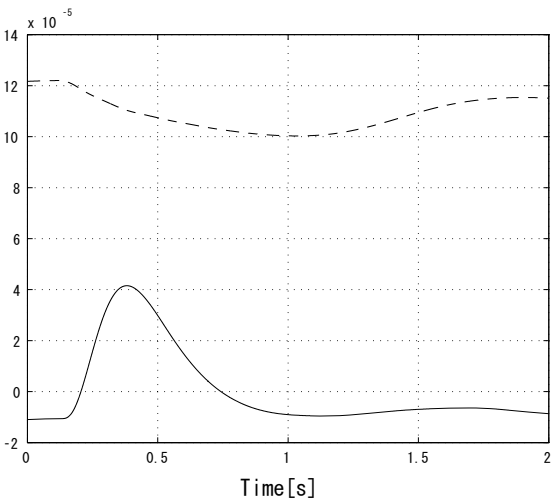


Fig. 10. Internal Signals $r(s)$ of GIMC structure with Detector and Switch

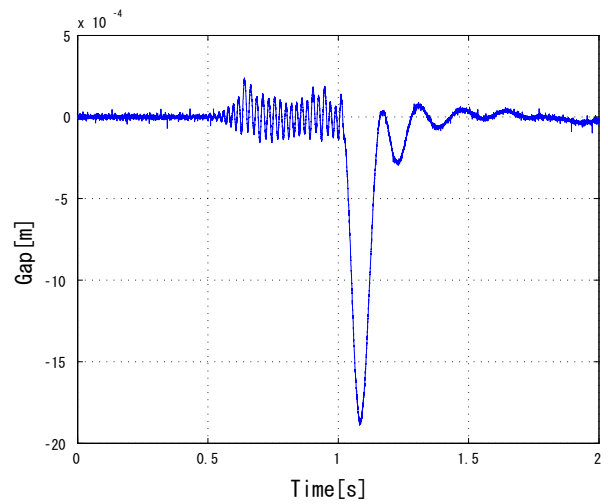


Fig. 13. Time Response of GIMC with Detector and Switch

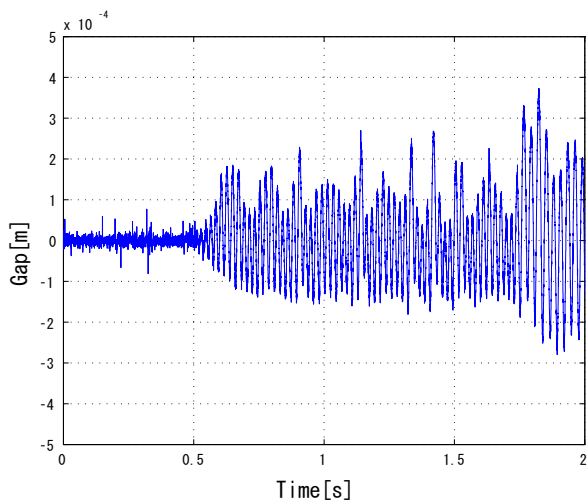


Fig. 11. Time Response of $K_0(s)$

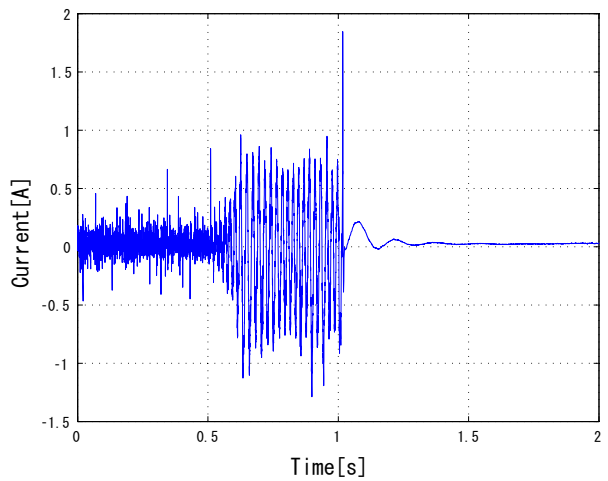


Fig. 14. Control Signal $u(s)$ of Time Response of GIMC