Robust $\mathcal{H}_\infty$ DIA Control of Levitated Steel Plates

Toru Namerikawa*† and Daisuke Mizutani*††

*Nagaoka University of Technology, 1603-1 Kamitomioka, Nagaoka, Niigata 940-2188, JAPAN, Phone +81-258-47-9767, Facsimile +81-258-47-9770

Email:†toru@nagaokaut.ac.jp, ††mizutani@stn.nagaokaut.ac.jp

Abstract This paper deals with an application of $\mathcal{H}_\infty$ DIA control to the magnetically levitated steel plates. Our goal in this research is to suspend two thin steel plates stably by using four electromagnets without any physical contacts. We apply the robust $\mathcal{H}_\infty$DIA control approach to the magnetically levitated steel plates. The $\mathcal{H}_\infty$DIA control problem treats a mixed Disturbance and an Initial State uncertainty Attenuation and it is expected to provide a good transient property to a control system. Experimental results show that the proposed robust control approach is effective for suppressing an elasticity vibration of steel plates.

Keywords: $\mathcal{H}_\infty$ DIA Control, Magnetic Suspension System, Robust Control, Levitated Steel Plate

1. Introduction

The magnetically levitated steel plate technology is expected to prevent a surface quality of steel plate from deteriorating in a manufacturing process [1][2]. In order to make this technology fit for practical use, a feedback controller should be able to suspend multiple steel plates, and to suppress an elasticity vibration of steel plates [3][4]. There have been a lot of related works concerning magnetically levitated steel plate technology, e.g., steel plate transport systems, et.al. The multiple steel plates levitation problem by a single controller has not been studied and the robust control system design for multiple plates is expected.

Our goal in this research is to suspend two thin steel plates stably by using four electromagnets without any physical contacts. We apply the robust $\mathcal{H}_\infty$DIA control approach [5] to the magnetically levitated steel plates. The $\mathcal{H}_\infty$DIA control problem treats a mixed Disturbance and an Initial State uncertainty Attenuation and it is expected to provide a good transient property to the control system.

2. Robust $\mathcal{H}_\infty$ DIA Control

Consider the linear time-invariant system.

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u, \quad x(0) = x_0 \\ z &= C_1 x + D_{12} u \\ y &= C_2 x + D_{21} w \quad \cdots\cdots\cdots (1) \end{aligned}$$

where $x \in R^n$ is the state and $x_0 = x(0)$ is the initial state; $u \in R^r$ is the control input; $y \in R^m$ is the observed output; $z \in R^q$ is the controlled output; $w \in R^p$ is the disturbance. For system (1), a control $u(t)$ is given by linear time-invariant system of the form

$$\begin{aligned} u &= J\zeta + Ky \\ \dot{\zeta} &= G\zeta + Hy, \quad \zeta(0) = 0 \quad \cdots\cdots\cdots (2) \end{aligned}$$

which makes the closed-loop system internally stable. For the system and the controls described above, consider a mixed-attenuation problem state as below.

Problem 1 $\mathcal{H}_\infty$ **DIA Control problem** [5]
Find a control $u(t)$ attenuating disturbances and initial state uncertainties in the way that, for given $N > 0$, z satisfies

$$\|z\|_2^2 < \|w\|_2^2 + x_0^T N^{-1} x_0 \cdots\cdots\cdots (3)$$

for all $w \in L^2[0, \infty)$ and all $x_0 \in R^n$, s.t., $(w, x_0) \neq 0$. Such an admissible control is called the **D**isturbance and **I**nitial state uncertainty **A**ttenuation (DIA) control.

In order to solve the problem, we require the following conditions.

(A1) There exists a solution $M > 0$ to the Riccati equation

$$\begin{aligned} &M(A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1) \\ &+(A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1)^T M \\ &-M(B_2(D_{12}^T D_{12})^{-1} B_2^T - B_1 B_1^T) M \\ &+C_1^T C_1 - C_1^T D_{12}(D_{12}^T D_{12})^{-1} D_{12}^T C_1 = 0 \cdots\cdots (4) \end{aligned}$$

s.t. $A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1 - B_2(D_{12}^T D_{12})^{-1} B_2^T M + B_1 B_1^T M$ is stable.

(A2) There exists a solution $P > 0$ to the Riccati equation

$$\begin{aligned} &(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2) P \\ &+P(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2)^T \\ &-P(C_2^T (D_{21} D_{21}^T)^{-1} C_2 - C_1^T C_1) P \\ &+B_1 B_1^T - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} D_{21} B_1^T = 0 \cdots\cdots (5) \end{aligned}$$

s.t. $A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2 - P C_2^T (D_{21} D_{21}^T)^{-1} C_2 + P C_1^T C_1$ is stable.

(A3) $\rho(PM) < 1$

where $\rho(X)$ denotes the spectral radius of matrix X. Then we obtained the following results.

Theorem 1 [5]
Suppose that the conditions (A1), (A2), (A3) are satisfied. Then the $\mathcal{H}_\infty$ central control is given as

$$\begin{aligned} u &= -(D_{12}^T D_{12})^{-1}(B_2^T M + D_{12}^T C_1)(I - PM)^{-1}\zeta \\ \dot{\zeta} &= A\zeta + B_2 u + P C_1^T (C_1 \zeta + D_{12} u) \\ &\quad +(P C_2^T + B_1 D_{21}^T)(D_{21} D_{21}^T)^{-1}(y - C_2 \zeta) \\ \zeta(0) &= 0. \cdots\cdots\cdots (6) \end{aligned}$$

The $\mathcal{H}_\infty$ central control (6) is a DIA control if and only if the condition (A4) is satisfied.

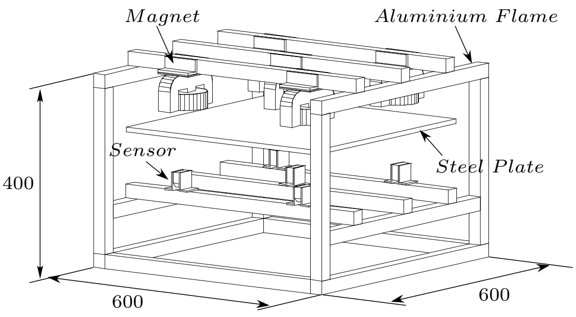

Fig. 1. System structure

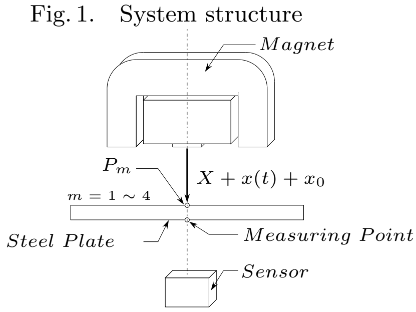

Fig. 2. Position relation between a electro magnet and a sensor

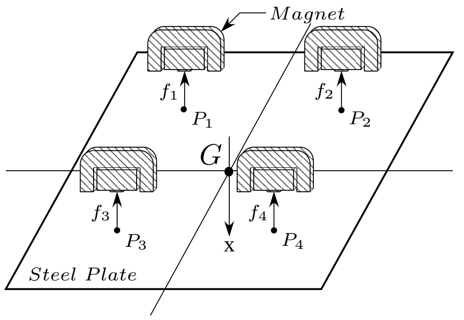

Fig. 3. Magnetically Levitated Steel Plates

(A4) $Q + N^{-1} - P^{-1} > 0$,

where Q is the maximal solution of the Riccati equation

$$\begin{aligned} & Q(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2 \\ & \quad + (B_1 B_1^T - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} D_{21} B_1^T) P^{-1}) \\ & + (A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2 \\ & \quad + (B_1 B_1^T - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} D_{21} B_1^T) P^{-1})^T Q \\ & - Q(B_1^T - D_{21}^T (D_{21} D_{21}^T)^{-1} (C_2 P + D_{21} B_1^T) L)^T \\ & \quad \times (B_1^T - D_{21}^T (D_{21} D_{21}^T)^{-1} (C_2 P + D_{21} B_1^T) L) Q \\ & = 0 \cdots\cdots\cdots (7) \end{aligned}$$

with $L := (I - PM)^{-1}$.

3. System Configuration and Modeling

We have constructed an experimental system for levitated steel plates shown in Fig.1. This system has five electromagnets and four of them are used in the feedback control. Each electromagnet has its own optical gap sensor across the steel plate as shown in Fig.2. Detail specifications of two steel plates: *Steel Plate* 1 and *Steel Plate* 2 used in experiments are described in Table 1. Here the *Steel Plate* 1 is a nominal suspended object.

Table 1. Specifications for the steel plates

	Steel Plate 1	*Steel Plate* 2
Wide	500[mm]	500[mm]
Depth	500[mm]	500[mm]
Thickness	0.3[mm]	0.5[mm]
Mass	0.537[kg]	0.937[kg]

In order to derive a model of the system by lows of physic, we introduce following assumptions.

- The whole system can be divided to four independent Single Input Single Output sub-systems.
- Four electromagnets are identical(see Fig.3).

Under these assumptions, we derive the equation of motion of the iron steel plate and the electromagnetic force equation as followed [6].

$$m \frac{d^2 x(t)}{dt^2} = mg - f(t) + v_m(t) \cdots\cdots (8)$$

$$f(t) = k \left(\frac{I + i(t)}{X + x(t) + x_0} \right)^2 \cdots\cdots (9)$$

where m as the mass of the 1/4 steel plate, X as a steady gap between the electromagnet and the steel plate, $x(t)$ as a deviation from X, I as a steady current, $i(t)$ as a deviation from I, $f(t)$ as an electromagnet force, k, x_0 are coefficients of $f(t)$, $v_m(t)$ as exogenous disturbance force.

The electromagnetic force (9) is linearized around the operating point by the Taylor series expansion as

$$f(t) = k \left(\frac{I}{X + x_0} \right)^2 - K_x x(t) + K_i i(t) \cdots (10)$$

$$K_x = \frac{2kI^2}{(X + x_0)^3}, \ K_i = \frac{2kI}{(X + x_0)^2}$$

The sensor provides the information for the gap $x(t)$. Hence the measurement equation can be written as

$$y_g(t) = x(t) + w_0(t) \cdots\cdots (11)$$

where $w_0(t)$ represents the sensor noise as well as the model uncertainties.

Thus, summing up the above results, the state equations for the system [6] are

$$\begin{aligned} \dot{x}_g &= A_g x_g + B_g u_g + D_g v_0 \\ y_g &= C_g x_g + w_0 \end{aligned} \cdots\cdots (12)$$

$$x_g := [x \ \ \dot{x}]^T, u_g := i, v_0 := v_m$$

$$A_g = \begin{bmatrix} 0 & 1 \\ K_x & 0 \end{bmatrix}, B_g \begin{bmatrix} 0 \\ -K_i \end{bmatrix}, D_g = \begin{bmatrix} 0 \\ 1/m \end{bmatrix}$$

$$C_g = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Table 2. System Parameter

Parameter	Symbol	Value
Mass of a quarter of Steel Plate	m	134[g]
Steady Gap	X	5.0×10^{-3}[m]
Coefficients of $f(t)$	k	8.50×10^{-4}
	x_0	2.80×10^{-2}
Steady Current	I	0.309[A]

4. Control System Design

Let v_0 and w_0 be of the form with the frequency weighting functions W_v and W_w as

$$v_0 = W_v(s) w_2 \quad (13)$$
$$W_v = \Phi C_v (sI - A_v)^{-1} B_v, \quad \Phi = [1 \quad 1]^T$$
$$w_0 = W_w w_1. \quad (14)$$

The gap $x(t)$ and the corresponding velocity $\dot{x}(t)$ are restricted by the weight Θ and the control input $u(t)$ also should be regulated to suppress the energy, where its weight is expressed by ρ; i.e.,

$$z_1 = \Theta x_g, \quad \Theta = \mathrm{diag}\left[\begin{array}{cc} \theta_1 & \theta_2 \end{array}\right] \quad (15)$$
$$z_2 = \rho u_g \quad (16)$$

Finally, we can construct the generalized plant (17) as in the following;

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{12} u \\ y &= C_2 x + D_{21} w \quad (17) \end{aligned}$$
$$A = \begin{bmatrix} A_g & D_g C_w \\ 0 & A_w \end{bmatrix}, B_1 = \begin{bmatrix} 0 & D_g D_w \\ 0 & B_w \end{bmatrix}$$
$$B_2 = \begin{bmatrix} B_g \\ 0 \end{bmatrix}, \ C_1 = \begin{bmatrix} \Theta & 0 \\ 0 & 0 \end{bmatrix}, \ D_{12} = \begin{bmatrix} 0 \\ \rho \end{bmatrix}$$
$$C_2 = \left[\begin{array}{cc} C_g & 0 \end{array}\right], \ D_{21} = \left[\begin{array}{cc} W_w & 0 \end{array}\right]$$

where $x := [x_g^T \ x_v^T]^T$ and x_v is a state of $W_v(s)$, $w := [w_1^T \ w_2^T]^T$ and $z := [z_1^T \ z_2^T]^T$.

Now our control problem setup is as followed.

Control Problem Setup: finding an admissible controller $K(s)$ that attenuates disturbance and initial state uncertainties to achieve DIA condition in (3).

After some iteration in MATLAB environment, the final $\mathcal{H}_\infty$ DIA controller was obtained, where the maximum value of the weighting matrix N is $N = 1.013 \times 10^{-2}$. The frequency response of the controller K_{DIA} is shown in Fig. 4. We designed a PID controller for the comparison and its frequency response is also shown.

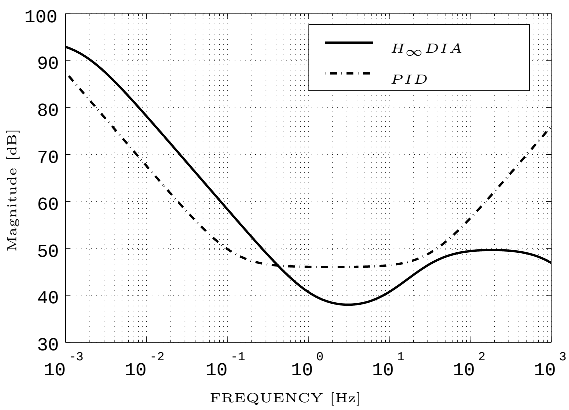

Fig. 4. Frequency Responses of Two Controllers

5. Experimental Evaluation

We conducted experiments to evaluate properties of the $\mathcal{H}_\infty$ DIA controller for steel plate levitation system.

5.1 Nominal Stability and Performance First we conducted experiments by using the nominal *Steel Plate* 1. Steady state position responses for stability evaluation of $\mathcal{H}_\infty$ DIA control and PID control are shown in Figs.5 and 8, respectively. These figures show that $\mathcal{H}_\infty$ DIA control suppress a vibration of the steel plate compared with PID.

Transient step responses for performance evaluation of both controllers are shown in Figs.6 and 9, respectively. $\mathcal{H}_\infty$DIA shows relatively bigger overshoot but the steady state vibration is suppressed.

5.2 Robustness for multiple steel plates Robust stability was checked by using the *Steel Plate* 2. Steady state response of $\mathcal{H}_\infty$DIA control and PID are shown in Fig.7 and Fig.10, respectively. Both controllers achieve robust stability and the closed-loop system with both controllers are stable for *Steel Plate* 2.

Our proposed $\mathcal{H}_\infty$DIA controller suppresses a vibration of the steel plates and it has a robust stability for multiple steel plates. $\mathcal{H}_\infty$DIA control restrains a deterioration and a vibration compared with PID control. The overshoot of the $\mathcal{H}_\infty$DIA control should be improved by MIMO DIA control system design.

6. Conclusion

This paper dealt with an application of $\mathcal{H}_\infty$ DIA control to the magnetically levitated steel plates. Our goal was to suspend two thin steel plates stably by using four electromagnets without any physical contacts and it was achieved.

We applied the robust $\mathcal{H}_\infty$DIA control approach to the magnetically levitated steel plates and achieved robustly stable suspension for multiple steel plates. Experimental results showed that the proposed robust control approach was effective for suppressing an elasticity vibration of steel plates.

References

(1) T. Nakagawa and M. Hama: "Study of Magnetic Levitation Control by Means of Correcting Gap Length Command for a Thin Steel Plate", *Trans. on IEE Japan*, Vol.120-D, No.4, pp.489-494,(2000) (in Japanese)

(2) M. Morishita and M. Akashi: "Inclination Guide Experiment for Magnetically Levitated Steel Plates", *1988 National Convention Record, IEEJ*, pp.5-183(1998)(in Japanese)

(3) Y. Oshinoya, K. Ishibashi and T. Sekihara: "Conveyance Control for an Electromagnetic Levitation Rectangular Sheet Steel (Proposition of Non-contact Control Mechanism for the Horizontal Inertial Force of the Steel Plate which Suppressed the Elastic Vibration)", *Trans. on JSME. Series C*, Vol.68, no.669, pp.86-92 (2002)(in Japanese)

(4) T. Ishiwatari, M. Watada, S. Torii and D. Ebihara: "Effect of suppressing vibration on thin steel sheet suspended by magnetic levitation control which includes two desired values", *Technical Meeting on Linear Drives, IEEJ, LD-98-37*, pp.43-48 (1998)(in Japanese)

(5) T. Namerikawa, M. Fujita, R.S. Smith and K. Uchida: "On the $\mathcal{H}_\infty$ Control System Design Attenuating Initial State Uncertainties", *Trans. on SICE,* vol.40, no.3. pp. 307-314 (2004)

(6) M. Fujita, T. Namerikawa, F. Matsumuta and K. Uchida: "*mu*-Synthesis of an Electomagnetic Suspension System", *IEEE Trans. on Automatic Control*, Vol.40, No.3, pp.530-536 (1995)

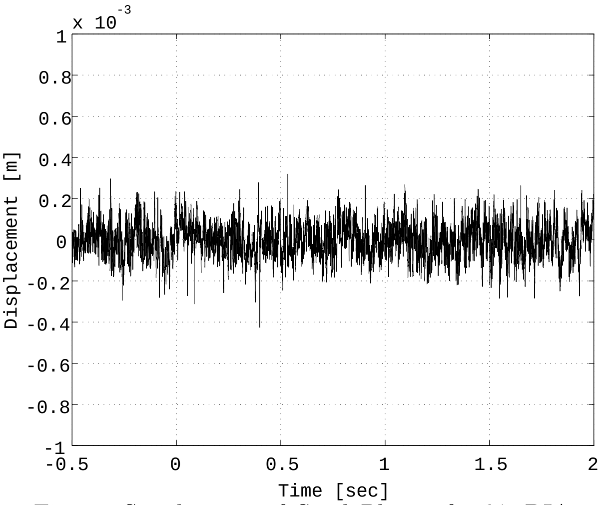

Fig. 5. Steady state of *Steel Plate* 1 for $\mathcal{H}_\infty$DIA

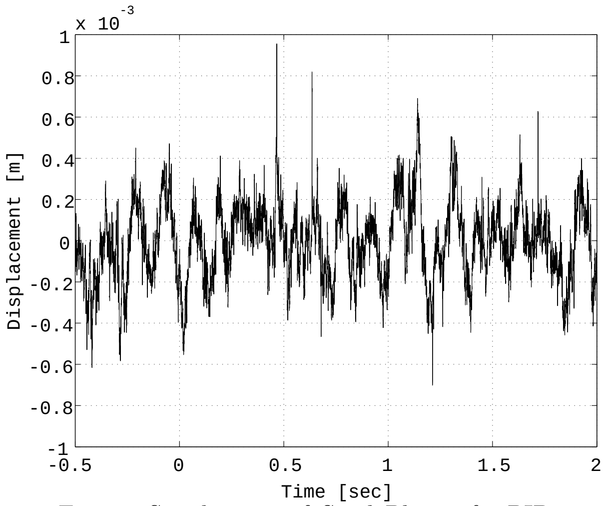

Fig. 8. Steady state of *Steel Plate* 1 for PID

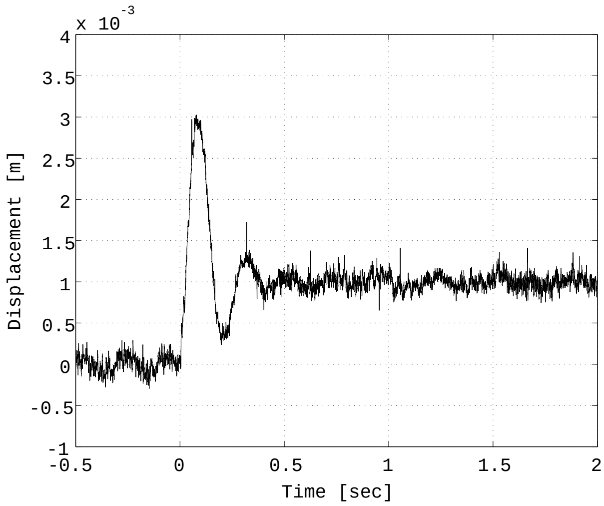

Fig. 6. Step response of *Steel Plate* 1 for $\mathcal{H}_\infty$DIA

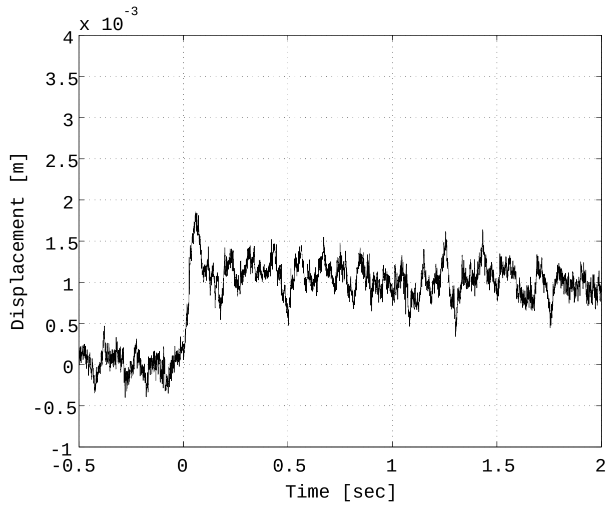

Fig. 9. Step response of *Steel Plate* 1 for PID

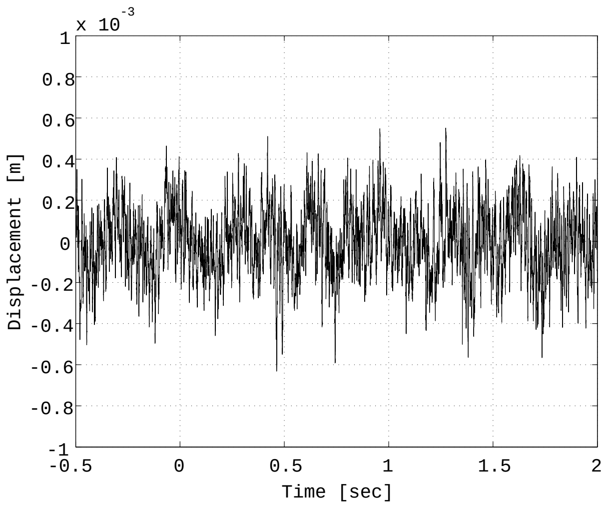

Fig. 7. Steady state of *Steel Plate* 2 for $\mathcal{H}_\infty$DIA

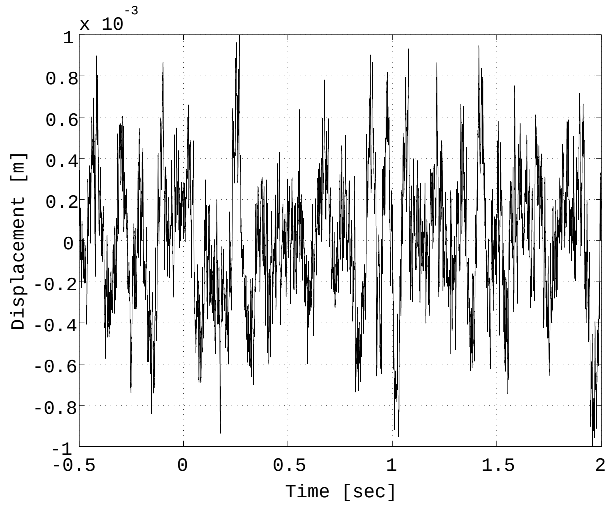

Fig. 10. Steady state of *Steel Plate* 2 for PID