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Robust Control of Master-Slave Robot System Considering Environmental Uncertainties

Ruiqu Lin and Toru Namerikawa

Abstract— **This paper deals with robust control of a master-slave teleoperation robotic system considering environmental uncertainties.**

We construct a master-slave system by using two 2-DOF Direct Drive robot manipulators and design a robust control system via impedance shaping and μ-Synthesis considering various uncertainties; e.g., environment and operator dynamics, perturbation of impedance model and time delay in telecommunications.

The proposed control methodology can guarantee the robust stability and the robust performance for all these uncertainties of the master-slave robotic system. Several experimental results show the effectiveness of our proposed approach for various environmental uncertainties.

I. INTRODUCTION

Master-slave system has been well known as one of the extension of a person's sensing and manipulation capability to a remote location[1], [2].

In bilateral master-slave system, the slave manipulator is controlled by an operator through the master manipulator, while the contact force of the slave manipulator with the environment is reflected back to the operator through the master manipulator[5], [6].

The goal of master-slave system control is to achieve transparency while maintaining stability under any operating condition and for any environment including communication delay[3], [4].

Several previous studies for this robust stability problem in teleoperation have been done[1], [7]et. al.. However the teleoperation problem including multi-degree of freedom robot dynamics and structure uncertainties have not been fully dealt with[5].

In this paper, a robust controller based on μ-synthesis is proposed for impedance shaped 2-DOF direct-drive robot manipulators in bilateral master-slave system with environmental uncertainties and communication delay.

A general condition based on the structured singular value μ for robustness of a bilateral manipulator is derived.

The proposed control methodology can guarantee the robust stability and the robust performance for all these uncertainties of the master-slave system.

Several experimental results show the effectiveness of our proposed approach for various environmental uncertainties.

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II. DYNAMICAL PROPERTY OF MASTER-SLAVE SYSTEM CONSTRUCTED WITH 2-DOF ROBOT MANIPULATORS

Our Master-Slave teleoperation system is illustrated in Fig.1. It is constructed with 2-DOF Direct-Drive robot master manipulator, 2-DOF DD robot slave manipulator, an operator and an Environment. We derive a mathematical model for the Master-Slave teleoperation system with various uncertainties below in this section.

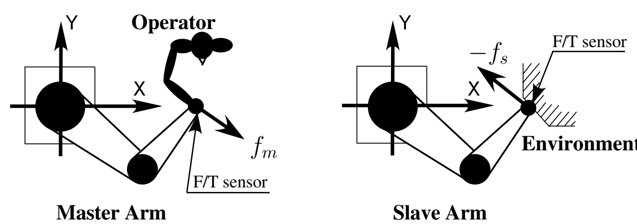

Fig. 1. Master-Slave System with 2 DOF Manipulators

A. Robot Dynamics of 2-DOF Robot Manipulator

Let us consider the robot dynamics first. The robot dynamics of master and slave manipulators are given in these equations respectively.

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta}) + G(\theta) = \tau_m + J^T(\theta)f_m \quad (1)$$

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta}) + G(\theta) = \tau_s - J^T(\theta)f_s \quad (2)$$

where θ, $\dot{\theta}$, $\ddot{\theta}$ denote the link position, velocity and acceleration vectors, respectively, $M(\theta)$ represents the link inertia matrix, $C(\theta, \dot{\theta})$ represents the centrifugal-Coriolis matrix, $G(\theta)$ represents the gravity effects, τ_m, τ_s represent the torque input vectors of master and slave manipulators, respectively, f_m, f_s are the operator force to the master manipulator and the slave force to the environment, respectively, and J is the geometric Jacobian matrix.

Let $X(t) = [x(t)\ y(t)]^T$ denote the vector of the position of the end-effector frame, then the kinematic model of the manipulator gives the relationship between $\theta = [\theta_1\ \theta_2]^T$ and X, and the differential kinematic model gives the relationship between $\dot{\theta}$ and $\dot{X}$, respectively

$$X = P(\theta) \quad (3)$$

$$\dot{X} = J(\theta)\dot{\theta} \quad (4)$$

where $J(\theta)$ is the Jacobian matrix.

The time derivative of (4) gives the acceleration relationship in the form

$$\ddot{\theta} = J^{-1}(\theta)(\ddot{X} - \dot{J}(\theta)\dot{\theta}). \quad (5)$$

B. Linearization of Robot Dynamics by Impedance Shaping

Impedance shaped representation of the master and the slave robot dynamics should be given as in [5].

$$\begin{aligned} M_m\ddot{X}_m + D_m\dot{X}_m + K_mX_m &= Z_m\dot{X}_m \\ &= f_{km} + f_m \end{aligned} \quad (6)$$

$$\begin{aligned} M_s\ddot{X}_s + D_s\dot{X}_s + K_sX_s &= Z_s\dot{X}_s \\ &= f_{ks} - f_s \end{aligned} \quad (7)$$

where the desired master and slave impedances Z_m and Z_s are defined, respectively as follows.

$$Z_m = M_ms + D_m + \frac{K_m}{s} \quad (8)$$

$$Z_s = M_ss + D_s + \frac{K_s}{s} \quad (9)$$

Here M_m, M_s, D_m, D_s, K_m and K_s are all 2×2 real matrices and they are, respectively, the master and slave masses, damping and stiffness matrices in the desired impedance model where we apply the standard assumption $K_m = K_s = 0$ in the following.

f_{km} and f_{ks} denote the master and slave actuated forces, and they are defined as $f_{km} = [f_{kxm}, f_{kym}]^T$ and $f_{ks} = [f_{kxs}, f_{kys}]^T$ in the task space, respectively.

Substitute the equations (6) and (5) to the equation (1) , then the impedance shaping law for the master manipulator is given by

$$\begin{aligned} \tau_m = \; & M(\theta)\{M_mJ(\theta)\}^{-1}\{-M_m\dot{J}(\theta)\dot{\theta} - D_mJ(\theta)\dot{\theta} \\ & + f_m + f_{km}\} + C(\theta,\dot{\theta}) + G(\theta) - J^T(\theta)f_m. \end{aligned} \quad (10)$$

The impedance shaping law for the slave manipulator is also given as

$$\begin{aligned} \tau_s = \; & M(\theta)\{M_sJ(\theta)\}^{-1}\{-M_s\dot{J}(\theta)\dot{\theta} - D_sJ(\theta)\dot{\theta} \\ & - f_s + f_{ks}\} + C(\theta,\dot{\theta}) + G(\theta) + J^T(\theta)f_s. \end{aligned} \quad (11)$$

These impedance shaping laws can realize the ideal mechanical impedance (6)-(9).

C. Operator and Environmental Dynamics

The dynamics of the operator interacting with the master and the dynamics of the environmental interacting with the slave are modeled by the following equations, respectively,

$$Z_{op}(s)\dot{X}_m = f_{op} - f_m \quad (12)$$

$$Z_{env}(s)\dot{X}_s = f_s, \quad (13)$$

where f_{op} is an external force supplied by the operator hand, and the operator hand impedance Z_{op} and the environment impedance Z_{env} are defined, respectively as follows,

$$Z_{op}(s) = M_{op}s + D_{op} + \frac{K_{op}}{s} \quad (14)$$

$$Z_{env}(s) = M_{env}s + D_{env} + \frac{K_{env}}{s}. \quad (15)$$

Here M_{op}, M_{env}, D_{op}, D_{env}, K_{op} and K_{env} are all 2×2 real matrices and they are, respectively, the masses, damping and stiffness matrices in the impedance models.

III. Control System Design

A. Configuration of Feedback Control System

For the feedback linearized and decoupled 2-DOF master and slave robot dynamics, the control system is constructed as shown in Fig.2.

The feedback controller $K_x(s)$ and $K_y(s)$ are independently designed for each X and Y axis in the task space. The all behavior in both X and Y dynamics are same in this problem setup. Then we design the feedback control system for only X axis and apply the obtained controller $K_x(s) = K_y(s)$ to the Y axis control system.

The following treats only the control system design for X axis.

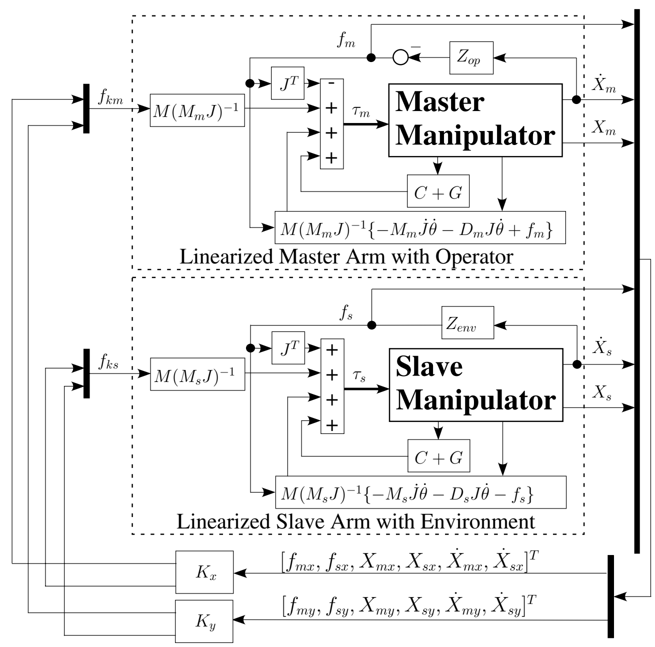

Fig. 2. Feedback Control System

B. Regulation Performance for position and velocity reference

The control performance for position and velocity control problem in the master-slave system is defined in this section.

First we denote the intervening impedance model of the master-slave system in Fig.3[6] and its mathematical model is given in (16).

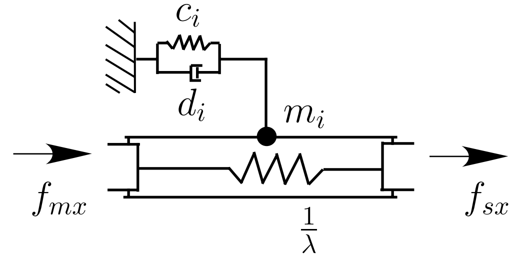

Fig. 3. Intervening Impedance Model

$$f_{mx} - f_{sx} = Z_i(s)v_{ms}$$
$$Z_i(s) = m_i s + d_i + \frac{c_i}{s}, \quad v_{ms} = \frac{v_m + v_s}{2} \tag{16}$$

where $Z_i(s)$ is a desired intervening impedance and m_i, d_i, c_i are the mass, damping and stiffness scalar in the impedance models. v_{ms} is an average of the master velocity and the slave velocity.

Consider the position error e and the desired error of the master-slave system

$$e = x_m - x_s, \quad e_d = \lambda \frac{f_{mx} + f_{sx}}{2}, \tag{17}$$

where x_m and x_s are the master position and the slave position, respectively.

Equation (17) represents a behavior of the error dynamics of the master and slave manipulators and $\lambda (\geq 0)$ is a compliance parameter which can adjust a relative position of the master and slave manipulator[6].

Equation (16) shows a behavior of the average velocity against external relative force, which define a characteristic of the desired intervening impedance $Z_i(s)$.

Let v_{msd} denote the desired velocity as

$$v_{msd} = \frac{1}{Z_i}(f_{mx} - f_{sx}). \tag{18}$$

Consider the control performance indexes e_{rel}, e_{abs} as below. Then the final control performance problem is to find a controller $K_x(s)$ which minimize these two indexes e_{rel}, e_{abs} simultaneously.

$$e_{rel} = e_d - e, \quad e_{abs} = v_{msd} - v_{ms} \tag{19}$$

C. Robustness for perturbation of the impedance model

Let $Z_{mx}(s)$ and $Z_{sx}(s)$ denote the X-axis components of master and slave impedance model $Z_m(s)$ and $Z_s(s)$, respectively.

$Z_{mx}(s)$ and $Z_{sx}(s)$ are defined as

$$Z_{mx}(s) = \hat{Z}_{mx}(s) + \delta Z_{mx}(s) \tag{20}$$
$$Z_{sx}(s) = \hat{Z}_{sx}(s) + \delta Z_{sx}(s), \tag{21}$$

where $\hat{Z}_{mx}(s)$ are $\hat{Z}_{sx}(s)$ nominal values and $\delta Z_{mx}(s)$ are $\delta Z_{sx}(s)$ are perturbations caused by neglected nonlinearities and exogenous disturbances.

Further, δZ_{mx}, δZ_{sx} are defined as

$$\delta Z_{mx}(s) = \delta m_m s + \delta b_m \tag{22}$$
$$\delta Z_{sx}(s) = \delta m_s s + \delta b_s, \tag{23}$$

where δm_m, δm_s, δb_m, δb_s are the mass, damping and stiffness scalar in the impedance models. Assume these values of impedance perturbation are bounded as $|\delta m_m| \leq \Delta m_m$, $|\delta m_s| \leq \Delta m_s$, $|\delta b_m| \leq \Delta b_m$, $|\delta b_s| \leq \Delta b_s$.

Let $W_m(s)$ and $W_s(s)$ denote the weighting functions as

$$W_m(s) = \Delta m_m s + \Delta b_m,$$
$$W_s(s) = \Delta m_s s + \Delta b_s. \tag{24}$$

Finally we have

$$\begin{aligned} |\delta Z_{mx}(j\omega)| &= \sqrt{\delta m_m^2 \omega^2 + \delta b_m^2} \leq \sqrt{\Delta m_m^2 \omega^2 + \Delta b_m^2} \\ &= |W_m(j\omega)|, \quad \forall \omega \in R \end{aligned} \tag{25}$$
$$\begin{aligned} |\delta Z_{sx}(j\omega)| &= \sqrt{\delta m_s^2 \omega^2 + \delta b_s^2} \leq \sqrt{\Delta m_s^2 \omega^2 + \Delta b_s^2} \\ &= |W_s(j\omega)|, \quad \forall \omega \in R \end{aligned} \tag{26}$$

These equations can be represented by using Δ_m, $\Delta_s (\|\Delta_m\|_\infty \leq 1, \|\Delta_s\|_\infty \leq 1)$ as

$$\begin{aligned} Z_{mx}(s) &= \hat{Z}_{mx}(s) + \delta Z_{mx}(s) \\ &= \hat{Z}_{mx} + W_m \Delta_m \end{aligned} \tag{27}$$
$$\begin{aligned} Z_{sx}(s) &= \hat{Z}_{sx}(s) + \delta Z_{sx}(s) \\ &= \hat{Z}_{sx} + W_s \Delta_s \end{aligned} \tag{28}$$

D. Robustness for perturbation of Operator and Environment

Let δZ_{op} and δZ_{env} define the perturbation impedances of the operator impedance $Z_{mx}(s)$ and the environment impedance $Z_{sx}(s)$ and assume they are norm bounded as

$$|\delta Z_{op}(j\omega)| \leq |W_{op}(j\omega)|, \quad \forall \omega \in R \tag{29}$$
$$|\delta Z_{env}(j\omega)| \leq |W_{env}(j\omega)|, \quad \forall \omega \in R. \tag{30}$$

Then $Z_{mx}(s)$ and $Z_{sx}(s)$ can be represented as

$$\begin{aligned} Z_{op}(s) &= \hat{Z}_{op}(s) + \delta Z_{op}(s) \\ &= \hat{Z}_{op} + W_{op} \Delta_{op} \end{aligned} \tag{31}$$
$$\begin{aligned} Z_{env}(s) &= \hat{Z}_{env}(s) + \delta Z_{env}(s) \\ &= \hat{Z}_{env} + W_{env} \Delta_{env} \end{aligned} \tag{32}$$

where $\hat{Z}_{op}(s)$ and $\hat{Z}_{env}(s)$ are nominal models, and Δ_{op} and Δ_{env} are bounded as $\|\Delta_{op}\|_\infty \leq 1$, $\|\Delta_{env}\|_\infty \leq 1$.

E. Robustness for Time Delay

It is well known that the time delay $e^{-j\omega T}$ is infinite-dimensional in polynomial space and cannot be represented exactly in the model. However it can be treated as a multiplicative perturbation of the plant model in the H_∞/μ control framework shown in Fig.4. For all ω and $0 < L < L_{max}$, the following inequality is achieved[3].

$$|e^{-j\omega L} - 1| \leq |\frac{2.1 j\omega}{j\omega + \frac{1}{L_{max}}}| \tag{33}$$

The time delay $e^{-Ls}(0 < L < L_{max})$ can be expressed as a multiplicative uncertainty by the weighting function W_t and the normalized uncertainty $\Delta_t (\|\Delta_t\|_\infty \leq 1)$, where W_t is given by

$$W_t(s) = \frac{2.1 s}{s + \frac{1}{L_{max}}}. \tag{34}$$

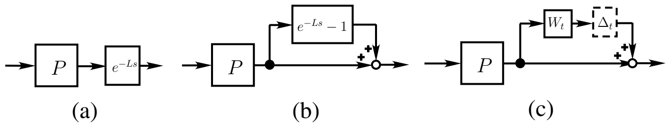

Fig. 4. Time Delay Uncertainty

F. Construction of the Generalized Plant

The above control objectives are itemized as

- Regulation Performance for Position and Velocity Reference
- Robustness for perturbation of the impedance model
- Robustness for perturbation of Operator and Environment
- Robustness for Time Delay

This multiple control objectives can be simultaneously specified in the robust μ-synthesis framework. Consider the generalized plant in Fig.5 to solve this problem, where d_m and d_s are exogenous force disturbances to master and slave manipulators and W_d is an weighting function, $W_{rel}(s)$ and $W_{abs}(s)$ are weighting functions for regulation performance, $W_{um}(s)$ and $W_{us}(s)$ are weights for control inputs and n_v is an exogenous sensor disturbance.

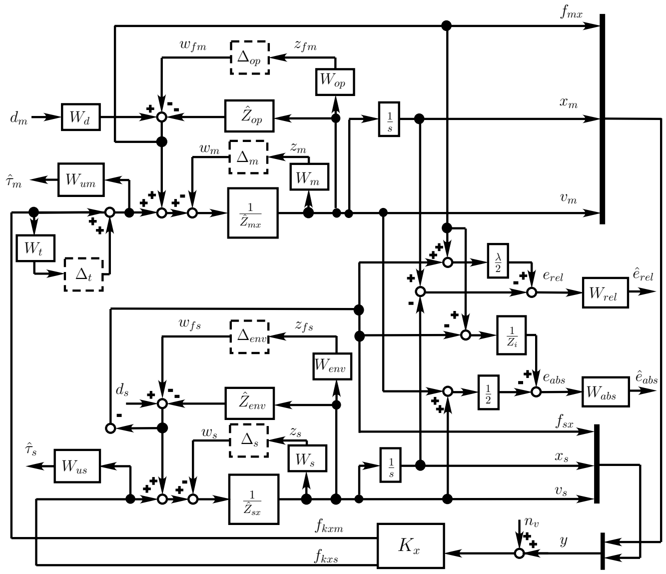

Fig. 5. Generalized Plant with Uncertainties

In order to formulate the robust performance problem, the fictitious performance uncertainty block $\Delta_{perf}(\|\Delta_{perf}\|_\infty \leq 1)$ is introduced, and the generalized plant in Fig.5 is transformed into the LFT form in Fig.6 with the structured uncertainty.

Let Δ_{mss} denote the block structure as

$$\Delta_{mss} = diag[\Delta_{op}, \Delta_{env}, \Delta_m, \Delta_s, \Delta_t, \Delta_{perf}], \tag{35}$$

where there six uncertainties have complex value and appropriate dimensions. The $\mathcal{H}_\infty$ norm of these six uncertainties are normalized.

Then the robust performance condition is given by the following structure singular value μ test.

$$\sup_{\omega \in R} \mu_{\boldsymbol{\Delta_{mss}}}[F_l(P(j\omega), K(j\omega))] < 1 \tag{36}$$

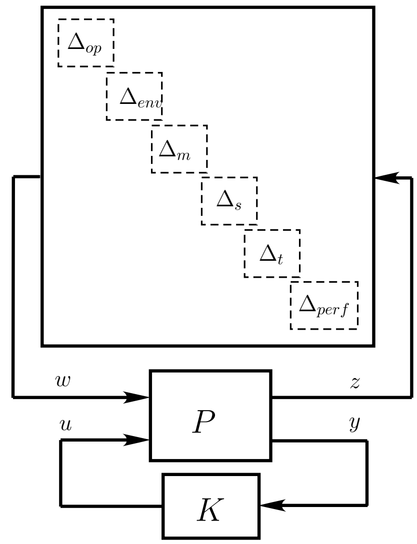

Fig. 6. Robust Performance Framework

IV. Evaluation by Experiments

A. Control System Design

We designed a robust controller by using MATLAB μ-Analysis and Synthesis Toolbox.

First, the parameters for the linearized impedance shaped model are chosen as follows.

$$\begin{aligned}
&m_m = m_s = 2.0[kg],\ d_m = d_s = 0.2[Ns/m],\\
&m_{op} = 1.0[kg],\ d_{op} = 2.0[Ns/m],\\
&k_{op} = 10.0[N/m],\\
&m_{env} = 3.0[kg],\ d_{env} = 1.0[Ns/m],\\
&k_{env} = 100.0[N/m],\\
&m_i = 1.0[kg],\ d_i = 0.01[Ns/m],\ c_i = 0[N/m],\\
&\lambda = 0[m/N]
\end{aligned}$$

The design parameters for the robust control system design are selected based on experimental trial and error and the final set of design parameters is as follows.

$$\begin{aligned}
&W_{rel} = \tfrac{2200}{s+10},\ W_{abs} = \tfrac{80}{s+10},\\
&W_{um} = W_{us} = \tfrac{0.1s+0.01}{s+1000},\\
&W_d = \tfrac{s^2+25}{s^2+2s+25}
\end{aligned}$$

The maximum time delay L_{max} is 10[msec] and 10 % perturbation of the impedance models and operator and environment are considered. We set that the generalized plant in Fig.5 includes this 10[msec] time delay and 10% parametric uncertainties.

After the 2nd D-K iteration, the value of μ of the closed-loop system is less than 1 and the robust performance condition is achieved. The μ plot is shown in Fig.7.

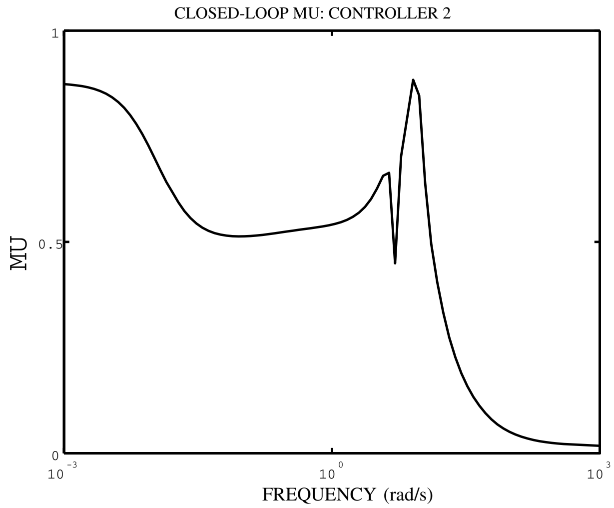

Fig. 7. μ plot of the closed-loop system

B. Experimental Conditions

Fig.8 shows the experimental setup of the spring environment on the slave manipulator. The master manipulator is not connected with the spring. All experiments have been done with 10[msec] time delay in data communication.

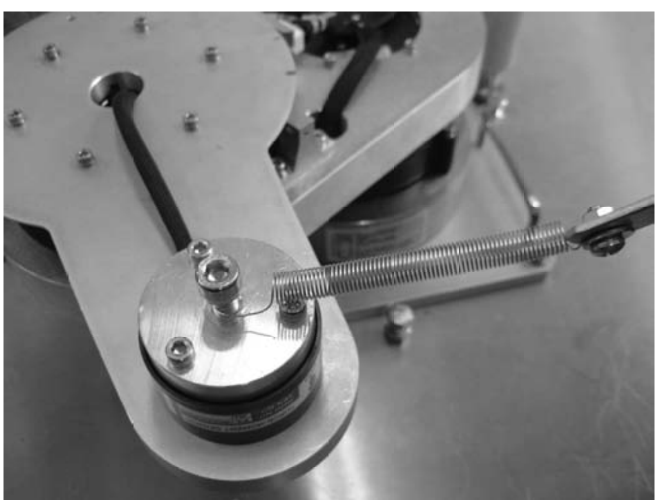

Fig. 8. Slave Manipulator and Spring Environment

Three kinds of experimental environments are given as follows.

- Type 1: Slave Manipulator is free.
- Type 2: Slave Manipulator is restricted by spring T($k_1 = 40.0[N/m]$).
- Type 3: Slave Manipulator is restricted by spring U($k_2 = 110.0[N/m]$).

C. Experimental Results and Discussion

For the comparison, the conventional force reflecting servo type PID controllers are employed[8].

Experimental results on X-axis have the same performance on Y axis in this system, then only results on Y-axis are shown.

Fig.9 shows the results with the Type 1 via the proposed μ controller and the conventional force reflecting servo controller. They show time responses of Y-axis position signals and Y-axis force signals of both of master and slave manipulators. This figure shows that the stability with both of two controllers is guaranteed. Force signals of the slave manipulator is zero all time, because the slave is free in Type 1 environment.

Figs.10 and 11 show the experimental results with Type 2 and Type 3 environment. The conventional force reflecting servo controller shows a big vibration when slave is restricted.

Note that the PID gain is tuned for each environment and three sets of PID gain are chosen in this experimental evaluation. On the other side, a μ controller is used for all three experiments.

Time responses depend on a choice of the intervening impedance model in Fig.3 and the feedback controller. The intervening impedance model with μ controller guarantees the stability by the time delay where the maximum time delay 50[msec] is compensated in Type 3 environment.

Fig.12 summarize all the results given in Figs9-10 and it shows the errors of position signals and force signals between master and slave to compare two controllers.

The μ controller shows a relatively small errors of force signal caused by environmental change. This means μ controller has a better robust performance for environmental uncertainties.

Actually the robust performance is theoretically guaranteed for these uncertainties via the μ controller.

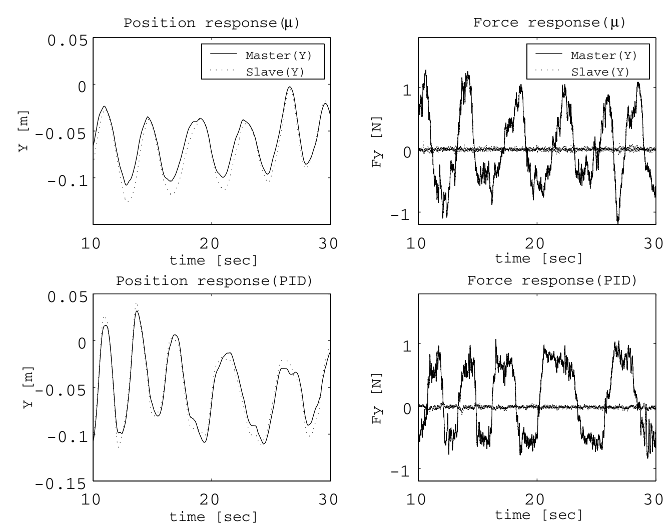

Fig. 9. Experimental Results : Type 1 (Slave Free)

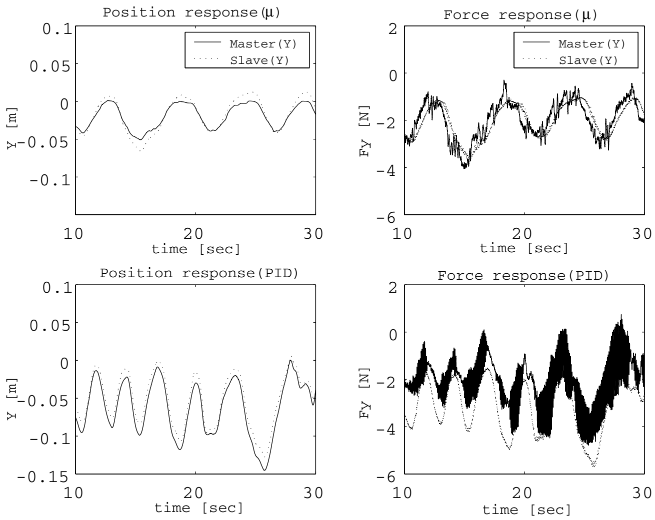

Fig. 10. Experimental Results : Type 2 (Slave Restricted by k_1)

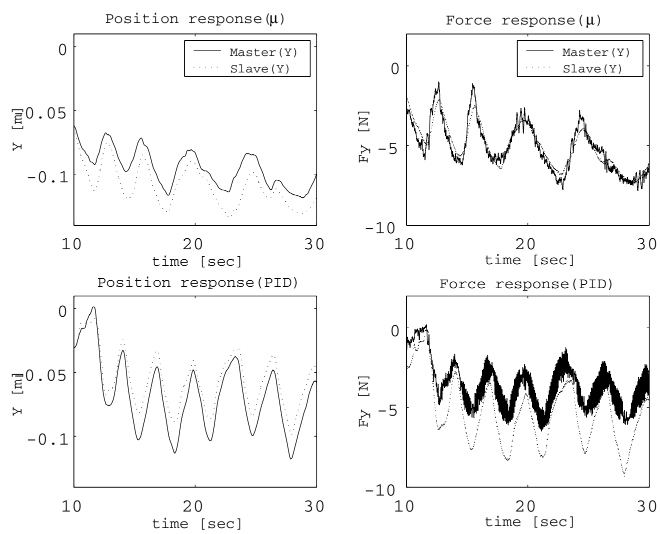

Fig. 11. Experimental Results : Type 3 (Slave Restricted by k_2)

V. Conclusion

In this paper, a robust controller based on μ-synthesis was proposed for impedance shaped 2-DOF direct-drive robot manipulators in bilateral master-slave teleoperation system with environmental uncertainties and communication delay.

A general condition based on the structured singular value μ for the robustness of a bilateral manipulator was derived.

The proposed control methodology can guarantee the robust stability and the robust performance for all 10% parametric model uncertainties and 10[msec] constant time delay of the master-slave system.

Experimental results showed the effectiveness of our proposed approach for various environmental uncertainties.

Future work is to extend this robust control method to guarantee the robust performance for a time varying communication delay[9], [10].

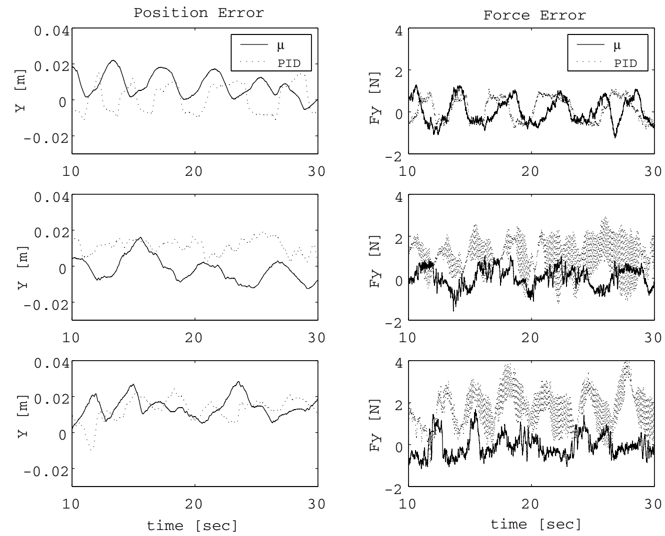

Fig. 12. All Experimental Error Data between Master and Slave Manipulators

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