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Improving the Transient Response of Magnetic Bearings by the $\mathcal{H}_\infty$ DIA Control

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***Abstract*— This paper deals with an application of $\mathcal{H}_\infty$ control attenuating initial-state uncertainties to the magnetic bearing and examines the $\mathcal{H}_\infty$ control problem, which treats a mixed Disturbance and an Initial state uncertainty Attenuation (DIA) control. The mixed $\mathcal{H}_\infty$ DIA problem supplies $\mathcal{H}_\infty$ controls with good transients and assures $\mathcal{H}_\infty$ controls of robustness against initial-state uncertainty. On the other hand, active magnetic bearings allow contract-free suspension of rotors and they are used for various industrial purposes. We derive a mathematical model of the magnetic bearing which has complicated rotor dynamics and nonlinear magnetic property. Then we apply this proposed $\mathcal{H}_\infty$ DIA control for the magnetic bearing, and design a robust $\mathcal{H}_\infty$ controller both for exogenous disturbances and for initial state uncertainties of the plant. Experimental results show that the proposed robust control approach is effective for improving transient response and robust performance.**

I. INTRODUCTION

$\mathcal{H}_\infty$ control problem has been proven an effective robust control design methodology and it has been applied to a variety of industrial products. On the other hand, recent precision control industries and manufacturing technologies requires not only robust stability of the control systems but also transient performance for reference signals. One of the major approach for this problem is a two-degree of freedom robust control, but this approach generally has a coupling problem of feedforward and feedback control design. An H_2/H_∞ control approach[1] seems to be effective, but it is not easy to design such controller for MIMO complex systems.

A mixed Disturbance and an Initial-state uncertainty Attenuation (DIA) control is expected to provide a good transient characteristic as compared with conventional $\mathcal{H}_\infty$ control[2], [3]. Recently, hybrid/switching control are actively studied, this method might be one of the most reasonable approach to implement them. In this paper, we apply the proposed $\mathcal{H}_\infty$ DIA control to the magnetic bearing, and designed a robust $\mathcal{H}_\infty$ controller both for exogenous disturbances and for initial state uncertainties of the plant.

Active magnetic bearings are used to support and maneuver a levitated object, often rotating, via magnetic force[4], [5]. Because magnetic bearings support rotors without physical contacts, they have many advantages, e.g. frictionless operation, less frictional wear, low vibration, quietness, high rotational speed, usefulness in special environments, and low maintenance. On the other hand, disadvantages of magnetic bearings include the expense of the equipment, the necessity of countermeasures in case of a power failure, and instability in their control systems. However, there are many real-world applications which utilize the advantages outlined above. Examples of these applications are : turbomolecular pumps, high-speed spindles for machine tools, flywheels for energy storage[4], reaction wheels for artificial satellites, gas turbine engines, blood pumps[6], and fluid pumps, etc. [5], [7].

In this paper, we apply the $\mathcal{H}_\infty$ control attenuating initial-state uncertainties to the magnetic bearing. First we derive a mathematical model of magnetic bearing systems considering rotor dynamics and nonlinearities of magnetic force. Then we set the generalized plant which contains design parameter for uncertainty and control performance. Experimental results show that the proposed robust control approach is effective for a mixed disturbance and an initial-state uncertainty attenuation and for improving transient response and robust performance.

II. $\mathcal{H}_\infty$ DIA CONTROL

Consider the linear time-invariant system which is defined on the time interval $[0,\infty)$.

$$\begin{aligned}\dot{x} &= Ax + B_1 w + B_2 u, \quad x(0) = x_0\\ z &= C_1 x + D_{12} u\\ y &= C_2 x + D_{21} w\end{aligned} \tag{1}$$

where $x \in R^n$ is the state and $x_0 = x(0)$ is the initial state; $u \in R^r$ is the control input; $y \in R^m$ is the observed output; $z \in R^q$ is the controlled output; $w \in R^p$ is the disturbance. The disturbance $w(t)$ is a square integrable function defined on $[0,\infty)$. A, B_1, B_2, C_1, C_2, D_{12} and D_{21} are constant matrices of appropriate dimensions and satisfies that

- (A, B_1) is stabilizable and (A, C_1) is detectable
- (A, B_2) is controllable and (A, C_2) is observable
- $D_{12}^T D_{12} \in R^{r\times r}$ is nonsingular
- $D_{21} D_{21}^T \in R^{m\times m}$ is nonsingular

For system (1), every admissible control $u(t)$ is given by linear time-invariant system of the form

$$\begin{aligned}u &= J\zeta + Ky\\ \dot{\zeta} &= G\zeta + Hy, \quad \zeta(0) = 0\end{aligned} \tag{2}$$

which makes the closed-loop system given internally stable, where $\zeta(t)$ is the state of the controller of a finite dimension; J, K, G and H are constant matrices of appropriate dimensions. For the system and the class of admissible controls

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described above, consider a mixed-attenuation problem state as below.

Problem 1: $\mathcal{H}_\infty$ **DIA control problem**

Find an admissible control attenuating disturbances and initial state uncertainties in the way that, for given $N > 0$, z satisfies

$$\|z\|_2^2 < \|w\|_2^2 + x_0^T N^{-1} x_0 \tag{3}$$

for all $w \in L^2[0,\infty)$ and all $x_0 \in R^n$, s.t., $(w, x_0) \neq 0$. Such an admissible control is called the **D**isturbance and **I**nitial state uncertainty **A**ttenuation (DIA) control.

In order to solve the DIA control problem, we require the so-called Riccati equation conditions:

(A1) There exists a solution $M > 0$ to the Riccati equation

$$\begin{aligned} &M(A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1) \\ &+(A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1)^T M \\ &-M(B_2(D_{12}^T D_{12})^{-1} B_2^T - B_1 B_1^T) M \\ &+C_1^T C_1 - C_1^T D_{12}(D_{12}^T D_{12})^{-1} D_{12}^T C_1 = 0 \end{aligned} \tag{4}$$

s.t. $A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1 - B_2(D_{12}^T D_{12})^{-1} B_2^T M + B_1 B_1^T M$ is stable.

(A2) There exists a solution $P > 0$ to the Riccati equation

$$\begin{aligned} &(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2) P \\ &+P(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2)^T \\ &-P(C_2^T (D_{21} D_{21}^T)^{-1} C_2 - C_1^T C_1) P \\ &+B_1 B_1^T - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} D_{21} B_1^T = 0 \end{aligned} \tag{5}$$

s.t. $A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2 - P C_2^T (D_{21} D_{21}^T)^{-1} C_2 + P C_1^T C_1$ is stable.

(A3) $\rho(PM) < 1$

where $\rho(X)$ denotes the spectral radius of matrix X, $\rho(X) = \max |\lambda_i(X)|$.

Then we can obtain the following result.

Theorem 1: [2]

Suppose that the conditions (A1), (A2) and (A3) are satisfied, then the central control is given by

$$\begin{aligned} u &= -(D_{12}^T D_{12})^{-1}(B_2^T M + D_{12}^T C_1)(I - PM)^{-1}\zeta \\ \dot{\zeta} &= A\zeta + B_2 u + P C_1^T (C_1 \zeta + D_{12} u) \\ &\quad +(P C_2^T + B_1 D_{21}^T)(D_{21} D_{21}^T)^{-1}(y - C_2 \zeta) \\ \zeta(0) &= 0 \end{aligned} \tag{6}$$

The central control (6) is a DIA control if and only if the condition (A4) is satisfied.

(A4) $Q + N^{-1} - P^{-1} > 0$,

where Q is the maximal solution of the Riccati equation

$$\begin{aligned} &Q(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2 \\ &\qquad + (B_1 B_1^T - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} D_{21} B_1^T) P^{-1}) \\ &\quad +(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2 \\ &\qquad + (B_1 B_1^T - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} D_{21} B_1^T) P^{-1})^T Q \\ &\quad -Q(B_1^T - D_{21}^T (D_{21} D_{21}^T)^{-1}(C_2 P + D_{21} B_1^T) L)^T \\ &\qquad \times (B_1^T - D_{21}^T (D_{21} D_{21}^T)^{-1}(C_2 P + D_{21} B_1^T) L) Q \\ &\quad = 0 \end{aligned} \tag{7}$$

with $L := (I - PM)^{-1}$.

III. SYSTEM DESCRIPTION AND MODELING

The experimental setup of the magnetic suspension system[8] is shown in Fig.1 and rotor coordinate is defined in Fig.2. The controlled plant is a 4-axis controlled type active magnetic bearing with symmetrical structure. The axial motion is not controlled actively. The electromagnets are located in the horizontal and the vertical direction of both sides of the rotor. Moreover, hall-device-type gap sensors are located in the both sides of the vertical and horizontal direction.

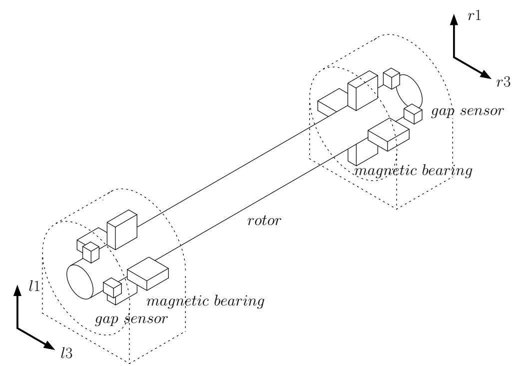

Fig. 1. Magnetic Bearing

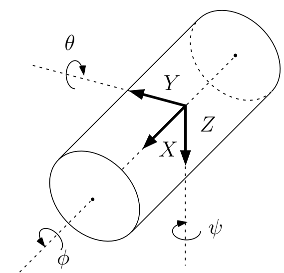

Fig. 2. Rotor

The equation of the motion of the rotor in Y and Z directions in Fig.2 has been derived as follows[5].

$$m\ddot{y}_s = -f_{l3} - v_{ml3} - f_{r3} - v_{mr3} \tag{8}$$

$$m\ddot{z}_s = mg - f_{l1} - v_{ml1} - f_{r1} - v_{mr1} \tag{9}$$

$$J_y\ddot{\theta} = -J_x p\dot{\psi} + lm(f_{l1} + v_{ml1} - f_{r1} - v_{mr1}) \tag{10}$$

$$J_y\ddot{\psi} = -J_x p\dot{\theta} + lm(-f_{l3} - v_{ml3} + f_{r3} + v_{mr3}) \tag{11}$$

where $y_s(t)$ and $z_s(t)$ are displacements of Y direction and Z direction respectively; $\theta(t)$ and $\psi(t)$ are angles about Y direction and Z direction respectively; m is mass of the rotor; g is gravity; l_m is distance between center and electromagnet; J_x and J_y are Moments of Inertia about X axis and Y axis respectively; p is rotation rate of the rotor; f_js are electromagnetic force; and v_{mj}s are exogenous disturbance. Here the subscript 'j' shows the each four directions:$\{l1,\ r1,\ l3,\ r3\}$ in Fig.1.

TABLE I
MODEL PARAMETER

Parameter	Symbol	Value
Mass of the Rotor	m	0.248[kg]
Length of the Rotor	L_R	0.269[m]
Distance between Center and Electromagnet	l_m	0.1105[m]
Moment of Inertia about X	J_x	$5.05 \cdot 10^{-6}$ [kgm^2]
Moment of Inertia about Y	J_y	$1.59 \cdot 10^{-3}$ [kgm^2]
Steady Gap	G	0.4×10^{-3}[m]
Coefficients of $f_j(t)$	k	2.8×10^{-7}
steady Current(vertical)	I_{l1}, I_{r1}	0.1425[A]
steady Current(horizontal)	I_{l3}, I_{r3}	0[A]
Resistance	R	4[Ω]
Inductance	L	8.8×10^{-4}[H]
Steady Voltage(vertical)	E_{l1}, E_{r1}	0.57[V]
Steady Voltage(horizontal)	E_{l3}, E_{r3}	0[V]

The position variables y_s and z_s and the rotational variables θ and ψ can be transformed by using gap lengths: $\{g_{l1}, g_{r1}, g_{l3}, g_{r3}\}$ which are small deviations from the equilibrium point as follows.

$$\begin{aligned} y_s &= -(g_{l3} + g_{r3})/2 && (12)\\ z_s &= -(g_{l1} + g_{r1})/2 && (13)\\ \theta &= (g_{l1} - g_{r1})/2l_m && (14)\\ \psi &= (-g_{l3} + g_{r3})/2l_m && (15) \end{aligned}$$

Next, attractive force of electromagnets is given as followed.

$$f_j = k\frac{(i_j + 0.5)^2}{(g_j - 0.0004)^2} - k\frac{(i_j - 0.5)^2}{(g_j + 0.0004)^2} \qquad (16)$$

The electric circuit equations are given as followed.

$$L\frac{di_j(t)}{dt} + R(I_j + i_j(t)) = E_j + e_j(t) + v_{Lj}(t) \qquad (17)$$

where $i_j(t)$ is a deviation form steady current; $e_j(t)$ is a deviation form steady voltage; v_{Lj} is noise.

The sensors provide the information for the gap lengths $g_j(t)$. Hence the measurement equations can be written as

$$y_j(t) = g_j(t) + w_j \qquad (18)$$

where $w_j(t)$ represents the sensor noise as well as the model uncertainties. Thus, summing up the above results (8)-(18), the state-space equations for the system are

$$\begin{aligned} \begin{bmatrix} \dot{x}_v \\ \dot{x}_h \end{bmatrix} &= \begin{bmatrix} A_v & pA_{vh} \\ -pA_{vh} & A_h \end{bmatrix} \begin{bmatrix} x_v \\ x_h \end{bmatrix} \\ &\quad + \begin{bmatrix} B_v & 0 \\ 0 & B_h \end{bmatrix} \begin{bmatrix} u_v \\ u_h \end{bmatrix} \\ &\quad + \begin{bmatrix} D_v & 0 \\ 0 & D_h \end{bmatrix} \begin{bmatrix} v_v \\ v_h \end{bmatrix} \\ \begin{bmatrix} y_v \\ y_h \end{bmatrix} &= \begin{bmatrix} C_v & 0 \\ 0 & C_h \end{bmatrix} \begin{bmatrix} x_v \\ x_h \end{bmatrix} + \begin{bmatrix} w_v \\ w_h \end{bmatrix} \qquad (19) \end{aligned}$$

$$\begin{aligned} x_v &= [g_{l1}\ g_{r1}\ \dot{g}_{l1}\ \dot{g}_{r1}\ i_{l1}\ i_{r1}]^T \\ x_h &= [g_{l3}\ g_{r3}\ \dot{g}_{l3}\ \dot{g}_{r3}\ i_{l3}\ i_{r3}]^T \\ u_v &= [e_{l1}\ e_{r1}]^T, \quad u_h = [e_{l3}\ e_{r3}]^T \\ v_v &= [v_{ml1}\ v_{mr1}\ v_{Ll1}\ v_{Lr1}]^T \\ v_h &= [v_{ml3}\ v_{mr3}\ v_{Ll3}\ v_{Lr3}]^T \\ y_v &= [y_{l1}\ y_{r1}]^T, \quad y_h = [y_{l3}\ y_{r3}]^T \\ w_v &= [w_{l1}\ w_{r1}]^T, \quad w_h = [w_{l3}\ w_{r3}]^T \\ A_v &:= \begin{bmatrix} 0 & I_2 & 0 \\ K_{x1}A_1 & 0 & K_{i1}A_1 \\ 0 & 0 & -(R/L)I_2 \end{bmatrix} \\ A_h &:= \begin{bmatrix} 0 & I_2 & 0 \\ K_{x3}A_1 & 0 & K_{i3}A_1 \\ 0 & 0 & -(R/L)I_2 \end{bmatrix} \\ A_{vh} &:= \begin{bmatrix} 0 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ B_v &= B_h := \begin{bmatrix} 0 \\ 0 \\ (1/L)I_2 \end{bmatrix} \\ C_v &= C_h := \begin{bmatrix} I_2 & 0 & 0 \end{bmatrix} \\ D_v &= D_h := \begin{bmatrix} 0 & 0 \\ A_1 & 0 \\ 0 & (1/L)I_2 \end{bmatrix} \\ A_1 &:= \begin{bmatrix} 1/m + l_m^2/J_y & 1/m - l_m^2/J_y \\ 1/m - l_m^2/J_y & 1/m + l_m^2/J_y \end{bmatrix} \\ A_2 &:= \begin{bmatrix} J_x/2J_y & -J_x/2J_y \\ -J_x/2J_y & J_x/2J_y \end{bmatrix} \end{aligned}$$

where $I_2 \in R^{2\times 2}$ is unit matrix, and $K_{x1} = K_{xl1} = K_{xr1}$, $K_{x3} = K_{xl3} = K_{xr3}$, $K_{i1} = K_{il1} = K_{ir1}$, $K_{i3} = K_{il3} = K_{ir3}$ in (16), and p is the rotor speed. Here p is equal to 0 and we do not consider a rotation of the rotor in this paper.

The equation (19) can is also expressed simply as

$$\begin{aligned} \dot{x}_g &= A_g x_g + B_g u_g + D_g v_0 \\ y_g &= C_g x_g + w_0 \qquad (20) \end{aligned}$$

where $x_g := [x_v^T\ x_h^T]^T$, $u_g := [u_v^T\ u_h^T]^T$, $v_0 := \begin{bmatrix} v_v^T & v_h^T \end{bmatrix}^T$, $w_0 = \begin{bmatrix} w_v^T & w_h^T \end{bmatrix}^T$ and A_g, B_g, C_g, D_g are constant matrices of appropriate dimensions.

IV. CONTROL SYSTEM DESIGN

Let us construct a generalized plant for the magnetic bearing control system. First, consider the system disturbance v_0. Since v_0 mainly acts on the plant in a low frequency range in practice, it is helpful to introduce a frequency weighting factor. Hence let v_0 be of the form

$$\begin{aligned} v_0 &= W_v(s) w_2 \qquad (21) \\ W_v(s) &= \begin{bmatrix} I_2 & 0 \\ I_2 & 0 \\ 0 & I_2 \\ 0 & I_2 \end{bmatrix} W_{v0}(s) \end{aligned}$$

$$W_{v0}(s) \quad = \quad C_{v0}\left(sI_4 - A_{v0}\right)^{-1} B_{v0}$$

where $W_v(s)$ is a frequency weighting whose gain is relatively large in a low frequency range, and w_2 is a $(1,2)$ element of w . These values, as yet unspecified, can be regarded as free design parameters.

Let us consider the system disturbance w_0 for the output. The disturbance w_0 shows an uncertain influence caused via unmodeled dynamics, and define

$$\begin{aligned} w_0 &= W_w(s)w_1 && (22)\\ W_w(s) &= I_4 W_{w0}(s) \\ W_{w0}(s) &= C_{w0}\left(sI_4 - A_{w0}\right)^{-1} B_{w0} \end{aligned}$$

where $W_w(s)$ is a frequency weighting function and w_1 is a $(1,1)$ element of w. Note that I_4 is unit matrix in $R^{4\times 4}$.

The frequency functions W_v and W_w in (21) and (22) are rewritten as equations in (23) and (24).

$$\begin{aligned} \dot{x}_v &= A_v x_v + B_v w_2 \\ v_0 &= C_v x_v + D_v w_2 && (23)\\ \dot{x}_w &= A_w x_w + B_w w_1 \\ w_0 &= C_w x_w + D_w w_1 && (24) \end{aligned}$$

where the state x_v and x_w are defined as $x_v := \left[x_{v1}^T\ x_{v2}^T\ x_{v3}^T\ x_{v4}^T\right]^T$, $x_w := \left[x_{w1}^T\ x_{w2}^T\ x_{w3}^T\ x_{w4}^T\right]^T$.

Next we consider the variables which we want to regulate. In this case, since our main concern is in the stabilization of the rotor, the gap and the corresponding velocity are chosen; i.e.,

$$\begin{aligned} z_g &= F_g x_g, && (25)\\ F_g &= \begin{bmatrix} I_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_2 & 0 \end{bmatrix} \\ z_1 &= \Theta z_g,\ \ \Theta = \mathrm{diag}\left[\ \theta_1 \quad \theta_2 \quad \theta_1 \quad \theta_2\ \right] && (26) \end{aligned}$$

where Θ is a weighting matrix on the regulated variables z_g, and z_1 is a $(1,1)$ element of z. This value Θ, as yet unspecified, are also free design parameters.

Furthermore the control input u_g should be also regulated, and we define

$$z_2 = \rho u_g \qquad (27)$$

where ρ is a weighting scalar, and z_2 is a $(1,2)$ element of z. Finally, let $x := \left[\ x_g^T \quad x_v^T \quad x_w^T\ \right]^T$, where x_v denotes the state of the function $W_v(s)$, x_w denotes the state of the function $W_w(s)$, and $w := \left[\ w_1^T \quad w_2^T\ \right]^T$, $z := \left[\ z_1^T \quad z_2^T\ \right]^T$, then we can construct the generalized plant as in Fig.3 with an unspecified controller K.

The state-space formulation of the generalized plant is given as follows.

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{12} u \\ y &= C_2 x + D_{21} w && (28) \end{aligned}$$

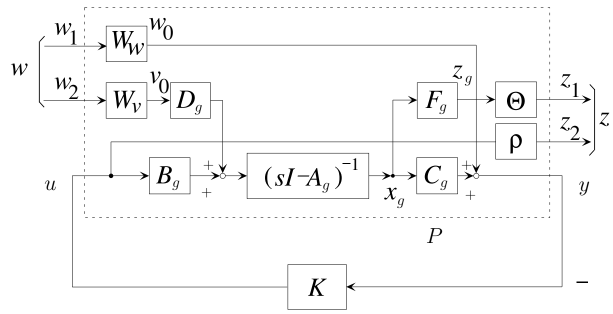

Fig. 3. Generalized Plant

where A, B_1, B_2, C_1, C_2, D_{12} and D_{21} are constant matrices of appropriate dimensions. Since the disturbances w represent the various model uncertainties, the effects of these disturbances on the error vector z should be reduced.

Next our control problem setup is defined as;

Control problemF find an admissible controller $K(s)$ that attenuates disturbances and initial state uncertainties to achieve DIA condition in (3) for generalized plant (28).

After some iteration in MATLAB environment, design parameters are chosen as follows;

$$\begin{aligned} W_{v0}(s) &= \frac{40000}{s+0.1} \\ W_{w0}(s) &= \frac{1.1s^3 + 1.4\cdot 10^4 s^2 + 7.3\cdot 10^7 s + 3.5\cdot 10^{11}}{0.2s^3 + 1.1\cdot 10^4 s^2 + 5.1\cdot 10^6 s + 2.7\cdot 10^{11}} \\ \Theta &= diag\left[\ \theta_{v1} \quad \theta_{v2} \quad \theta_{h1} \quad \theta_{h2}\ \right] \\ \theta_{v1} &= diag\left[\ 0.4 \quad 0.4\ \right], \\ \theta_{h1} &= diag\left[\ 0.5 \quad 0.5\ \right] \\ \theta_{v2} &= \theta_{h2} = diag\left[\ 0.0005 \quad 0.0005\ \right] \\ \rho &= 8.0\cdot 10^{-7} I_4 \end{aligned}$$

Frequency responses of $W_{w0}(s)$ is shown in Fig.4. $W_{w0}(s)$ represents an uncertainty for the 1st bending mode of the rotor at the resonance frequency 800[Hz].

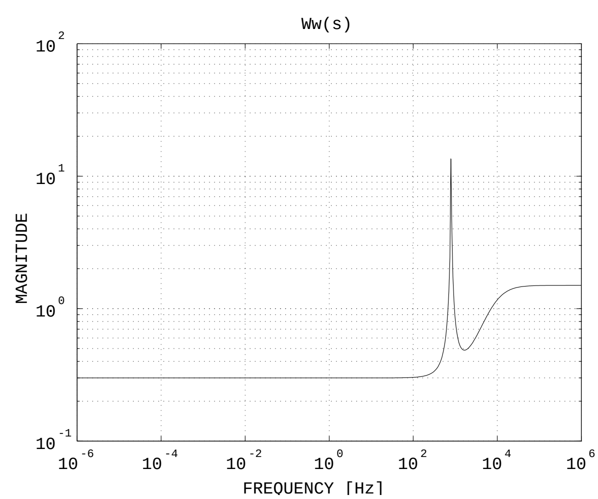

Fig. 4. Frequency Response of $W_{w0}(s)$

Direct calculations yield the 24-order $\mathcal{H}_\infty$ DIA central controller K_{DIA} and its frequency response is shown in Fig.5.

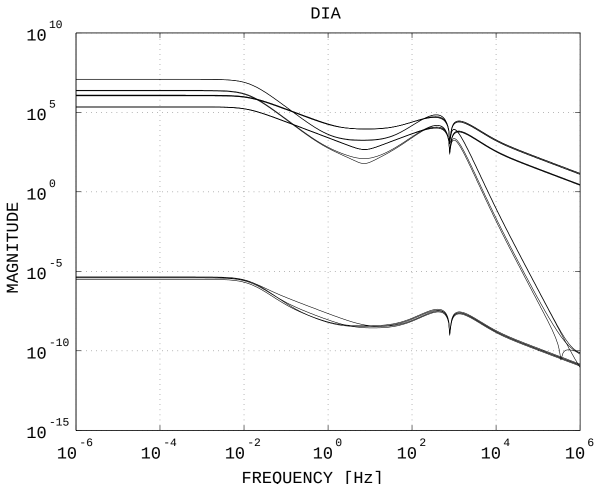

Fig. 5. Frequency Responses of $\mathcal{H}_\infty$DIA Controllers

The maximum value of the weighting matrix N in the DIA condition (3) is given by

$$N = 3.3176 \cdot 10^{-6} \cdot I_{24}. \tag{29}$$

V. EVALUATION BY EXPERIMENTS

We conducted control experiments to evaluate properties of the designed $\mathcal{H}_\infty$ DIA controller compared with an integral-type Optimal State Feedback Control with a state observer and a notch filter. We define this controller as "**LQ** Controller".

The notch filter has a notch at 2000[Hz] and its transfer function is as follows.

$$\frac{s^2 + 1.5791 \times 10^8}{s^2 + 12566s + 1.5791 \times 10^8} \tag{30}$$

The objective of this experimental comparison is to evaluate control performance for transient property, robust performance and initial response for uncertain initial state. The experimental results are shown in Figs. 6-11.

A. Step Responses

Step responses for a reference signal are shown in Fig.6 and Fig.9, where the step size is 0.05[mm] and the steady-state gap is 0.4[mm]. Compared with $\mathcal{H}_\infty$ DIA control, LQ control shows a quick response without any overshoot because LQ control utilizes full state information.

B. Disturbance Responses

Disturbance responses for a step-type disturbance signal with/without model parameter perturbation are shown in Fig.7 and Fig.10. A 60[g] weight is attached to the center of the rotor as a model perturbation and a step-type force disturbance is added to $-l1$ and $-r1$ directions in Fig.1, where the magnitude of the disturbance is $1/6$ steady-state vertical attractive force.

DIA controller shows good disturbance responses and also good robust performance for step-type disturbance and model perturbation.

C. Initial Responses

In Figs.8 and 11, initial responses of two controllers are shown respectively. The initial state is chosen that the rotor is touched down. Four gap lengths are shown in these figures and the $\mathcal{H}_\infty$ DIA controller shows better initial performance.

Finally, compared with LQ control with notch filter, we can see that $\mathcal{H}_\infty$ DIA control has a good robust performance and transient response except for nominal step response from Figs.6-11.

VI. CONCLUSION

This paper dealt with an application of $\mathcal{H}_\infty$ control attenuating initial-state uncertainties to the magnetic bearing and examined the $\mathcal{H}_\infty$ DIA control problem.

First we derived a mathematical model of magnetic bearing systems considering rotor dynamics and nonlinearities of magnetic force. Then we set the generalized plant which contains design parameter for uncertainty and control performance.

Finally, several experimental results of step responses and disturbance responses with model perturbation and initial responses showed that the proposed $\mathcal{H}_\infty$ DIA robust control approach is effective for a mixed disturbance and an initial-state uncertainty attenuation and for improving transient response and robust performance.

Future work is an evaluation of the proposed $\mathcal{H}_\infty$ DIA control via rotational experiments.

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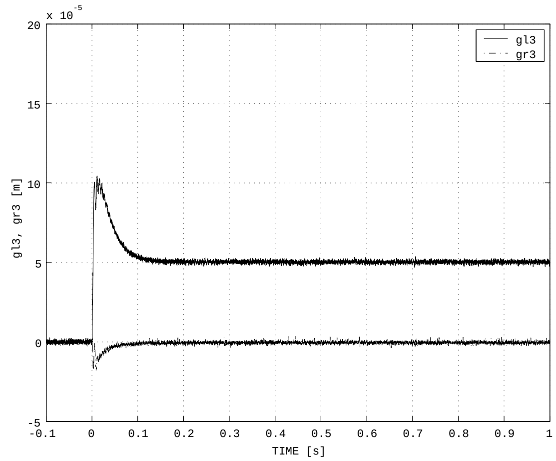

Fig. 6. Step Response of $\mathcal{H}_\infty$ DIA Controller

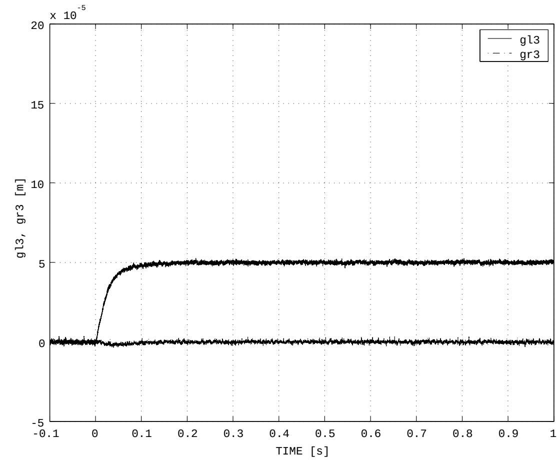

Fig. 9. Step Response of LQ Controller

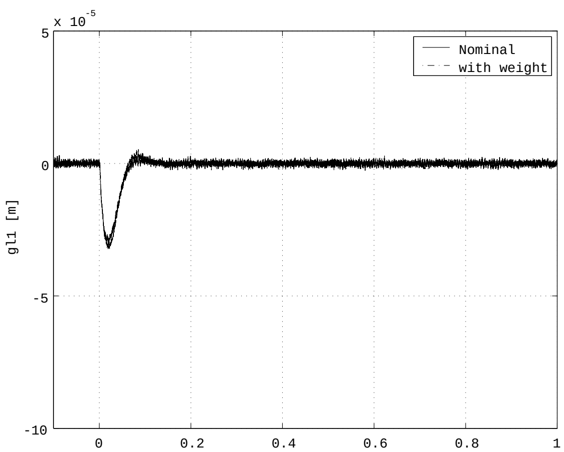

Fig. 7. Disturbance Response of $\mathcal{H}_\infty$DIA Controller with perturbation

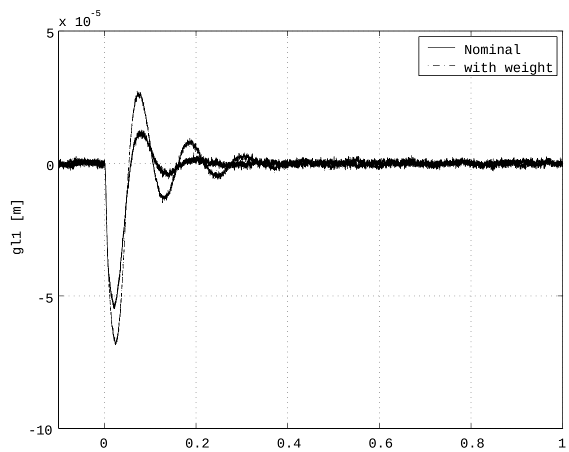

Fig. 10. Disturbance Response of LQ Controller with perturbation

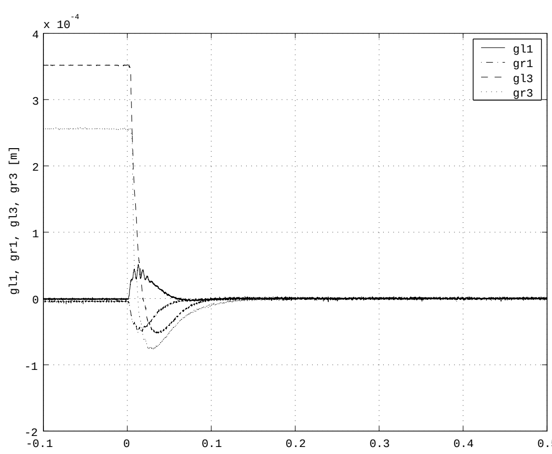

Fig. 8. Initial Response of $\mathcal{H}_\infty$ DIA Controller

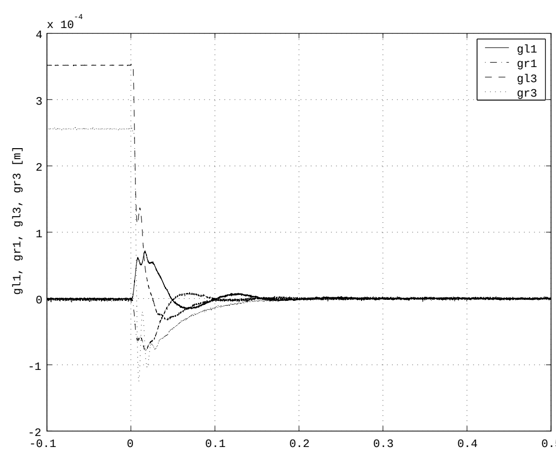

Fig. 11. Initial Response of LQ Controller