

Robust Performance and Transient Response of the H_∞/μ DIA Control for Magnetic Suspension Systems

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Abstract—This paper deals with robust performance and transient response of H_∞/μ DIA control for magnetic suspension systems. H_∞ DIA control is an H_∞ control problem which treats a mixed Disturbance and an Initial-state uncertainty Attenuation(DIA) and supplies H_∞ controls with good transients. H_∞ DIA controller has a good initial response property, however its robust performance might be improved. We propose H_∞/μ DIA Control which is to find a multi-objective controller to achieve both the H_∞ DIA condition for good initial responses/transient responses and the structured singular value μ condition for robust performance. We apply this proposed approach to magnetic suspension systems, and design a robust controller which has both good properties. Finally simulation and experimental results show effectiveness of the proposed control system design framework.

I. INTRODUCTION

Conventional H_∞ control attenuates the only effect of disturbances on controlled outputs and is originally defined under the assumption that the initial states of the system are zero. If the initial states are non-zero, some transient properties with the system applying an H_∞ control will deteriorate. We proposed an H_∞ control which achieves a mixed Disturbance and Initial-state uncertainty Attenuation in controlled outputs[1]. This mixed attenuation H_∞ control (H_∞ DIA Control) has good initial/transient response properties[2], however its robust performance might be improved. There are some previous works on studying about control performance of transient property and robust performance[3], [4]. Yang et. al.[3] utilized adaptive robust nonlinear control and Uchiyama et. al.[4] applied 2-degree of freedom control with μ -synthesis. Both results are effective, but the only problem here is a complexity of their implementations and a fragileness to initial state uncertainties.

To achieve the good transient property and robust performance under the initial state uncertainties of the plant, we apply a D - K iteration technique[5] for improving robust performance to H_∞ DIA control. Here, H_∞/μ DIA control is to find a multi-objective controller to achieve H_∞ DIA condition for good initial responses/transient responses and the structured singular value μ condition for robust performance[6].

This proposed approach is applied to magnetic suspension systems[7] and its effectiveness is evaluated via some control experiments and simulations. Magnetic suspension

systems can suspend a magnetic body by magnetic force without any contact[7], which requires feedback control in order to be workable. Recently, magnetic suspension systems including active magnetic bearings[8] and also magnetic filed control[9] seem to be one of the hot topics in control application field. Nonlinear control approaches are recently focused in this field[3], [8], [9], but our approach taken here is a reliable linear robust control methodology[1], [5].

Finally, compared with the conventional H_∞ DIA controller, usefulness and effectiveness of the proposed H_∞/μ DIA control design framework considering initial-state uncertainty will be shown via some simulation and experimental results for transient responses and for improving robust performance.

II. PROBLEM STATEMENT

Consider the linear time-invariant system which is defined on the time interval $[0, \infty)$ and described by

$$\begin{aligned}\dot{x} &= Ax + B_1w + B_2u, & x(0) &= x_0 \\ z &= C_1x + D_{12}u \\ y &= C_2x + D_{21}w\end{aligned}\quad (1)$$

where $x \in R^n$ is the state and x_0 is the unknown initial state; $u \in R^r$ is the control input; $y \in R^m$ is the observed output; $z \in R^q$ is the controlled output; $w \in R^p$ is the disturbance. Without loss of generality, we regard x_0 as the initial-state uncertainty, and x_0 as a known initial-state case. The disturbance $w(t)$ is a square integrable function defined on $[0, \infty)$. A , B_1 , B_2 , C_1 , C_2 , D_{12} and D_{21} are constant matrices of appropriate dimensions and satisfy that

- (A, B_1) is stabilizable and (A, C_1) is detectable
- (A, B_2) is controllable and (A, C_2) is observable
- $D_{12}^T D_{12} \in R^{r \times r}$ is nonsingular
- $D_{21} D_{21}^T \in R^{m \times m}$ is nonsingular

For system (1), every admissible control $u(t)$ is given by a linear time-invariant system of the form

$$\begin{aligned}\dot{\zeta} &= A_k \zeta + B_k y, & \zeta(0) &= 0 \\ u &= C_k \zeta + D_k y\end{aligned}\quad (2)$$

which makes the closed-loop system given by (1) and (2) internally stable, where $\zeta(t)$ is the state of the controller of a finite dimension; A_k , B_k , C_k and D_k are constant matrices of appropriate dimensions.

For the system and the class of admissible controls described above, consider the H_∞/μ DIA control problem to consider both of transient response and robust performance under the initial state uncertainty of the plant.

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A. H_∞ DIA Control Problem

Find an admissible control attenuating disturbances and initial state uncertainties in the way that, for given $N > 0$, z satisfies

$$\|z\|_2^2 < \|w\|_2^2 + x_0^T N^{-1} x_0 \quad (3)$$

for all $\omega \in L^2[0, \infty)$ and all $x_0 \in R^n$, s.t., $(w, x_0) \neq 0$.

We call such an admissible control the **D**isturbance and **I**nitial state uncertainty **A**ttenuation (**DIA**) control. The weighting matrix N on x_0 is a measure of relative importance of the initial-state uncertainty attenuation to the disturbance attenuation[1].

B. μ -Synthesis Problem

LFT and μ -synthesis[5], [6] have come to play an important role in control system design and proved a uniform frame work for realization, analysis and synthesis for uncertain systems. The block structure Δ is generally defined as

$$\Delta = \text{diag}[\delta_1 I_{r_1}, \dots, \delta_S I_{r_S}, \Delta_1, \dots, \Delta_F] \quad (4)$$

$\delta_i \in \mathcal{R}, \Delta_j \in \mathcal{C}^{m_j \times m_j}$

Here it is well known that the structured singular value $\mu_\Delta(M)$ is defined for matrices $M \in \mathbf{C}^{n \times n}$ with the block structure Δ as

$$\mu_\Delta(M) = \frac{1}{\min\{\bar{\sigma}(\Delta) : \Delta \in \Delta, \det(I - M\Delta) = 0\}} \quad (5)$$

unless no $\Delta \in \Delta$ makes $(I - M\Delta)$ singular, in which case $\mu_\Delta(M) := 0$ [5]. Then the control problem is to find the controller $K(s)$ which achieves the following robust performance condition.

$$\sup_{\omega \in R} \mu_\Delta[F_l(P(j\omega), K(j\omega))] < 1 \quad (6)$$

C. Final Control Problem

Our final control problem is to find an admissible controller to satisfy both H_∞ DIA control condition in (3) and robust performance condition in (6).

III. SYSTEM DESCRIPTION AND MODELING

Consider the electromagnetic suspension system shown schematically in Fig. 1. An electromagnet is located at the top of the experimental system. The control problem is to levitate the iron ball stably utilizing the electromagnetic force. The mass M of the iron ball is 286[g], and steady state gap X is 3[mm]. Note that this simple electromagnetic suspension system is unstable without feedback control. A standard optical gap sensor is placed both sides of the ball to detect the distance between the iron ball and the electromagnet.

Under some assumptions around the steady state operation[7], we can derive the following three equations, which show an equation of the motion of the iron ball(7),

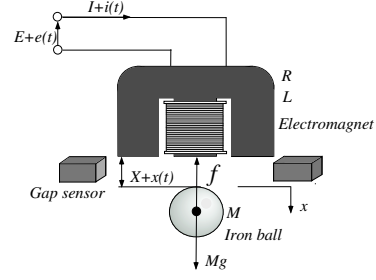


Fig. 1. Magnetic Suspension System

electromagnetic force(8) and equation of an electric circuit of the electromagnet(9) respectively.

$$M \frac{d^2 x(t)}{dt^2} = Mg - f + v_m(t) \quad (7)$$

$$f(t) = k \left(\frac{I + i(t)}{X + x(t) + x_0} \right)^2 \quad (8)$$

$$L \frac{di(t)}{dt} + R(I + i(t)) = E + e(t) + v_L(t) \quad (9)$$

where M is the mass of the iron ball, X is the steady gap between the electromagnet(EM) and the iron ball, $x(t)$ is the deviation from X , I is the steady current, $i(t)$ is the deviation from I , E is the steady voltage, $e(t)$ is the deviation from E , $f(t)$ is the electromagnetic force, k and x_0 are coefficients of $f(t)$ which are determined by experiments, L is an inductance of the EM, R is a resistance of the EM, and $v_m(t), v_L(t)$ are exogenous disturbance and uncertainty. The nominal model parameters of the plant are given in Table I.

In the case we apply the linear control theory with respect to this system and the problem is that the equation of the electromagnetic force(8) is nonlinear concerning $x(t)$ and $i(t)$. Here we utilize the standard linearization approach based on the Taylor series expansion around the operating point.

$$f(t) = k \left(\frac{I}{X + x_0} \right)^2 - K_x x(t) + K_i i(t) \quad (10)$$

where $K_x = 2kI^2/(X + x_0)^3, K_i = 2kI/(X + x_0)^2$.

TABLE I
PHYSICAL MODEL PARAMETERS

Symbol	Parameter Name	Value	Unit
M	Mass of the ball	0.286	kg
X	Steady Gap	3.000×10^{-3}	m
I	Steady Current	0.843	A
E	Steady Voltage	8.47	V
k	coefficient of f	2.14×10^{-4}	Nm ² /A ²
x_0	coefficient of f	4.36×10^{-3}	m
R	Resistance	9.50	Ω
L	Inductance	0.300	H

The gap sensor provides the information for the gap $x(t)$

a noise. Hence the measurement equation for $y_g(t)$ can be written as

$$y_g(t) = x(t) + w_0(t) \quad (11)$$

where $w_0(t)$ represents the sensor noise as well as the model uncertainties.

Moreover, the steady state equations are given by $M_g = k \left(\frac{I}{X+x_0} \right)^2$ and $RI = E$, then summing up the above results, the state equations for the system are

$$\begin{aligned} \dot{x}_g &= A_g x_g + B_g u_g + D_g v_0 \\ y_g &= C_g x_g + w_0 \end{aligned} \quad (12)$$

where $x_g := [x \ \dot{x} \ i]^T$, $u_g := e$, $v_0 := [v_m \ v_L]^T$,

$$A_g = \begin{bmatrix} 0 & 1 & 0 \\ 2670 & 0 & -23.3 \\ 0 & 0 & -31.6 \end{bmatrix}, \quad B_g = [0 \ 0 \ 3.33]^T$$

$$C_g = [1 \ 0 \ 0], \quad D_g = \begin{bmatrix} 0 & 0 \\ 3.50 & 0 \\ 0 & 3.33 \end{bmatrix}$$

Here (A_g, B_g) and (A_g, D_g) are controllable, and (A_g, C_g) is observable.

IV. CONTROL SYSTEM DESIGN

In this section, we apply the H_∞/μ DIA control to the magnetic suspension system and design a control system.

A. Construction of the generalized plant

First let us consider the system disturbance v_0 . Since v_0 mainly acts on the plant in a low frequency range in practice, it is helpful to introduce a frequency weighting factor. Hence let v_0 be of the form

$$v_0 = W_v(s) w_2 \quad (13)$$

$$W_v = \Phi C_v (sI - A_v)^{-1} B_v, \quad \Phi = [1 \ 1]^T \quad (14)$$

where $W_v(s)$ is a frequency weighting whose gain is relatively large in a low frequency range, and w_2 is a (1,2) element of w . Consider the system disturbance w_0 for the output. The disturbance w_0 shows an uncertain influence caused via unmodeled dynamics, and define

$$w_0 = W_w w_1 \quad (15)$$

where W_w is a weighting scalar, and w_1 is a (1,1) element of w . Note that W_w is sometimes frequency dependent, but it is selected as scalar for the sake of simplicity.

Next we consider the variables which we want to regulate. In this case, since our main concern is in the stabilization of the iron ball, the gap $x(t)$ and the corresponding velocity $\dot{x}(t)$ are chosen; i.e.,

$$z_g = F_g x_g, \quad F_g = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (16)$$

Then, as the error vector, let us define as follows

$$z_2 = \rho u_g, \quad \Theta = \text{diag} [\theta_1 \ \theta_2] \quad (17)$$

where Θ is a weighting matrix on the regulated variables

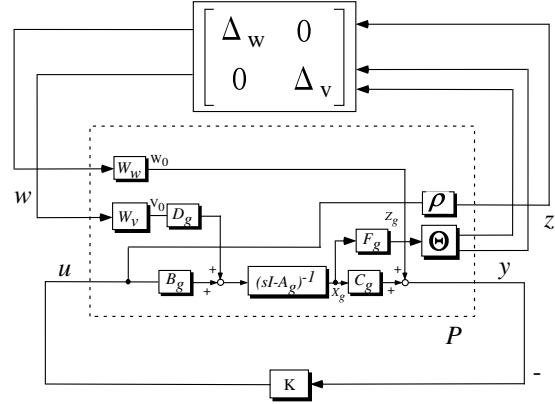


Fig. 2. Generalized Plant with Uncertainty

z_g , and z_2 is a (1,2) element of z . This value Θ , as yet unspecified, are also free design parameters.

Furthermore the control input u should be also regulated, and we define

$$z_1 = \Theta z_g \quad (18)$$

where ρ is a weighting scalar, and z_2 is a (1,2) element of z . Finally, let $x := [x_g^T \ x_v^T]^T$, where x_v denotes the state of the frequency weighting $W_v(s)$, and $w := [w_1^T \ w_2^T]^T$, $z := [z_1^T \ z_2^T]^T$, then we can construct the generalized plant as in the following;

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{12} u \\ y &= C_2 x + D_{21} w \end{aligned} \quad (19)$$

$$\begin{aligned} A &= \begin{bmatrix} A_g & D_g C_v \\ 0 & A_v \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 & 0 \\ 0 & B_v \end{bmatrix}, \\ B_2 &= \begin{bmatrix} B_g \\ 0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0 & 0 \\ \Theta F_g & 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} \rho \\ 0 \end{bmatrix}, \\ C_2 &= \begin{bmatrix} C_g & 0 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} W_w & 0 \end{bmatrix} \end{aligned}$$

B. Problem Setup for Control System Design

Next, for the robust performance synthesis, we define the definite block structure Δ in this system as follows.

$$\Delta := \left\{ \begin{bmatrix} \Delta_w & 0 \\ 0 & \Delta_v \end{bmatrix} : \Delta_w \in \mathcal{C}^{1 \times 1}, \Delta_v \in \mathcal{C}^{1 \times 2} \right\} \quad (20)$$

where Δ_w is an additive perturbation including parametric uncertainty, linearization error and unmodeled dynamics; and Δ_v is defined as $\Delta_v = [\Delta_{v1} \ \Delta_{v2}]$ and is fictitious uncertainty for control performance. The final interconnection structure with an unspecified controller K by LFT representation is shown in Fig. 2.

Control Problem Setup: Find an admissible controller $K(s)$ that achieves both of the DIA condition in (3) and the structured singular value μ condition (6) for the interconnection structure. We call such an admissible controller $K(s)$ μ -DIA controller.

C. Design Procedure of the μ -DIA Controller

We design the μ -DIA controller based on the following Nine-step procedure. Iterative calculations concerning to design parameters and D -scaling matrix are done to obtain appropriate numerical sets on MATLAB, then we obtain a numerical μ -DIA controller $K(s)$ directly.

[Step 1] Select a weighting function $W_v(s)$:

$W_v(s)$ is a frequency weighting function which its gain is relatively large in a low frequency range. This parameter is mutually related to a low gain of the controller K and the controller performance.

[Step 2] Select a weighting function $W_w(s)$:

W_w is a frequency weighting function and this is related to robustness. Bigger choice of W_w could mean allowing bigger uncertainties. Here we selected W_w as a scalar for simplicity, but it can be chosen as a frequency function.

[Step 3] Select a weighting matrix Θ :

Θ is a weighting matrix on the regulated variables z_g which means that θ_1 and θ_2 regulate $x(t)$ and $\dot{x}(t)$ in $x_g(t)$ respectively.

[Step 4] Select a weighting scalar ρ :

ρ is a weighting scalar on the input variable u and ρ regulates input $u(t)$.

[Step 5] Construct the generalized plant and an H_∞ DIA controller:

With a specified set of design parameters from [Step 1] to [Step 4], a generalized plant is constructed. The DIA controller is designed for this plant, and its state-space description is given by easy algebraic calculation.

[Step 6] μ -Analysis:

Calculate μ for $F_l(P_i, K_i)$ and the block structure Δ . Next we can get the scaling matrix $\bar{D}_{i+1}(j\omega)$ to minimize the following function on every frequency ω

$$\bar{\sigma}[\hat{D}_{i+1}(j\omega)F_l(P_i, K_i)(j\omega)\hat{D}_{i+1}^{-1}(j\omega)] \quad (21)$$

Then, evaluate the condition;

$$\sup_{\omega \in \mathcal{R}} \bar{\sigma}[\hat{D}_{i+1}(j\omega)F_l(P_i, K_i)(j\omega)\hat{D}_{i+1}^{-1}(j\omega)] < 1 \quad (22)$$

If the condition(22) is achieved, then this procedure is completed and stopped. Otherwise go to the next step.

[Step 7] Calculate the maximum matrix N :

Calculating the maximum N satisfies the condition(3). For the sake of simplicity, the structure of the matrix N is limited as

$$N = nI \quad (23)$$

where n is a positive scalar number and I is a unit matrix of appropriate dimensions. This limitation on the positive definite matrix N is for easy evaluation after the DIA analysis.

[Step 8] Fix the scaling matrix $D(s)$:

The scaling matrix $\bar{D}_{i+1}(j\omega)D_i(j\omega)$ pointwise across frequency is transformed to the real rational matrix function $D_{i+1}(s)$. This step can be done by graphical matching using lower-order transfer functions.

[Step 9] Reconstruct the generalized plant:

Construct a new state-space model for the new generalized plant

$$P_{i+1} = \begin{bmatrix} D_{i+1} & 0 \\ 0 & 1 \end{bmatrix} P \begin{bmatrix} D_{i+1}^{-1} & 0 \\ 0 & 1 \end{bmatrix} \quad (24)$$

and return to **[Step 5]** and repeat the procedure until the controller K to achieve the conditionD

D. Design of μ -DIA controller

After some iteration in MATLAB environment, these parameters are chosen by the above 9-step design procedure as follows;

$$\begin{aligned} W_v(s) &= \frac{5.0 \times 10^4}{s + 0.010}, & W_w &= 0.3 \\ \Theta &= \begin{bmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{bmatrix} = \begin{bmatrix} 1.0 & 0 \\ 0 & 0.00010 \end{bmatrix} \\ \rho &= 4.0 \times 10^{-7} \end{aligned} \quad (25)$$

We obtained a following controller $K(s)$ after the 2nd D - K iteration, where the peak value of $\mu_\Delta[F_l(P, K)]$ is 0.743 and a constant scaling matrix D is employed.

$$\begin{aligned} K(s) &= \frac{8.496 \times 10^8 (s + (48.68 + 20.39i))}{(s + (330.59 + 655.7i))} \\ &\times \frac{(s + (48.68 - 20.39i))(s + 7.1955)}{(s + (330.59 - 655.7i))(s + 811.19)(s + 0.01)} \end{aligned} \quad (26)$$

The maximum value of the weighting matrix N in (3) is given by

$$N = 4.561157 \times 10^{-3} \times I_4 \quad (27)$$

Calculated upper and lower bounds of $\mu_\Delta[F_l(P, K)]$ and $\bar{\sigma}[DF_l(P_i, K_i)(j\omega)D^{-1}]$ with the controller $K(s)$ in (27) are shown in Fig. 3, where two solid lines show upper and lower bounds of μ and the dashed line shows the maximum singular value respectively. Since the peak value if the upper bound of μ is less than 1 in Fig. 3, the closed-loop system with uncertainties achieves the robust performance condition (6) and also achieve the “**Control Problem Setup**” condition for $N = 4.561157 \times 10^{-3} \times I_4$.

The frequency responses of the controller μ -DIA controller and the conventional H_∞ DIA controller shown in Fig. 4 by a solid line and a dashed line respectively. Fig. 4 shows that both controllers have high gain at the low frequency and good roll-off property at high frequency range. These two controllers are obtained by using the same set of design parameters (25).

V. EVALUATION BY SIMULATIONS AND EXPERIMENTS

In order to evaluate the proposed control design methodology, we implemented the obtained both of μ -DIA and H_∞ DIA controllers via digital control system, and carried out control experiments. The iron ball as a standstill has been suspended stably with both controllers.

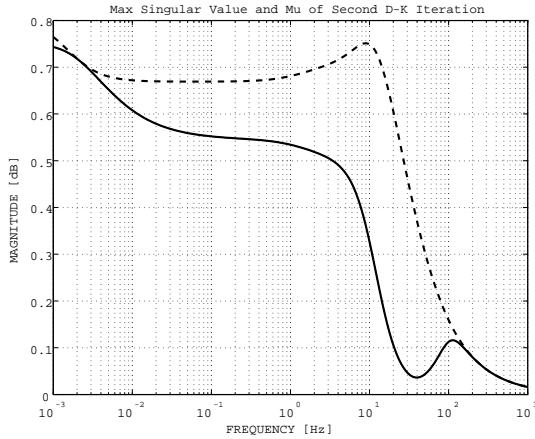


Fig. 3. $\bar{\sigma}$ and μ plots of the second iteration

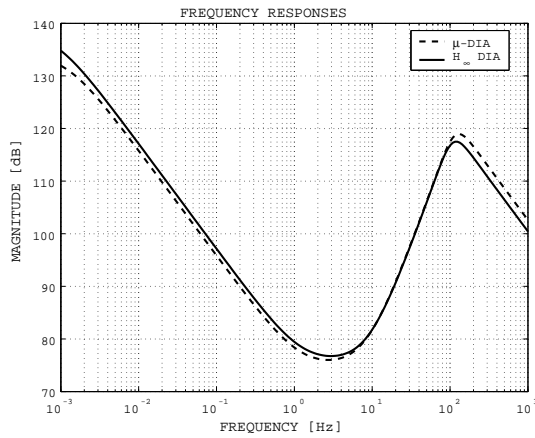


Fig. 4. Frequency Responses

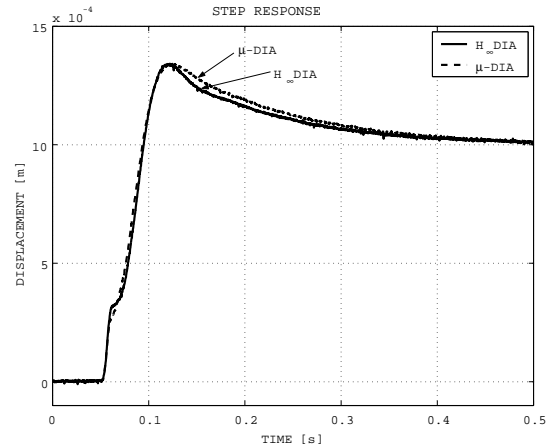


Fig. 5. Step Responses

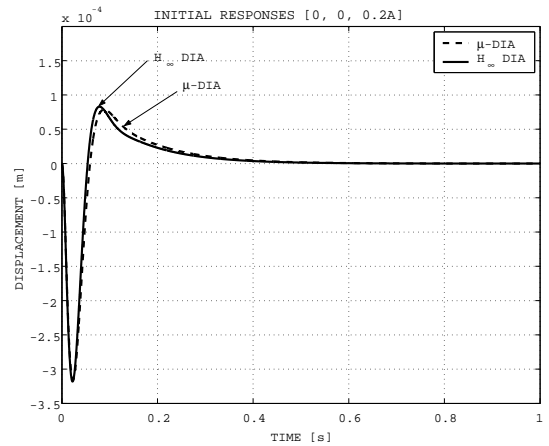


Fig. 6. Initial Responses with Initial-state uncertainty

Transient Response

For evaluation of transient response, a step reference signal is added to the system around 0.05[s], where the magnitude of the step signal is 1.0[mm] and the steady state displacement from the electromagnet to the iron ball is 3[mm]. Experimental results with μ -DIA and H_∞ DIA are shown respectively in Fig. 5.

Compared μ -DIA controller with H_∞ DIA controller, we can see that overshoots are almost same with both controllers, but two settling times are different and H_∞ DIA controller shows a better transient performance. On the other hand, the transient response of μ -DIA controller is getting worse in exchange for robust performance.

Next, we obtain the simulation results of initial responses where initial current is 0.2[A] as an initial-state uncertainty, and results are shown in Fig. 6. We consider an initial-state uncertainty which is the magnitude of maximum control input while the iron ball is suspended stably. From this figure, we can see that rise time and settling time of H_∞ DIA controller are shorter than μ -DIA controller's. It is

obvious that H_∞ DIA controller shows better performance than μ -DIA controller in Fig. 5 and 6.

Robust Performance

Next our concerns are the robust performance comparison of these two controllers. μ -DIA controller is expected to have better robust performance than H_∞ DIA because of the control problem setup in this study. To check robust performance, we changed the suspended iron ball. Three iron balls including the original ball in Table II were used to make model perturbation of the plant.

For the robust performance comparison, step responses of both controllers using these three iron balls are measured

TABLE II
MASS CHANGE OF THE IRON BALL

	Mass of the Ball [g]	Varying Rate
0	286(nominal val.)	0%
1	440	+54%
2	534	+87%

TABLE III
OVERSHOOT COMPARISON IN TWO CONTROLLERS

	Varying Rate[%]	
	440[g]	534[g]
H_∞ DIA	4.48	5.97
μ -DIA	1.49	2.99

and the obtained experimental results are shown in Fig. 7 and 8.

Then we find that the overshoot of the H_∞ DIA controller is getting bigger than μ -DIA controller according to an increase in mass of the iron ball. However, the influence of mass change is kept down relatively in μ -DIA case. The overshoots changed with both controllers are indicated in Table III.

Each numerical value shows a rate[%] of change of the overshoot based on the nominal response. The μ -DIA is robust to change in the mass M of the iron ball as recorded in Table III. Thus μ -DIA would be considered to achieve robust performance.

From the above 2-types of control experiments, μ -DIA controller would not have a bad transient response property and have a better robust performance compared with the conventional H_∞ DIA controller. It can be considered that μ -DIA controller have both a good transient performance of H_∞ DIA control and a good robust performance of μ -synthesis.

VI. CONCLUSION

In this paper, we applied a D - K iteration technique for improving robust performance to H_∞ DIA control and employed a μ -analysis to check the robust performance condition. Here, H_∞/μ DIA control is to find a multi-objective controller to achieve both the H_∞ DIA condition for good initial responses/transient responses and the structured singular value μ condition for robust performance. This proposed approach was applied to the magnetic suspension system and its transient response, initial response and robust performance were evaluated via several control experiments and simulations.

Finally, compared with the H_∞ DIA controller, usefulness and effectiveness of the proposed H_∞/μ DIA control design framework considering initial-state uncertainty were shown for transient responses and for improving robust performance.

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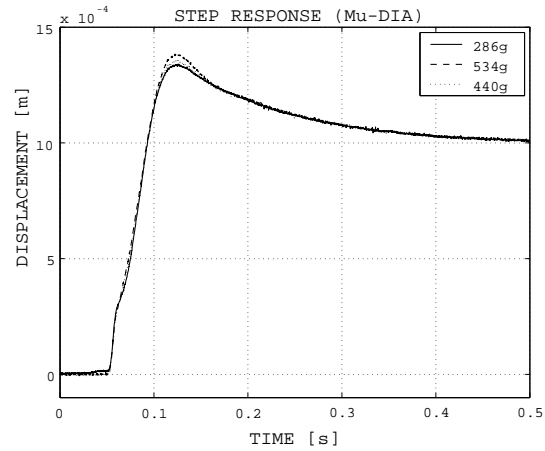


Fig. 7. Step Responses with Mass Change(μ -DIA)

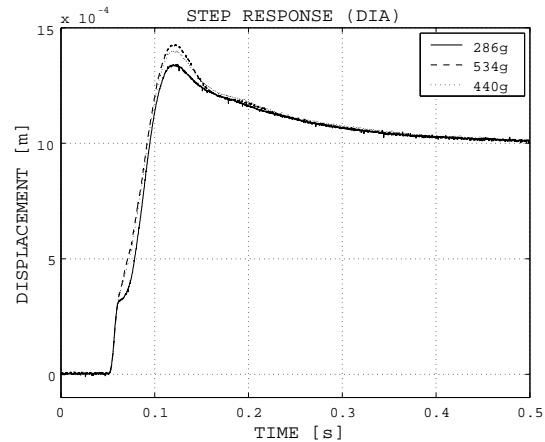


Fig. 8. Step Responses with Mass Change(H_∞ DIA)

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