# $H_\infty$ Control of the Magnetic Bearing Considering Initial State Uncertainties

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***Abstract*— This paper deals with an application of $H_\infty$ control attenuating initial-state uncertainties to the magnetic bearing and examines the $H_\infty$ control problem, which treats a mixed Disturbance and an Initial-state uncertainty Attenuation(DIA) control. The mixed $H_\infty$ DIA problem supplies $H_\infty$ controls with good transients and assures $H_\infty$ controls of robustness against initial-state uncertainty. On the other hand, active magnetic bearings allow contract-free suspension of rotors and they are used for various industrial purposes. We derive a mathematical model of the magnetic bearing which has complicated rotor dynamics and nonlinear magnetic property. Then we apply this proposed $H_\infty$DIA control for the magnetic bearing, and design a robust $H_\infty$ controller both for exogenous disturbances and for initial state uncertainties of the plant. Experimental results show that the proposed robust control approach is effective for improving transient response and robust performance.**

## I. Introduction

It has been proven that $H_\infty$ control problem is an effective robust control design methodology and applied to a variety of industrial products. On the other hand, recent precision control industries and manufacturing technologies requires not only robust stability of the control systems but also transient performance for reference signals. One of the major approach for this problem is a two-degree of freedom robust control. But this approach generally has a coupling problem of feedforward and feedback control design. An $H_2/H_\infty$ control approach[1] seems to be effective, but it is not easy to design such controller for MIMO complex systems.

A mixed Disturbance and an Initial-state uncertainty Attenuation (DIA) control is expected to provide a good transient characteristic as compared with conventional $H_\infty$ control[2], [3]. Recently, hybrid/switching control are actively studied, this method might be one of the most reasonable approach to implement them. In this paper, we apply the proposed $H_\infty$ DIA control to the magnetic bearing, and designed a robust $H_\infty$ controller both for exogenous disturbances and for initial state uncertainties of the plant.

Active magnetic bearings are used to support and maneuver a levitated object, often rotating, via magnetic forces[4], [5]. Because magnetic bearings support rotors without physical contact, they have many advantages, e.g. frictionless operation, less frictional wear, low vibration, quietness, high rotational speed, usefulness in special environments, and low maintenance. On the other hand, disadvantages of magnetic bearings include the expense of the equipment, the necessity of countermeasures in case of a power failure, and instability in their control systems. However, there are many real-world applications which utilize the advantages outlined above. Examples of these applications are : turbo-molecular pumps, high-speed spindles for machine tools, flywheels for energy storage[4], reaction wheels for artificial satellites, gas turbine engines, blood pumps[6], and fluid pumps, etc. [5], [7].

In this paper, we apply the $H_\infty$ control attenuating initial-state uncertainties to the magnetic bearing. First we derive a mathematical model of magnetic bearing systems considering rotor dynamics and nonlinearities of magnetic force. Then we set the generalized plant which contains design parameter for uncertainty and control performance. Experimental results show that the proposed robust control approach is effective for a mixed disturbance and an initial-state uncertainty attenuation and for improving transient response and robust performance.

## II. $H_\infty$DIA control

Consider the linear time-invariant system which is defined on the time interval $[0, \infty)$.

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u, \quad x(0) = x_0 \\ z &= C_1 x + D_{12} u \\ y &= C_2 x + D_{21} w \end{aligned} \tag{1}$$

where $x \in R^n$ is the state and $x_0 = x(0)$ is the initial state; $u \in R^r$ is the control input; $y \in R^m$ is the observed output; $z \in R^q$ is the controlled output; $w \in R^p$ is the disturbance. The disturbance $w(t)$ is a square integrable function defined on $[0, \infty)$. $A$, $B_1$, $B_2$, $C_1$, $C_2$, $D_{12}$ and $D_{21}$ are constant matrices of appropriate dimensions and satisfies that

- $(A, B_1)$ is stabilizable and $(A, C_1)$ is detectable
- $(A, B_2)$ is controllable and $(A, C_2)$ is observable
- $D_{12}^T D_{12} \in R^{r\times r}$ is nonsingular
- $D_{21} D_{21}^T \in R^{m\times m}$ is nonsingular

For system (1), every admissible control $u(t)$ is given by linear time-invariant system of the form

$$u = J\zeta + Ky$$

$$\dot{\zeta} = G\zeta + Hy, \quad \zeta(0) = 0 \tag{2}$$

which makes the closed-loop system given internally stable, where $\zeta(t)$ is the state of the controller of a finite dimension; $J$, $K$, $G$ and $H$ are constant matrices of appropriate dimensions. For the system and the class of admissible controls described above, consider a mixed-attenuation problem state as below.

**Problem 1** $H_\infty$ **DIA control problem**
*Find an admissible control attenuating disturbances and initial state uncertainties in the way that, for given $N > 0$, $z$ satisfies*

$$\|z\|_2^2 < \|w\|_2^2 + x_0^T N^{-1} x_0 \tag{3}$$

*for all $w \in L^2[0,\infty)$ and all $x_0 \in R^n$, s.t., $(w, x_0) \neq 0$.*

Such an admissible control is called the **D**isturbance and **I**nitial state uncertainty **A**ttenuation (DIA) control.

In order to solve the DIA control problem, we require the so-called Riccati equation conditions:

**(A1)** There exists a solution $M > 0$ to the Riccati equation

$$\begin{aligned} &M(A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1) \\ &+(A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1)^T M \\ &-M(B_2(D_{12}^T D_{12})^{-1} B_2^T - B_1 B_1^T) M \\ &+C_1^T C_1 - C_1^T D_{12}(D_{12}^T D_{12})^{-1} D_{12}^T C_1 = 0 \end{aligned} \tag{4}$$

such that

$$\begin{aligned} &A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1 \\ &\quad -B_2(D_{12}^T D_{12})^{-1} B_2^T M + B_1 B_1^T M \end{aligned} \tag{5}$$

is stable.
**(A2)** There exists a solution $P > 0$ to the Riccati equation

$$\begin{aligned} &(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2) P \\ &+P(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2)^T \\ &-P(C_2^T (D_{21} D_{21}^T)^{-1} C_2 - C_1^T C_1) P \\ &+B_1 B_1^T - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} D_{21} B_1^T = 0 \end{aligned} \tag{6}$$

such that

$$\begin{aligned} &A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2 \\ &\quad -PC_2^T (D_{21} D_{21}^T)^{-1} C_2 + PC_1^T C_1 \end{aligned} \tag{7}$$

is stable.
**(A3)** $\rho(PM) < 1$
where $\rho(X)$ denotes the spectral radius of matrix $X$, $\rho(X) = \max |\lambda_i(X)|$.

Then we can obtain the following result.

**Theorem 1** *[2]*
*Suppose that the conditions (A1), (A2) and (A3) are satisfied, then the central control is given by*

$$\begin{aligned} u &= -(D_{12}^T D_{12})^{-1}(B_2^T M + D_{12}^T C_1)(I - PM)^{-1}\zeta \\ \dot{\zeta} &= A\zeta + B_2 u + PC_1^T (C_1\zeta + D_{12} u) \\ &\quad +(PC_2^T + B_1 D_{21}^T)(D_{21} D_{21}^T)^{-1}(y - C_2\zeta) \\ \zeta(0) &= 0 \end{aligned} \tag{8}$$

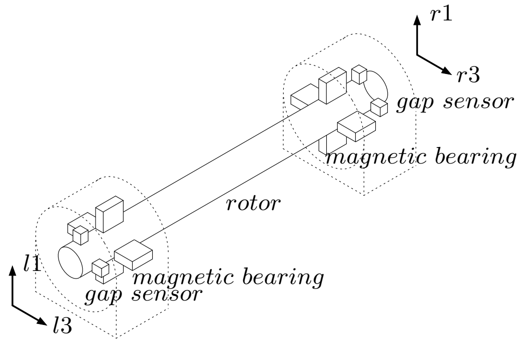


Fig. 1. Magnetic Bearing.

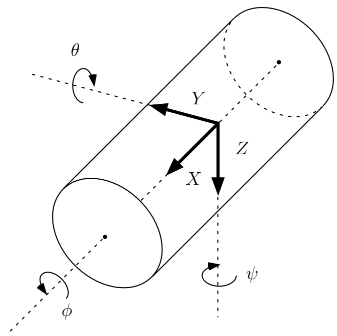


Fig. 2. Rotor

*The central control (8) is a DIA control if and only if the condition (A4) is satisfied.*

**(A4)** $Q + N^{-1} - P^{-1} > 0$,

where $Q$ is the maximal solution of the Riccati equation

$$\begin{aligned} &Q(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2 \\ &\qquad + (B_1 B_1^T - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} D_{21} B_1^T) P^{-1}) \\ &\quad +(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2 \\ &\qquad + (B_1 B_1^T - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} D_{21} B_1^T) P^{-1})^T Q \\ &\quad -Q(B_1^T - D_{21}^T (D_{21} D_{21}^T)^{-1}(C_2 P + D_{21} B_1^T) L)^T \\ &\qquad \times (B_1^T - D_{21}^T (D_{21} D_{21}^T)^{-1}(C_2 P + D_{21} B_1^T) L) Q \\ &\quad = 0 \end{aligned} \tag{9}$$

with $L := (I - PM)^{-1}$.

## III. System Description and Modeling

The experimental setup of the magnetic suspension system[8] is shown in Fig.1 and rotor coordinate is defined in Fig.2. The controlled plant is a 4-axis controlled type active magnetic bearing with symmetrical structure. The axial motion is not controlled actively. The electromagnets are located in the horizontal and the vertical direction of both sides of the rotor. Moreover, hall-device-type gap sensors are located in the both sides of the vertical and horizontal direction.

In order to derive a nominal model of the system, the following assumptions are introduced[5].

- The rotor is rigid and has no unbalance.
- All electromagnets are identical.
- Attractive force of an electromagnet is in proportion to (electric current / gap length)$^2$.

TABLE I

MODEL PARAMETER

| Parameter | Symbol | Value |
|---|---|---|
| Mass of the Rotor | $m$ | 0.248[kg] |
| Length of the Rotor | $L_R$ | 0.269[m] |
| Distance between Center and Electromagnet | $l_m$ | 0.1105[m] |
| Moment of Inertia about $X$ | $J_x$ | $5.053 \times 10^{-6}$[kgm$^2$] |
| Moment of Inertia about $Y$ | $J_y$ | $1.585 \times 10^{-3}$[kgm$^2$] |
| Steady Gap | $G$ | $0.4 \times 10^{-3}$[m] |
| Coefficients of $f_j(t)$ | $k$ | $2.8 \times 10^{-7}$ |
| steady Current(vertical) | $I_{l1}, I_{r1}$ | 0.1425[A] |
| steady Current(horizontal) | $I_{l3}, I_{r3}$ | 0[A] |
| Resistance | $R$ | 4[Ω] |
| Inductance | $L$ | $8.8 \times 10^{-4}$[H] |
| Steady Voltage(vertical) | $E_{l1}, E_{r1}$ | 0.57[V] |
| Steady Voltage(horizontal) | $E_{l3}, E_{r3}$ | 0[V] |

- The resistance and the inductance of the electromagnet coil are constant and independent of the gap length.
- Small deviations from the equilibrium point are treated.

These assumptions are not strong and suitable around the steady state operation, but if the rotor spins at super-high speed, these assumption will be failed. Based on the above assumptions, the equation of the motion of the rotor in $Y$ and $Z$ directions in Fig.2 has been derived as follows[5].

$$m\ddot{y}_s = -f_{l3} - v_{ml3} - f_{r3} - v_{mr3} \quad (10)$$

$$m\ddot{z}_s = mg - f_{l1} - v_{ml1} - f_{r1} - v_{mr1} \quad (11)$$

$$J_y\ddot{\theta} = -J_x p\dot{\psi} + lm(f_{l1} + v_{ml1} - f_{r1} - v_{mr1}) \quad (12)$$

$$J_y\ddot{\psi} = -J_x p\dot{\theta} + lm(-f_{l3} - v_{ml3} + f_{r3} + v_{mr3}) \quad (13)$$

where $y_s(t)$ and $z_s(t)$ are displacements of $Y$ direction and $Z$ direction respectively; $\theta(t)$ and $\psi(t)$ are angles about $Y$ direction and $Z$ direction respectively; $m$ is mass of the rotor; $g$ is gravity; $l_m$ is distance between center and electromagnet; $J_x$ and $J_y$ are Moments of Inertia about $X$ axis and $Y$ axis respectively; $p$ is rotation rate of the rotor; $f_j$s are electromagnetic force; and $v_{mj}$s are exogenous disturbance. Here the subscript '$j$' shows the each four directions:$\{l1, r1, l3, r3\}$ in Fig.1.

The position variables $y_s$ and $z_s$ and the rotational variables $\theta$ and $\psi$ can be transformed by using gap lengths: $\{g_{l1}, g_{r1}, g_{l3}, g_{r3}\}$ which are small deviations from the equilibrium point as follows.

$$y_s = -(g_{l3} + g_{r3})/2 \quad (14)$$

$$z_s = -(g_{l1} + g_{r1})/2 \quad (15)$$

$$\theta = (g_{l1} - g_{r1})/2l_m \quad (16)$$

$$\psi = (-g_{l3} + g_{r3})/2l_m \quad (17)$$

The straightforward calculations of the above equations (10), (11), (12) and (13) give the following.

$$\ddot{g}_{l1} = -\ddot{z} + l_m\ddot{\theta} \quad (18)$$

$$\ddot{g}_{r1} = -\ddot{z} - l_m\ddot{\theta} \quad (19)$$

$$\ddot{g}_{l3} = -\ddot{y} + l_m\ddot{\psi} \quad (20)$$

$$\ddot{g}_{r3} = -\ddot{y} - l_m\ddot{\psi} \quad (21)$$

Attractive force of electromagnets is given by assumptions.

$$f_j = k\frac{(i_j + 0.5)^2}{(g_j - 0.0004)^2} - k\frac{(i_j - 0.5)^2}{(g_j + 0.0004)^2} \quad (22)$$

Next we linearize the electromagnetic force (22) around the operating point by the Taylor series expansions as

$$\begin{aligned} f_j &= k\frac{(I_j + 0.5)^2 - (I_j - 0.5)^2}{1.6 \times 10^{-7}} \\ &\quad + K_{xj} g_j + K_{ij} i_j \\ K_{xj} &= -2k\left(\frac{(I_j + 0.5)^2}{(-4 \times 10^{-4})^3} + \frac{(I_j - 0.5)^2}{(4 \times 10^{-4})^3}\right) \\ K_{ij} &= 2k\left(\frac{(I_j + 0.5)}{(-4 \times 10^{-4})^2} - \frac{(I_j - 0.5)}{(4 \times 10^{-4})^2}\right). \end{aligned} \quad (23)$$

The electric circuit equations are given as followed.

$$L\frac{di_j(t)}{dt} + R(I_j + i_j(t)) = E_j + e_j(t) + v_{Lj}(t) \quad (24)$$

where $i_j(t)$ is a deviation form steady current; $e_j(t)$ is a deviation form steady voltage; $v_{Lj}$ is noise.

The sensors provide the information for the gap lengths $g_j(t)$. Hence the measurement equations can be written as

$$y_j(t) = g_j(t) + w_j \quad (25)$$

where $w_j(t)$ represents the sensor noise as well as the model uncertainties.

Thus, summing up the above results (18)-((25), the state-space equations for the system are

$$\begin{aligned} \begin{bmatrix} \dot{x}_v \\ \dot{x}_h \end{bmatrix} &= \begin{bmatrix} A_v & pA_{vh} \\ -pA_{vh} & A_h \end{bmatrix}\begin{bmatrix} x_v \\ x_h \end{bmatrix} \\ &\quad + \begin{bmatrix} B_v & 0 \\ 0 & B_h \end{bmatrix}\begin{bmatrix} u_v \\ u_h \end{bmatrix} \\ &\quad + \begin{bmatrix} D_v & 0 \\ 0 & D_h \end{bmatrix}\begin{bmatrix} v_v \\ v_h \end{bmatrix} \\ \begin{bmatrix} y_v \\ y_h \end{bmatrix} &= \begin{bmatrix} C_v & 0 \\ 0 & C_h \end{bmatrix}\begin{bmatrix} x_v \\ x_h \end{bmatrix} + \begin{bmatrix} w_v \\ w_h \end{bmatrix} \end{aligned} \quad (26)$$

$$\begin{aligned} x_v &= [g_{l1}\ g_{r1}\ \dot{g}_{l1}\ \dot{g}_{r1}\ i_{l1}\ i_{r1}]^T \\ x_h &= [g_{l3}\ g_{r3}\ \dot{g}_{l3}\ \dot{g}_{r3}\ i_{l3}\ i_{r3}]^T \\ u_v &= [e_{l1}\ e_{r1}]^T, \quad u_h = [e_{l3}\ e_{r3}]^T \\ v_v &= [v_{ml1}\ v_{mr1}\ v_{Ll1}\ v_{Lr1}]^T \\ v_h &= [v_{ml3}\ v_{mr3}\ v_{Ll3}\ v_{Lr3}]^T \\ y_v &= [y_{l1}\ y_{r1}]^T, \quad y_h = [y_{l3}\ y_{r3}]^T \\ w_v &= [w_{l1}\ w_{r1}]^T, \quad w_h = [w_{l3}\ w_{r3}]^T \\ A_v &:= \begin{bmatrix} 0 & I_2 & 0 \\ K_{x1}A_1 & 0 & K_{i1}A_1 \\ 0 & 0 & -(R/L)I_2 \end{bmatrix} \\ A_h &:= \begin{bmatrix} 0 & I_2 & 0 \\ K_{x3}A_1 & 0 & K_{i3}A_1 \\ 0 & 0 & -(R/L)I_2 \end{bmatrix} \\ A_{vh} &:= \begin{bmatrix} 0 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

$$
\begin{aligned}
B_v &= B_h := \begin{bmatrix} 0 \\ 0 \\ (1/L)I_2 \end{bmatrix} \\
C_v &= C_h := \begin{bmatrix} I_2 & 0 & 0 \end{bmatrix} \\
D_v &= D_h := \begin{bmatrix} 0 & 0 \\ A_1 & 0 \\ 0 & (1/L)I_2 \end{bmatrix} \\
A_1 &:= \begin{bmatrix} 1/m + l_m^2/J_y & 1/m - l_m^2/J_y \\ 1/m - l_m^2/J_y & 1/m + l_m^2/J_y \end{bmatrix} \\
A_2 &:= \begin{bmatrix} J_x/2J_y & -J_x/2J_y \\ -J_x/2J_y & J_x/2J_y \end{bmatrix}
\end{aligned}
$$

where $I_2 \in R^{2\times 2}$ is unit matrix, and $K_{x1} = K_{xl1} = K_{xr1}$, $K_{x3} = K_{xl3} = K_{xr3}$, $K_{i1} = K_{il1} = K_{ir1}$, $K_{i3} = K_{il3} = K_{ir3}$ in (22), and $p$ is the rotor speed. Here $p$ is equal to 0 and we do not consider a rotation of the rotor in this paper.

The equation (26) can is also expressed simply as

$$
\begin{aligned}
\dot{x}_g &= A_g x_g + B_g u_g + D_g v_0 \\
y_g &= C_g x_g + w_0
\end{aligned} \tag{27}
$$

where $x_g := [x_v^T \ x_h^T]^T$, $u_g := [u_v^T \ u_h^T]^T$, $v_0 := \begin{bmatrix} v_v^T & v_h^T \end{bmatrix}^T$, $w_0 = \begin{bmatrix} w_v^T & w_h^T \end{bmatrix}^T$ and $A_g$, $B_g$, $C_g$, $D_g$ are constant matrices of appropriate dimensions.

## IV. Control System Design

In this section, we design an $H_\infty$ DIA controller for the magnetic bearing system based on the derived state-space formula.

Let us construct a generalized plant for the magnetic bearing control system. First, consider the system disturbance $v_0$. Since $v_0$ mainly acts on the plant in a low frequency range in practice, it is helpful to introduce a frequency weighting factor. Hence let $v_0$ be of the form

$$
\begin{aligned}
v_0 &= W_v(s) w_2 \qquad (28) \\
W_v(s) &= \begin{bmatrix} I_2 & 0 \\ I_2 & 0 \\ 0 & I_2 \\ 0 & I_2 \end{bmatrix} W_{v0}(s) \\
W_{v0}(s) &= C_{v0}\left(sI_4 - A_{v0}\right)^{-1} B_{v0}
\end{aligned}
$$

where $W_v(s)$ is a frequency weighting whose gain is relatively large in a low frequency range, and $w_2$ is a $(1,2)$ element of $w$ . These values, as yet unspecified, can be regarded as free design parameters.

Let us consider the system disturbance $w_0$ for the output. The disturbance $w_0$ shows an uncertain influence caused via unmodeled dynamics, and define

$$
\begin{aligned}
w_0 &= W_w(s) w_1 \qquad (29) \\
W_w(s) &= I_4 W_{w0}(s) \\
W_{w0}(s) &= C_{w0}\left(sI_4 - A_{w0}\right)^{-1} B_{w0}
\end{aligned}
$$

where $W_w(s)$ is a frequency weighting function and $w_1$ is a $(1,1)$ element of $w$. Note that $I_4$ is unit matrix in $R^{4\times 4}$.

The frequency functions $W_v$ and $W_w$ in (28) and (29) are rewritten as equations in (30) and (30).

$$
\begin{aligned}
\dot{x}_v &= A_v x_v + B_v w_2 \\
v_0 &= C_v x_v + D_v w_2
\end{aligned}
$$

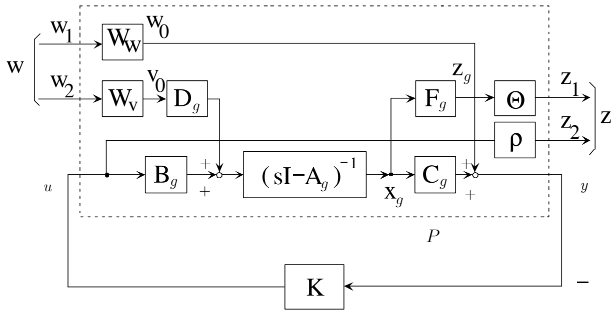


Fig. 3. Generalized Plant

$$
\begin{aligned}
\dot{x}_w &= A_w x_w + B_w w_1 \\
w_0 &= C_w x_w + D_w w_1
\end{aligned} \tag{30}
$$

where the state $x_v$ and $x_w$ are defined as $x_v := \begin{bmatrix} x_{v1}^T \ x_{v2}^T \ x_{v3}^T \ x_{v4}^T \end{bmatrix}^T$, $x_w := \begin{bmatrix} x_{w1}^T \ x_{w2}^T \ x_{w3}^T \ x_{w4}^T \end{bmatrix}^T$.

Next we consider the variables which we want to regulate. In this case, since our main concern is in the stabilization of the rotor, the gap and the corresponding velocity are chosen; i.e.,

$$
z_g = F_g x_g, \tag{31}
$$

$$
F_g = \begin{bmatrix} I_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_2 & 0 \end{bmatrix}
$$

$$
z_1 = \Theta z_g, \ \ \Theta = \mathrm{diag}\begin{bmatrix} \theta_1 & \theta_2 & \theta_1 & \theta_2 \end{bmatrix} \tag{32}
$$

where $\Theta$ is a weighting matrix on the regulated variables $z_g$, and $z_1$ is a $(1,1)$ element of $z$. This value $\Theta$, as yet unspecified, are also free design parameters.

Furthermore the control input $u_g$ should be also regulated, and we define

$$
z_2 = \rho u_g \tag{33}
$$

where $\rho$ is a weighting scalar, and $z_2$ is a $(1,2)$ element of $z$. Finally, let $x := [\, x_g^T \quad x_v^T \quad x_w^T \,]^T$, where $x_v$ denotes the state of the function $W_v(s)$, $x_w$ denotes the state of the function $W_w(s)$, and $w := [\, w_1^T \quad w_2^T \,]^T$, $z := [\, z_1^T \quad z_2^T \,]^T$, then we can construct the generalized plant as in Fig.3 with an unspecified controller $K$.

The state-space formulation of the generalized plant is given as follows.

$$
\begin{aligned}
\dot{x} &= Ax + B_1 w + B_2 u \\
z &= C_1 x + D_{12} u \\
y &= C_2 x + D_{21} w
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
A &= \begin{bmatrix} A_g & D_g G_v & 0 \\ 0 & A_v & 0 \\ 0 & 0 & A_w \end{bmatrix} \\
B_1 &= \begin{bmatrix} 0 & D_g D_v \\ 0 & B_v \\ B_w & 0 \end{bmatrix}, \ B_2 = \begin{bmatrix} B_g \\ 0 \\ 0 \end{bmatrix} \\
C_1 &= \begin{bmatrix} \Theta F_g & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ D_{12} = \begin{bmatrix} 0 \\ \rho \end{bmatrix}, \\
C_2 &= \begin{bmatrix} C_g & 0 & C_w \end{bmatrix}, \ D_{21} = \begin{bmatrix} W_w & 0 \end{bmatrix}
\end{aligned}
$$

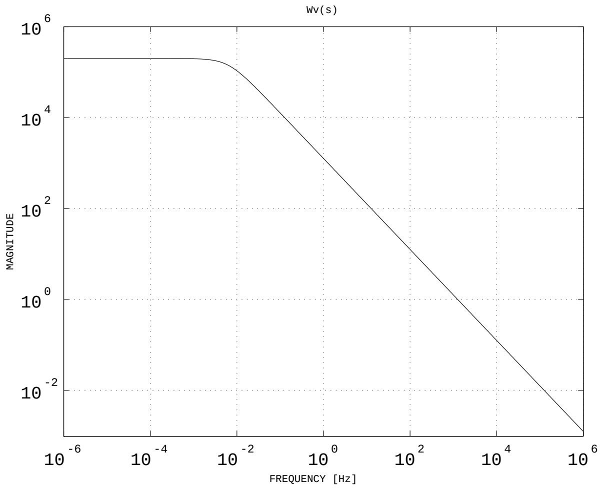


Fig. 4. $W_{v0}(s)$

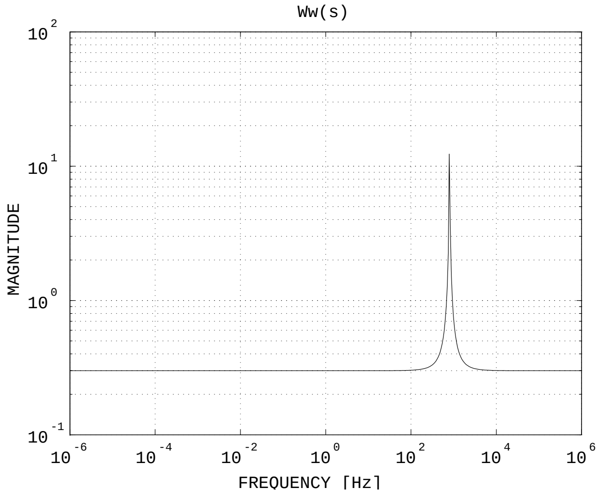


Fig. 5. $W_{w0}(s)$

Since the disturbances $w$ represent the various model uncertainties, the effects of these disturbances on the error vector $z$ should be reduced.

Next our control problem setup is defined as;

**Control problem**! ′find an admissible controller $K(s)$ that attenuates disturbances and initial state uncertainties to achieve DIA condition in (3) for generalized plant (34).

After some iteration in MATLAB environment, design parameters are chosen as follows;

$$\begin{aligned} W_{v0}(s) &= \frac{8000}{s+0.04} \\ W_{w0}(s) &= \frac{0.3s^2+1479.7s+7.2983\times10^6}{s^2+s+2.4328\times10^7} \\ \Theta &= diag\begin{bmatrix}\theta_1 & \theta_2 & \theta_1 & \theta_2\end{bmatrix} \\ \theta_1 &= diag\begin{bmatrix}0.5 & 0.5\end{bmatrix} \\ \theta_2 &= diag\begin{bmatrix}0.0005 & 0.0005\end{bmatrix} \\ \rho &= 8.0\times10^{-7}I_4 \end{aligned}$$

Frequency responses of $W_{v0}(s)$ and $W_{w0}(s)$ are shown in Fig.4 and Fig.5 respectively. $W_{w0}(s)$ represents an uncertainty for the 1st bending mode of the rotor at the resonance frequency $800$[Hz].

Direct calculations yield the 24-order $\mathcal{H}_\infty$ DIA central controller $K_{DIA}$ and its frequency response is shown in Fig.6.

The maximum value of the weighting matrix $N$ in the DIA condition (3) is given by

$$N = 1.3265979325391\times10^{-7}\times I_{24}. \tag{35}$$

## V. Evaluation by Experiments

We conducted control experiments to evaluate properties of the designed $H_\infty$ DIA controller comparing to a PID controller with a notch filter. The PID gain is chosen as

$$\begin{aligned} K_p &= 8400\times I_4 \\ K_i &= 15000\times I_4 \\ K_d &= 6\times I_4. \end{aligned}$$

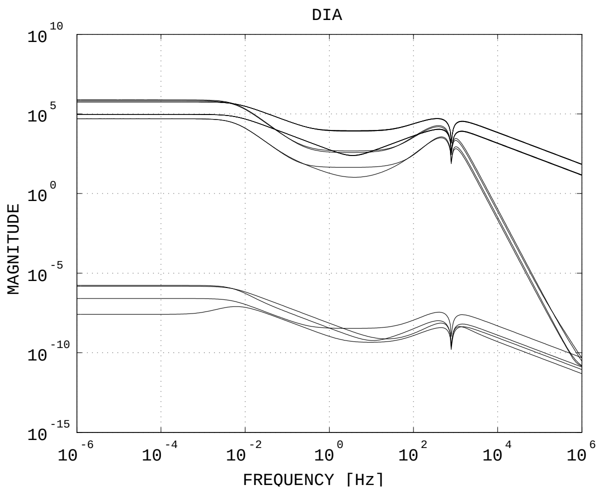


Fig. 6. Frequency Responses of $H_\infty$DIA Controllers

The notch filter has a notch at 2000[Hz] and its transfer function is as follows.

$$\frac{s^2+1.5791\times10^8}{s^2+12566s+1.5791\times10^8} \tag{36}$$

The objective of this experimental comparison is to evaluate control performance for transient property and robust performance. The experimental results are shown in Figs. 7-10. In Figs.7-8, step responses for a reference signal are shown, where the step size is 0.05[mm] and the steady-state gap is 0.4[mm]. Comparing with PID control $H_\infty$ DIA control shows a quick response and a good disturbance attenuation property. Figs. 9-10 show properties of robust performance for step-type disturbance. A 60[g] weight is attached to the center of the rotor as a model perturbation and a step-type force disturbance is added to $-l1$ and $-r1$ directions in Fig.1, where the magnitude of the disturbance is $1/6$ steady-state vertical attractive force.

From Figs.9-10, we can see that $H_\infty$DIA control has a good transient response and robust performance comparing with PID control with notch filter.

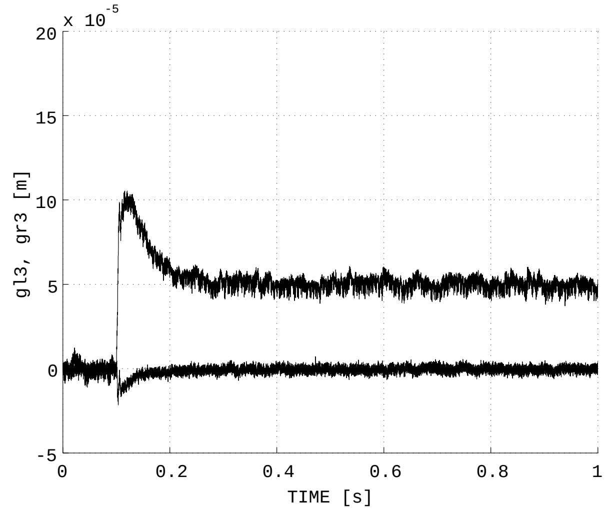


Fig. 7. Step Response of $H_\infty$DIA Controller

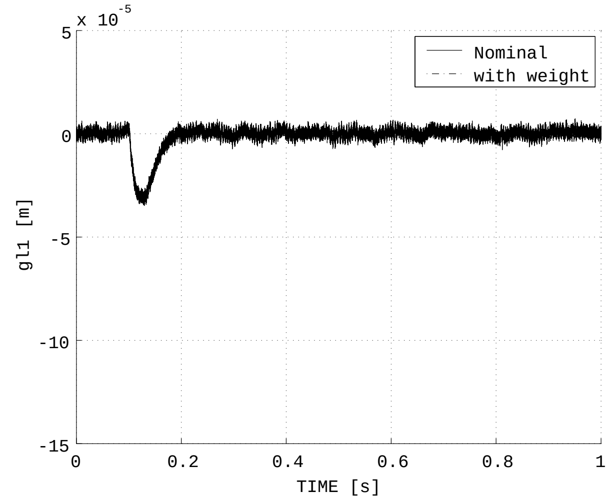


Fig. 9. Disturbance Response of $H_\infty$DIA Controller with/without perturbation

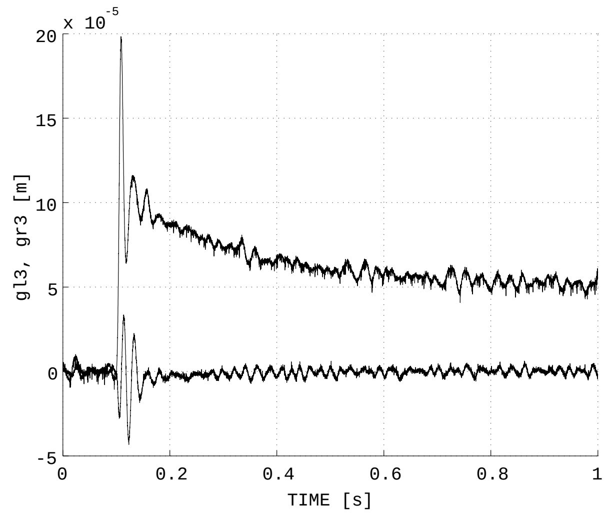


Fig. 8. Step Response of PID Controller

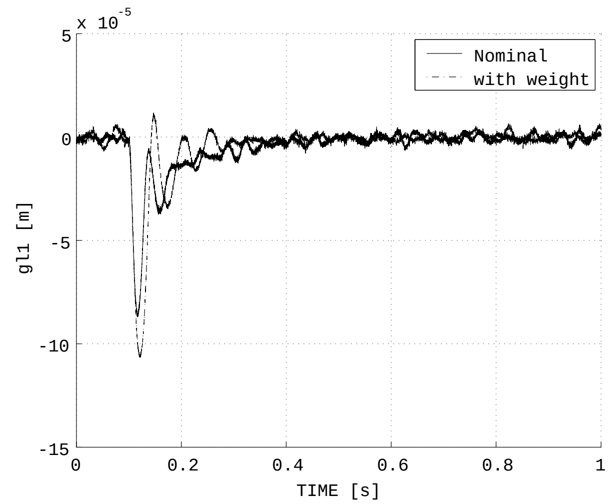


Fig. 10. Disturbance Response of PID Controller with/without perturbation

## VI. CONCLUSION

This paper dealt with an application of $H_\infty$ control attenuating initial-state uncertainties to the magnetic bearing and examined the $H_\infty$ DIA control problem.

First we derived a mathematical model of magnetic bearing systems considering rotor dynamics and nonlinearities of magnetic force. Then we set the generalized plant which contains design parameter for uncertainty and control performance.

Finally, several experimental results of step responses and disturbance responses with model perturbation showed that the proposed $H_\infty$ DIA robust control approach is effective for a mixed disturbance and an initial-state uncertainty attenuation and for improving transient response and robust performance.

Future work is an evaluation of the proposed $H_\infty$ DIA control via rotational experiments.

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