H_∞ CONTROL ATTENUATING INITIAL STATE UNCERTAINTIES AND ITS APPLICATION TO THE MAGNETIC SUSPENSION SYSTEM

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ABSTRACT

This paper deals with a generalized H_∞ control attenuating initial-state uncertainties. An H_∞ control problem, which treats a mixed attenuation of disturbance and initial-state uncertainty for linear time-invariant systems in the infinite-horizon case, is examined. We derived a necessary and sufficient condition of the generalized mixed attenuation problem[7]. In this paper, we apply this proposed approach to a magnetic suspension system, and evaluate the effectiveness of the proposed approach by using a magnetic suspension system. Comparing the proposed controller with previous results[6], we show the property and effectiveness of the proposed generalized H_∞ control attenuating initial state uncertainties.

keywords: H_∞ Control, DIA Control, Initial-State Uncertainties, Magnetic Suspension Systems

INTRODUCTION

Usual notation of the H_∞ control is a time-invariant control which attenuates the effect of disturbances on controlled outputs and is originally defined under the assumption that the initial states of the system are zero. Initial states are often uncertain and might be zero or non-zero. If the initial states are non-zero, the system adopting an H_∞ control will present some transients as the effect of the non-zero initial states, to which the H_∞ control is not responsible. On the other hand, it is expected that the mixed attenuation supplies H_∞ controls with some good transients and assures H_∞ controls of robustness against initial-state uncertainty. Recently, hybrid/switching control are actively studied, this method might be one of the reasonable approach to implement them.

In the finite-horizon case, a generalized type of H_∞ control problem which was formulated and solved by Uchida and Fujita[1] and Khargonekar et al.[2]. This problem was extended to the infinite-horizon case[2, 3]. Furthermore a generalized type of H_∞ control problem which considers a mixed attenuation of disturbance and initial-state uncertainty was derived[4]. The problem discussed in [4], however, was limited to time-invariant systems satisfying the orthogonality assumptions [5]. This is an immensely serious problem as a matter of fact, if we apply this problem setup to the real physical control system design. The previous mixed attenuation of disturbance and initial-state uncertainty in the infinite-horizon case is not sufficient[6] in practice, because time-invariant systems satisfying the orthogonality assumptions restrict the degrees of freedom of the control system design, and have difficulty in regulating control inputs[6]. Then we formulated an infinite horizon disturbance and initial state uncertainty attenuation control problem without the orthogonality assumptions[7].

In this paper, we evaluate the effectiveness of the proposed approach[7] by using a magnetic suspension system. We apply this proposed approach to a magnetic suspension system. Comparing with the standard H_∞ controller and the other controllers via previous results[6], we show the property and effectiveness of the proposed generalized H_∞ control attenuating initial state uncertainties.

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MIXED ATTENUATION OF DISTURBANCE AND INITIAL-STATE UNCERTAINTY

Consider the linear time-invariant system which is defined on the time interval $[0,\infty)$ and described by

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u, \quad x(0) = x_0 \\ z &= C_1 x + D_{12} u \\ y &= C_2 x + D_{21} w \end{aligned} \tag{1}$$

where $x \in R^n$ is the state and x_0 is the initial state; $u \in R^r$ is the control input; $y \in R^m$ is the observed output; $z \in R^q$ is the controlled output; $w \in R^p$ is the disturbance. Without loss of generality, we regard x_0 as the initial-state uncertainty, and $x_0 = 0$ as known initial-state case. The disturbance $w(t)$ is a square integrable function defined on $[0,\infty)$. Note that this system does not have the orthogonality assumptions[5], and one of the linear time-invariant systems with the orthogonality assumptions in this framework is written in [4, 6].

A, B_1, B_2, C_1, C_2, D_{12} and D_{21} are constant matrices of appropriate dimensions and satisfies that

- (A, B_1) is controllable and (A, C_1) is observable
- (A, B_2) is controllable and (A, C_2) is observable
- $D_{12}^T D_{12} \in R^{r\times r}$ is nonsingular
- $D_{21} D_{21}^T \in R^{m\times m}$ is nonsingular

For system (1), every admissible control $u(t)$ is given by a linear time-invariant system of the form

$$\begin{aligned} u &= J\zeta + Ky \\ \dot{\zeta} &= G\zeta + Hy, \quad \zeta(0) = 0 \end{aligned} \tag{2}$$

which makes the closed-loop system given by (1) and (2) internally stable, where $\zeta(t)$ is the state of the controller of a finite dimension; J, K, G and H are constant matrices of appropriate dimensions.

For the system and the class of admissible controls described above, consider a mixed-attenuation problem stated as below.

Problem 1 DIA Control Problem
Find an admissible control attenuating disturbances and initial state uncertainties in the way that, for given $N > 0$, z satisfies

$$\|z\|_2^2 < \|w\|_2^2 + x_0^T N^{-1} x_0 \tag{3}$$

for all $w \in L^2[0,\infty)$ and all $x_0 \in R^n$, s.t., $(w, x_0) \neq 0$.

We call such an admissible control the **D**isturbance and **I**nitial state uncertainty **A**ttenuation (DIA) control. The weighting matrix N on x_0 is a measure of relative importance of the initial-state uncertainty attenuation to the disturbance attenuation. A larger choice of N in the sense of matrix inequality order means finding an admissible control which attenuates the initial-state uncertainty more.

From the definition, a DIA control should be an H_∞ control when the initial state is known($x_0 = 0$). This implies that, in order to solve the DIA control problem, we require the so-called Riccati equation conditions:

(A1) There exists a solution $M > 0$ to the Riccati equation

$$\begin{aligned} &M(A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1) \\ &+(A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1)^T M \\ &-M(B_2(D_{12}^T D_{12})^{-1} B_2^T - B_1 B_1^T) M \\ &+C_1^T C_1 - C_1^T D_{12}(D_{12}^T D_{12})^{-1} D_{12}^T C_1 = 0 \end{aligned} \tag{4}$$

such that $A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1$
$-B_2(D_{12}^T D_{12})^{-1} B_2^T M + B_1 B_1^T M$ is stable.

(A2) There exists a solution $P > 0$ to the Riccati equation

$$\begin{aligned} &(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2) P \\ &+P(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2)^T \\ &-P(C_2^T (D_{21} D_{21}^T)^{-1} C_2 - C_1^T C_1) P \\ &+B_1 B_1^T - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} D_{21} B_1^T = 0 \end{aligned} \tag{5}$$

such that $A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2$
$-P C_2^T (D_{21} D_{21}^T)^{-1} C_2 + P C_1^T C_1$ is stable.

(A3) $\rho(PM) < 1$,
where $\rho(X)$ denotes the spectral radius of matrix X, and $\rho(X) = \max |\lambda_i(X)|$.

Next, the following condition is assumed.
(A4) $Q + N^{-1} - P^{-1} > 0$,
where Q is the maximal solution of the Riccati equation

$$\begin{aligned} &Q(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2 \\ &\quad + (B_1 B_1^T - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} D_{21} B_1^T) P^{-1}) \\ &+(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2 \\ &\quad + (B_1 B_1^T - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} D_{21} B_1^T) P^{-1})^T Q \\ &-Q(B_1^T - D_{21}^T (D_{21} D_{21}^T)^{-1} (C_2 P + D_{21} B_1^T) L)^T \\ &\quad \times (B_1^T - D_{21}^T (D_{21} D_{21}^T)^{-1} (C_2 P + D_{21} B_1^T) L) Q \\ &= 0 \end{aligned} \tag{6}$$

with $L := (I - PM)^{-1}$.

Then, we obtained the following main results[7].

Theorem 1 *[7] Suppose that the conditions (A1), (A2), and (A3) are satisfied. The central control (7) is a DIA control if and only if the condition (A4) is satisfied, where the central control is given by*

$$\begin{aligned} u &= -(D_{12}^T D_{12})^{-1} (B_2^T M + D_{12}^T C_1)(I - PM)^{-1} \zeta \\ \dot{\zeta} &= A\zeta + B_2 u + P C_1^T (C_1 \zeta + D_{12} u) \\ &\quad +(P C_2^T + B_1 D_{21}^T)(D_{21} D_{21}^T)^{-1} (y - C_2 \zeta) \\ \zeta(0) &= 0 \end{aligned} \tag{7}$$

and $S := M(I - PM)^{-1}$.

APPLICATION TO MAGNETIC SUSPENSION SYSTEM

We apply the proposed approach[7] to a magnetic suspension system, and evaluate its effectiveness.

CONSTRUCTION

The experimental setup is shown in Fig.1[8]. An electromagnet is located at the top of the experimental system. The control problem is to levitate the iron ball stably utilizing the electromagnetic force, where a mass M of the iron ball is 1.75 kg, and steady state gap X is 5 mm.

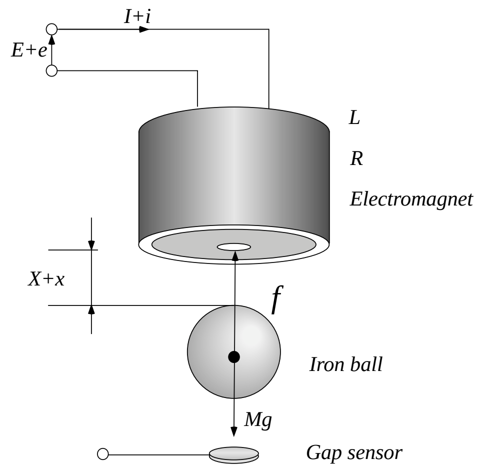

FIGURE 1: Magnetic Suspension System

MATHEMATICAL MODEL

In order to derive a model of the system by physical laws, we introduce following assumptions[8].

- **[a1]** Magnetic flux density and magnetic field do not have any hysteresis, and they are not saturated.
- **[a2]** There are no leakage flux in the magnetic circuit.
- **[a3]** Magnetic permeability of the electromagnet is infinity.
- **[a4]** Eddy current in the magnetic pole can be neglected.
- **[a5]** Coil inductance is constant around the operating point, and an electromotive force due to a motion of the iron ball can be neglected.

These assumptions are almost essential to model this system. Under these assumptions, we derived equations of the motion, the electromagnetic force, and the electric circuit as

$$M\frac{d^2x(t)}{dt^2} = Mg - f(t) + v_m \tag{8}$$

$$f(t) = k\left(\frac{I + i(t)}{X + x(t) + x_0}\right)^2 \tag{9}$$

$$L\frac{di(t)}{dt} + R(I + i(t)) = E + e(t) + v_L \tag{10}$$

where M is a mass of the iron ball, X is a steady gap between the electromagnet(EM) and the iron ball, $x(t)$ is a deviation from X, I is a steady current, $i(t)$ is a deviation from I, E is a steady voltage, $e(t)$ is a deviation from E, $f(t)$ is EM force, k, x_0 are coefficients of $f(t)$, L is an inductance of EM, and R is a resistance of EM, v_m and v_L are exogenous disturbance inputs.

Next we linearize the electromagnetic force (9) around the operating point by the Taylor series expansion as

$$f(t) = k\left(\frac{I}{X + x_0}\right)^2 - K_x x(t) + K_i i(t), \tag{11}$$

where $K_x = 2kI^2/(X + x_0)^3$ and $K_i = 2kI/(X + x_0)^2$. The sensor provides the information for the gap $x(t)$. Hence the measurement equation can be written as

$$y(t) = x(t) + w_0 \tag{12}$$

where w_0 represents the sensor noise as well as the model uncertainties. Thus, summing up the above results, the state equations for the system are

$$\begin{aligned} \dot{x}_g &= A_g x_g + B_g u_g + D_g v_0 \\ y_g &= C_g x_g + w_0 \end{aligned} \tag{13}$$

where $x_g := [x \;\; \dot{x} \;\; i]'$, $u_g := e$, $v_0 := [v_m \;\; v_L]'$,

$$A_g = \begin{bmatrix} 0 & 1 & 0 \\ 4481 & 0 & -18.4 \\ 0 & 0 & -45.7 \end{bmatrix}, \; B_g = \begin{bmatrix} 0 & 0 & 1.97 \end{bmatrix}^T$$

$$C_g = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad D_g = \begin{bmatrix} 0 & 0 \\ 0.57 & 0 \\ 0 & 1.97 \end{bmatrix}$$

Here $(A_g,\ B_g)$ and $(A_g,\ D_g)$ are controllable, and $(A_g,\ C_g)$ is observable.

PROBLEM SETUP FOR CONTROL SYSTEM DESIGN

For the magnetic suspension system described and modeled in the previous section, our principal control objective is its stabilization. Further, as we have clarified in the modeling of the disturbances, it should be stabilized robustly against v_0 and w_0. Moreover the closed-loop system is expected to have a better transient performance. To this end, we will setup the control problem within the framework of the H_∞ DIA control.

First let us consider the system disturbance v_0. Since v_0 mainly acts on the plant in a low frequency range in practice, it is helpful to introduce a frequency weighting factor. Hence let v_0 be of the form

$$v_0 = W_v(s)\, w_2 \tag{14}$$

$$W_v(s) = \Phi W(s) = \Phi C_w (sI - A_w)^{-1} B_w \tag{15}$$

$$\Phi = \begin{bmatrix} 1 & 1 \end{bmatrix}^T \tag{16}$$

where $W_v(s)$ is a frequency weighting whose gain is relatively large in a low frequency range, and w_2 is a $(1,2)$ element of w. These values, as yet unspecified, can be regarded as free design parameters. It is noted that, in (14), we have not made explicit distinction in the notation between a time domain function and its Laplace transform. And let us consider the system disturbance w_0 for the output. The disturbance w_0 shows an uncertain influence caused via unmodeled dynamics, and define

$$w_0 = W_w w_1 \quad (17)$$

where W_w is a weighting scalar, and w_1 is a $(1,1)$ element of w. Note that W_w is sometimes frequency dependent, but it is selected as a scalar for the sake of simplicity. Next we consider the variables which we want to regulate. In this study, since our main concern is in the stabilization of the iron ball, the gap and the corresponding velocity are chosen; i.e.,

$$z_g = F_g x_g, \quad F_g = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (18)$$

Then, as the error vector, let us define as follows

$$z_1 = \Theta z_g, \quad \Theta = \text{diag}\begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix} \quad (19)$$

where Θ is a weighting matrix on the regulated variables z_g, and z_1 is a $(1,1)$ element of z. This value Θ, as yet unspecified, are also free design parameters. Furthermore the control input u should be also regulated, and we define

$$z_2 = \rho\ u \quad (20)$$

where ρ is a weighting scalar, and z_2 is a $(1,2)$ element of z. Finally, let $x := \begin{bmatrix} x_g^T & x_w^T \end{bmatrix}^T$, where x_w denotes the state of the frequency weighting $W_w(s)$, and $w := \begin{bmatrix} w_1^T & w_2^T \end{bmatrix}^T$, $z := \begin{bmatrix} z_1^T & z_2^T \end{bmatrix}^T$, then we can construct the generalized plant as the following;

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{12} u \\ y &= C_2 x + D_{21} w \end{aligned} \quad (21)$$

where

$$A = \begin{bmatrix} A_g & D_g C_w \\ 0 & A_w \end{bmatrix}, B_1 = \begin{bmatrix} 0 & D_g D_w \\ 0 & B_w \end{bmatrix},$$
$$B_2 = \begin{bmatrix} B_g \\ 0 \end{bmatrix}, \; C_1 = \begin{bmatrix} \Theta F_g & 0 \\ 0 & 0 \end{bmatrix}, \; D_{12} = \begin{bmatrix} 0 \\ \rho \end{bmatrix},$$
$$C_2 = \begin{bmatrix} C_g & 0 \end{bmatrix}, \; D_{21} = \begin{bmatrix} W_w & 0 \end{bmatrix}$$

The block diagram of the generalized plant with an unspecified controller K is shown in Fig.2. Since the disturbances w represent the various model uncertainties, the effects of these disturbances on the error vector z should be reduced. Note that this generalized plant does not have the orthogonality

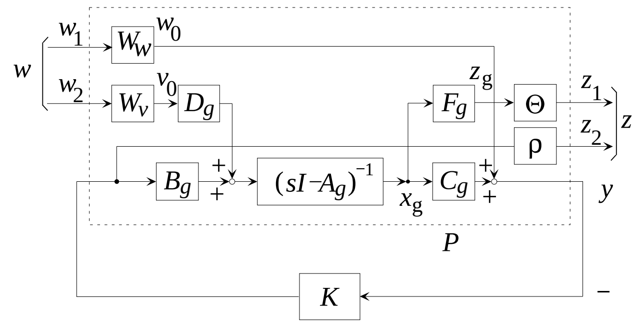

FIGURE 2: Generalized Plant

assumptions[5], and one of the generalized plants with the orthogonality assumptions in this framework is written in [4, 6]. Now our control design problem setup is: Find an admissible controller $K(s)$ that attenuates disturbances and initial state uncertainties to achieve DIA condition in (3).

DIA CONTROLLER

We design controllers for the generalized plant in the previous subsection based on the following 4-Step procedure.

[Step 1] Selection of the weighting function $W_v(s)$ and W_w:
$W_w(s)$ is a frequency weighting whose gain is relatively large in a low frequency range, and W_w is a weighting scalar.

[Step 2] Selection of the weighting Matrix Θ and ρ:
Θ is a weighting matrix for the regulated variables z_g, and ρ is for the input variable u.

[Step 3] Construction of generalized plant and a DIA controller:
With the specified design parameters in [**Step 1**] and [**Step 2**], the generalized plant is constructed. The DIA controller is designed for this plant, and its state-space description is given by easy algebraic calculation from (7).

$$\dot{x}_c = A_c x_c + B_c y, \quad u = C_c x_c \quad (22)$$

where x_c is the state of the DIA controller and A_C, B_C and C_C are given as

$$\begin{aligned} A_c &= A + PC_1^T C_1 - (PC_2^T + B_1 D_{21}^T)(D_{21} D_{21}^T)^{-1} C_2 \\ &\quad -(B_2 + PC_1^T D_{12})(D_{12}^T D_{12})^{-1} \\ &\quad \times (B_2^T M + D_{12}^T C_1) L \\ B_c &= (PC_2^T + B_1 D_{21}^T)(D_{21} D_{21}^T)^{-1} \\ C_c &= -(D_{12}^T D_{12})^{-1}(B_2^T M + D_{12}^T C_1) L. \end{aligned}$$

[Step 4] Calculation of the max. matrix N:
Calculate the maximum N satisfies the condition **(A4)**. For the sake of simplicity, the structure of the matrix N is limited in $N = nI$, where n is a positive scalar number.

After some iteration in MATLAB environment, these design parameters are chosen by the above 4-step procedure as follows;

$$\begin{aligned} W_v(s) &= \frac{2.5\times 10^6}{s+0.010} \\ W_w &= 0.5 \\ \Theta &= \text{diag}\left[\begin{array}{cc} 1.01 & 1.0\times 10^{-5} \end{array}\right] \\ \rho &= 1.0\times 10^{-10} \end{aligned} \qquad (23)$$

Direct calculations yield the central controller;

$$K(s) := C_K(sI - A_K)^{-1}B_K \qquad (24)$$

where

$$\begin{aligned} A_K &= \left[\begin{array}{cccc} -269 & 1.00 & 0 & 0 \\ -31700 & 2.29\cdot 10^{-4} & -18.4 & 1.43\cdot 10^{6} \\ 2.05\cdot 10^{10} & 5.65\cdot 10^{6} & -14400 & 1.12\cdot 10^{9} \\ 1.72 & -1.56\cdot 10^{-8} & 0 & -0.010 \end{array}\right] \\ B_K &= \left[\begin{array}{cccc} 361 & 48600 & -3.04\cdot 10^5 & -2.31 \end{array}\right]^T \\ C_K &= \left[\begin{array}{cccc} 1.04\cdot 10^{10} & 2.87\cdot 10^6 & -7290 & 5.64\cdot 10^8 \end{array}\right] \end{aligned}$$

The frequency response of the controller $K(s)$ is shown in Fig. 3 by a solid line. And the maximum value of the weighting matrix N is given by $N = 2.7735\times 10^{-2}\times I$.

We designed the standard H_∞ controller for the comparison, where the H_∞ controller[8] was designed via the MATLAB command `hinfsyn.m`. We denote the state-space realization of the obtained H_∞ controller as K_∞. The frequency response of the controller K_∞ is shown in Fig. 3 by a dotted line. Furthermore we also show the previous DIA controllers $K_{DIA_1}(s)$ and $K_{DIA_2}(s)$ in Fig.3, by a dashed line and a dash-dot line, respectively [6].

Comparing these four controllers, $K(s)$ has a high gain at the low frequency and a good roll-off property at the high frequency, and the comprehensive frequency response looks like a modified PID controller. In the previous DIA design framework, it was difficult to let controllers get hold an integral property[6].

SIMULATION RESULTS

We have conducted simulations to evaluate properties of the controller K. The iron ball at a standstill has been suspended stably with either the controller K, K_∞, K_{DIA_1} and K_{DIA_2}. To ascertain transient responses, we input a step reference signal to a suspended iron ball with a nonzero initial state x_0. It is expected that K will show a better initial response and also a better step response for the reference signal.

A step reference signal is added to the system around 1.0[s], where the magnitude of the step signal is 0.1[mm], and the initial state is $x_0 = [0.0\ \ 0.0\ \ 0.1]^T$.

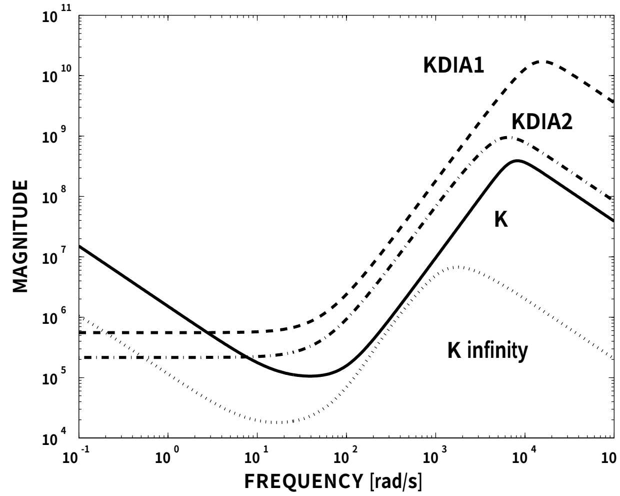

FIGURE 3: Frequency Response of the controller K with K_{DIA_1} , K_{DIA_2} and K_∞

Simulation results are shown in Fig.4. An enlargement of Fig.4 at the initial response is Fig.5, and an enlargement at the step response is Fig.6, respectively. Note that they are definitely same with Fig.4.

We first evaluate the attenuating property of the initial state uncertainty in Fig.5. From the results, we can see that the K_{DIA_1} and K_{DIA_2} show relatively better performance than K for the initial state uncertainty. However K has a better transient performance than K_∞, which shows K has a better property than K_∞, but not better than K_{DIA_1} and K_{DIA_2}.

Since our concerns are not only in the attenuation of the initial state uncertainty, but also in the basic control performance of the controllers, we then wonder whether the controller has a good performance for the step reference signal. Fig.6 shows an enlarged step response of Fig.4. Controller K shows better and quicker transient response than K_∞. Controllers K_{DIA_1} and K_{DIA_2} shows pretty quick response around 1.0[s] because of their high gain at the high frequency in Fig.3, however we must give careful attention for steady-state error with those both controllers.

K_{DIA_1} and K_{DIA_2} leave steady-state errors because of their low gain at the low frequency in Fig.3. In the previous problem setup, the degrees of freedom in the design parameters are limited, so that it is difficult to shape a good controller frequency response[6].

Considering all the factors, we reached the conclusion that K has a better performance for all control requirements, and has a potential ability to be improved by using the degrees of freedom in the design parameters.

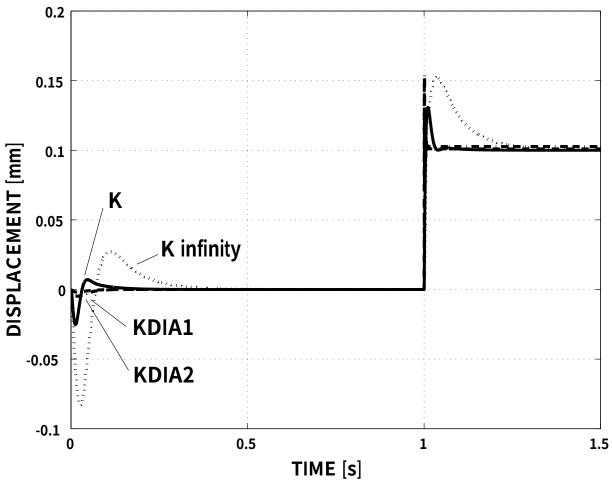

FIGURE 4: Time Responses for a Step Reference with an Initial State $x_0 = [0.0 \;\; 0.0 \;\; 0.1]^T$

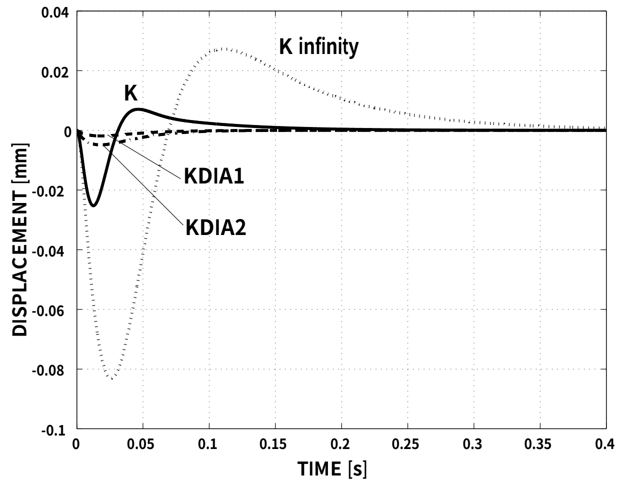

FIGURE 5: Initial Responses for $x_0 = [0.0 \;\; 0.0 \;\; 0.1]^T$

CONCLUSION

In this paper, we formulated a generalized type of H_∞ control problem which considers a mixed attenuation of disturbance and initial-state uncertainty in the infinite-horizon case, without the orthogonality assumptions. We applied an infinite horizon disturbance and initial state uncertainty attenuation control problem without the orthogonality assumptions to a magnetic suspension system, and evaluated the effectiveness of the proposed approach. Comparing the proposed controller with the standard H_∞ controller and the other controllers based on previous results, we showed the property and effectiveness of the proposed mixed attenuation controller.

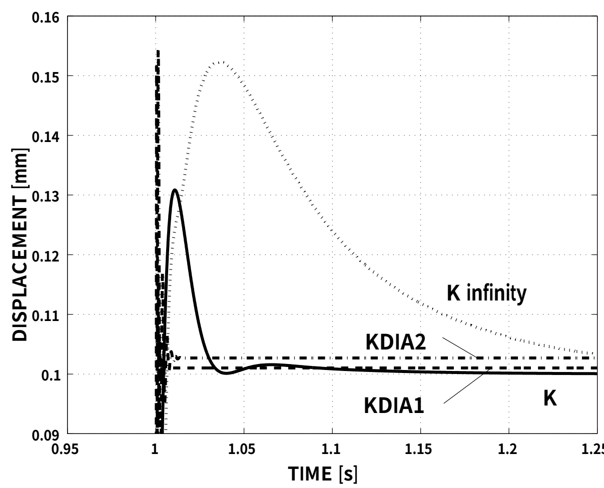

FIGURE 6: Step Responses

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