A Generalized H_∞ Control System Design Attenuating Initial State Uncertainties

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Abstract

This paper deals with a generalized H_∞ control attenuating initial-state uncertainties. An H_∞ control problem, which treats a mixed attenuation of disturbance and initial-state uncertainty for linear time-invariant systems in the infinite-horizon case, is examined. The mixed attenuation supplies H_∞ controls with good transients and assures H_∞ controls of robustness against initial-state uncertainty. We derive a necessary and sufficient condition of the generalized mixed attenuation problem. Furthermore we apply this proposed method to a magnetic suspension system, and evaluate attenuation property of the proposed generalized H_∞ control approach.

keywords: H_∞ Control, DIA Control, Initial-State Uncertainties, Magnetic Suspension Systems

1 Introduction

H_∞ control for linear time-invariant systems attenuates the effect of disturbances on controlled outputs and is originally defined under the assumption that the initial states of the system are zero. Initial states are often uncertain and might be zero or non-zero. If the initial states are non-zero, the system adopting an H_∞ control will present some transients as the effect of the non-zero initial states, to which the H_∞ control is not intrinsically responsible. It is expected that the mixed attenuation of disturbance and initial-state uncertainty in controlled outputs supplies H_∞ controls with some good transients and assures H_∞ controls of robustness against initial-state uncertainty. Recently, hybrid/switching control are actively studied, this method might be one of the reasonable approach to implement them. In the finite-horizon case, a generalized type of H_∞ control problem which formulated and solved by Uchida and Fujita[1] and Khargonekar et al.[2]. This problem was extended to the infinite-horizon case, and a result was derived by Uchida et al.[3](see also Khargonekar et al.[2]). The problem discussed in [3] was, however, limited to time-invariant systems satisfying the orthogonality assumptions [4, 5]. This is an immensely serious problem as a matter of fact, if we apply this problem setup to the real physical control system design. The previous mixed attenuation of disturbance and initial-state uncertainty in the infinite-horizon case is not sufficient[6] in practice, because time-invariant systems satisfying the orthogonality assumptions restrict the degrees of freedom of the control system design, and have difficulties in regulating control inputs[6].

In this paper, we have formulated an infinite horizon disturbance and initial state uncertainty attenuation control problem without the orthogonality assumptions. The solution based on [4] is given as a natural but complicated extension of the previous results in [3, 6]. A necessary and sufficient condition for a solution to exist, together with an explicit formula of the solution, is derived. Based on the condition, a robustness property of H_∞ controls against initial-state uncertainty is discussed. Moreover, we apply the proposed approach to a magnetic suspension system, and evaluate the effectiveness of the generalized H_∞ control attenuating initial state uncertainties comparing with the previous results[6].

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2 Problem Statement

Consider the linear time-invariant system which is defined on the time interval $[0, \infty)$ and described by

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u, \quad x(0) = x_0 \\ z &= C_1 x + D_{12} u \\ y &= C_2 x + D_{21} w \end{aligned} \qquad (1)$$

where $x \in R^n$ is the state and x_0 is the initial state; $u \in R^r$ is the control input; $y \in R^m$ is the observed output; $z \in R^q$ is the controlled output; $w \in R^p$ is the disturbance. Without loss of generality, we regard x_0 as the initial-state uncertainty, and $x_0 = 0$ as a known initial-state case. The disturbance $w(t)$ is a square integrable function defined on $[0, \infty)$. Note that this system does not have the orthogonality assumptions[5]. A, B_1, B_2, C_1, C_2, D_{12} and D_{21} are constant matrices of appropriate dimensions and satisfies that

- (A, B_1) is controllable and (A, C_1) is observable
- (A, B_2) is controllable and (A, C_2) is observable
- $D_{12}^T D_{12} \in R^{r \times r}$ is nonsingular
- $D_{21} D_{21}^T \in R^{m \times m}$ is nonsingular

For system (1), every admissible control $u(t)$ is given by a linear time-invariant system of the form

$$\begin{aligned} u &= J\zeta + Ky \\ \dot{\zeta} &= G\zeta + Hy, \quad \zeta(0) = 0 \end{aligned} \qquad (2)$$

which makes the closed-loop system given by (1) and (2) internally stable, where $\zeta(t)$ is the state of the controller of a finite dimension; J, K, G and H are constant matrices of appropriate dimensions.

For the system and the class of admissible controls described above, consider a mixed-attenuation problem stated as below.

Problem 1 DIA Control Problem
Find an admissible control attenuating disturbances and initial state uncertainties in the way that, for given $N > 0$, z satisfies

$$\|z\|_2^2 < \|w\|_2^2 + x_0^T N^{-1} x_0 \qquad (3)$$

for all $w \in L^2[0, \infty)$ and all $x_0 \in R^n$, s.t., $(w, x_0) \neq 0$.

We call such an admissible control the **D**isturbance and **I**nitial state uncertainty **A**ttenuation (**DIA**) control. The weighting matrix N on x_0 is a measure of relative importance of the initial-state uncertainty attenuation to the disturbance attenuation. A larger choice of N in the sense of matrix inequality order means finding an admissible control which attenuates the initial-state uncertainty more. In the special case when the initial state is known, that is $x_0 = 0$, the problem is reduced to finding an admissible control which assures that

$$\|z\|_2^2 < \|w\|_2^2 \qquad (4)$$

for all $w \in L^2[0, \infty)$. Then, we call the admissible control the H_∞ control as usual.

3 Mixed attenuation of disturbance and initial-state uncertainty

From the definition, a DIA control should be an H_∞ control when the initial state is known($x_0 = 0$). This implies that, in order to solve the DIA control problem, we require the so-called Riccati equation conditions:

(A1) There exists a solution $M > 0$ to the Riccati equation

$$\begin{aligned} &M(A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1) \\ &+(A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1)^T M \\ &-M(B_2(D_{12}^T D_{12})^{-1} B_2^T - B_1 B_1^T) M \\ &+C_1^T C_1 - C_1^T D_{12}(D_{12}^T D_{12})^{-1} D_{12}^T C_1 = 0 \end{aligned} \qquad (5)$$

such that

$$\begin{aligned} &A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1 \\ &\quad -B_2(D_{12}^T D_{12})^{-1} B_2^T M + B_1 B_1^T M \end{aligned} \qquad (6)$$

is stable.

(A2) There exists a solution $P > 0$ to the Riccati equation

$$\begin{aligned} &(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2) P \\ &+P(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2)^T \\ &-P(C_2^T (D_{21} D_{21}^T)^{-1} C_2 - C_1^T C_1) P \\ &+B_1 B_1^T - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} D_{21} B_1^T = 0 \end{aligned} \qquad (7)$$

such that

$$A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2$$

$$-PC_2^T(D_{21}D_{21}^T)^{-1}C_2 + PC_1^TC_1 \qquad (8)$$

is stable.

(A3) $\rho(PM) < 1$,
where $\rho(X)$ denotes the spectral radius of matrix X, and $\rho(X) = \max|\lambda_i(X)|$.

Then we can obtain the following result.

Lemma 1 *Suppose that the conditions (A1), (A2) and (A3) are satisfied, then the central control satisfies the following inequality.*

$$\|z\|_2^2 \leq \|w\|_2^2 + x_0^TP^{-1}x_0 \qquad (9)$$

for all $w \in L^2[0,\infty)$, and all $x_0 \in R^n$, s.t., $(w, x_0) \neq 0$, where the central control is given by

$$\begin{aligned} u &= -(D_{12}^TD_{12})^{-1}(B_2^TM + D_{12}^TC_1)(I-PM)^{-1}\zeta \\ \dot{\zeta} &= A\zeta + B_2u + PC_1^T(C_1\zeta + D_{12}u) \\ &\quad +(PC_2^T + B_1D_{21}^T)(D_{21}D_{21}^T)^{-1}(y - C_2\zeta) \\ \zeta(0) &= 0 \end{aligned} \qquad (10)$$

and $S := M(I-PM)^{-1}$.

Proof: First note that $S = M(I-PM)^{-1}$ satisfies the Riccati equation

$$\begin{aligned} &S(A + PC_1^TC_1 - (B_2 + PC_1^TD_{12})(D_{12}^TD_{12})^{-1}D_{12}^TC_1) \\ &\quad +(A + PC_1^TC_1 \\ &\quad\quad - (B_2 + PC_1^TD_{12})(D_{12}^TD_{12})^{-1}D_{12}^TC_1)^TS \\ &\quad -S((B_2 + PC_1^TD_{12})(D_{12}^TD_{12})^{-1}(B_2 + PC_1^TD_{12})^T \\ &\quad\quad - (PC_2^T + B_1D_{21}^T)(D_{21}D_{21}^T)^{-1}(PC_2^T + B_1D_{21}^T)^T)S \\ &\quad +C_1^TC_1 - C_1^TD_{12}(D_{12}^TD_{12})^{-1}D_{12}^TC_1 = 0 \end{aligned} \qquad (11)$$

Consider the functional $V(t)$,

$$V(t) := \zeta^TS\zeta + (x-\zeta)^TP^{-1}(x-\zeta) \qquad (12)$$

then, differentiating both sides with respect to t, and inserting conditions (A1)-(A3) into the right hand side, we have

$$\begin{aligned} \dot{V}(t) &= -\|z\|^2 + \|w\|^2 \\ &\quad +\|(D_{12}^TD_{12})^{1/2}u + (D_{12}^TD_{12})^{-1/2} \\ &\quad\quad \times (B_2^TM + D_{12}^TC_1)(I-PM)^{-1}\zeta\|^2 \\ &\quad -\|w - D_{21}^T(D_{21}D_{21}^T)^{-1}(C_2P + D_{21}B_1^T)S\zeta \\ &\quad\quad - (B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1}(C_2P + D_{21}B_1^T)) \\ &\quad\quad \times P^{-1}(x-\zeta)\|^2 \end{aligned} \qquad (13)$$

integrating both sides with respect to t over the interval $[0,\infty)$, we obtain the left hand side as

$$V(\infty) - V(0) = -x_0^TP^{-1}x_0$$

implying the control input $u(t)$ as (10), and

$$\begin{aligned} -x_0^TP^{-1}x_0 &= -\|z\|_2^2 + \|w\|_2^2 \\ &\quad -\|w - D_{21}^T(D_{21}D_{21}^T)^{-1}(C_2P + D_{21}B_1^T)S\zeta \\ &\quad\quad - (B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1}(C_2P + D_{21}B_1^T)) \\ &\quad\quad \times P^{-1}(x-\zeta)\|_2^2 \end{aligned} \qquad (14)$$

then we finally obtain as

$$\|z\|_2^2 \leq \|w\|_2^2 + x_0^TP^{-1}x_0$$

■

This Lemma is concerned with the condition for P, not for N. This conclusion does not solve the infinite horizon DIA control problem, because the inequality (9) does not generally imply the inequality (3). Next, the following condition is assumed.

(A4) $N < P, \quad (N^{-1} > P^{-1})$.

If the condition (A4) holds, the inequality (3) follows from the inequality (9), and the central control (10) is a DIA control.

$$\|z\|_2^2 \leq \|w\|_2^2 + x_0^TP^{-1}x_0 < \|w\|_2^2 + x_0^TN^{-1}x_0 \qquad (15)$$

Since N is regarded as a measure of initial state uncertainties, e.g., a variance matrix, we can state that, if the initial state uncertainty is sufficiently small (so that (A4) holds), the central control has robustness against the initial state uncertainty. In view of the discussion above, the condition (A4) seems necessary for the central control to be a DIA control. We will show that this is not true by presenting a necessary and sufficient condition, which is the main result of this paper. In order to state the result, let us introduce the following condition:

(A5) $Q + N^{-1} - P^{-1} > 0$,
where Q is the maximal solution of the Riccati equation

$$\begin{aligned} &Q(A - B_1D_{21}^T(D_{21}D_{21}^T)^{-1}C_2 \\ &\quad\quad + (B_1B_1^T - B_1D_{21}^T(D_{21}D_{21}^T)^{-1}D_{21}B_1^T)P^{-1}) \\ &\quad +(A - B_1D_{21}^T(D_{21}D_{21}^T)^{-1}C_2 \\ &\quad\quad + (B_1B_1^T - B_1D_{21}^T(D_{21}D_{21}^T)^{-1}D_{21}B_1^T)P^{-1})^TQ \end{aligned}$$

$$
\begin{aligned}
&-Q(B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1}(C_2P + D_{21}B_1^T)L)^T \\
&\quad \times (B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1}(C_2P + D_{21}B_1^T)L)Q \\
&= 0 \qquad (16)
\end{aligned}
$$

with $L := (I - PM)^{-1}$.

Theorem 1 *Suppose that the conditions (A1), (A2), and (A3) are satisfied. The central control (10) is a DIA control if and only if the condition (A5) is satisfied.*

Remark 1 *The Riccati equation (16) has the maximal solution $Q \geq 0$ for any P^{-1}. such that*

$$
\begin{aligned}
&A - B_1D_{21}^T(D_{21}D_{21}^T)^{-1}C_2 \\
&\quad +(B_1B_1^T - B_1D_{21}^T(D_{21}D_{21}^T)^{-1}D_{21}B_1^T)P^{-1} \\
&\quad -(B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1}(C_2P + D_{21}B_1^T)L)^T \\
&\quad (B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1}(C_2P + D_{21}B_1^T)L)Q \qquad (17)
\end{aligned}
$$

is stable, since (A, B_1) is stabilizable. Hence, (A4) is a sufficient condition for the condition (A5) to be satisfied.

4 Proof of Theorem 1

We prove Lemma 2 and Lemma 3. Then Theorem 1 follows. Lemma 2 and Lemma 3 require the following condition:

(A6) : For all $(w) \in L^2[0, \infty)$ and all $x_0 \in R^n$, s.t., $(w, x_0) \neq 0$, the inequality

$$
\|w - w_0\|_2^2 + x_0^T\left(N^{-1} - P^{-1}\right)x_0 > 0 \qquad (18)
$$

holds, where w_0 is given by

$$
\begin{aligned}
w_0 &= D_{21}^T(D_{21}D_{21}^T)^{-1}(C_2P + D_{21}B_1^T)S\zeta \\
&\quad +(B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1}(C_2P + D_{21}B_1^T)) \\
&\quad \times P^{-1}(x - \zeta) \qquad (19) \\
\dot{\zeta} &= (A + PC_1^TC_1 - (B_2 + PC_1^TD_{12})(D_{12}^TD_{12})^{-1} \\
&\quad \times (B_2^TM + D_{12}^TC_1)L \\
&\quad +(PC_2^T + B_1D_{21}^T)(D_{21}D_{21}^T)^{-1} \\
&\quad \times (C_2P + D_{21}B_1^T)S)\zeta \\
&\quad +(PC_2^T + B_1D_{21}^T)(D_{21}D_{21}^T)^{-1}D_{21} \\
&\quad \times (w - w_0) \qquad (20)
\end{aligned}
$$

Lemma 2 *Suppose that the conditions (A1), (A2), and (A3) are satisfied. The central control (10) is a DIA control if and only if the condition (A6) is satisfied.*

Proof: Consider the functional $V(t) = \zeta^TS\zeta + (x - \zeta)^TP^{-1}(x - \zeta)$, then, differentiating both sides with respect to t, and inserting (1) and (10) into the right hand side, and integrating both sides with respect to t over the interval $[0, \infty)$, we obtain

$$
-x_0^TP^{-1}x_0 = -\|z\|_2^2 + \|w\|_2^2 - \|w - w_0\|_2^2 \qquad (21)
$$

Insert (21) into (18), then, we have

$$
\|z\|_2^2 < \|w\|_2^2 + x_0^TN^{-1}x_0. \qquad (22)
$$

Converse, insert (21) into (3), then, we have

$$
\|w - w_0\|_2^2 + x_0^T\left(N^{-1} - P^{-1}\right)x_0 > 0 \qquad (23)
$$

■

Lemma 3 *Suppose that the conditions (A1), (A2), and (A3) are satisfied. The condition (A6) is equivalent to the condition (A5)*

Proof: Consider the functional $U(t) := f^TQf$, where $f(t) := x(t) - L\zeta(t)$ is given by

$$
\begin{aligned}
\dot{f}(t) &= (A + B_1(B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1} \\
&\quad \times (C_2P + D_{21}B_1^T))P^{-1})f \\
&\quad + \left(B_1 - L(PC_2^T + B_1D_{21}^T)(D_{21}D_{21}^T)^{-1}D_{21}\right) \\
&\quad \times (w - w_0), \quad f(0) = x_0. \qquad (24)
\end{aligned}
$$

Differentiating both sides with respect to t and completing the square argument as

$$
\begin{aligned}
\dot{U}(t) &= \|w - w_0 + (B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1} \\
&\quad \times (C_2P + D_{21}B_1^T)L)Qf\|^2 \\
&\quad -\|w - w_0\|^2 \qquad (25)
\end{aligned}
$$

then, integrating both sides with respect to t over the interval $[0, \infty)$, we finally obtain

$$
\begin{aligned}
-x_0^TQx_0 &= \|w - w_0 + (B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1} \\
&\quad \times (C_2P + D_{21}B_1^T)L)Qf\|_2^2 \\
&\quad -\|w - w_0\|_2^2 \qquad (26)
\end{aligned}
$$

Inserting (26) into the condition (A6), then we have

$$
\begin{aligned}
&\|w - w_0 + (B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1} \\
&\quad \times (C_2P + D_{21}B_1^T)L)Qf\|_2^2 \\
&\quad + x_0^T\left(Q + N^{-1} - P^{-1}\right)x_0 > 0. \qquad (27)
\end{aligned}
$$

The 1st term in the left hand side are positive, hence

$$
Q + N^{-1} - P^{-1} > 0.
$$

■

5 Application to Magnetic Suspension Systems

The state-space representation of the magnetic suspension system is given as follows[6, 7].

$$\begin{aligned} \dot{x}_g &= A_g x_g + B_g u_g + D_g v_0 \\ y_g &= C_g x_g + w_0 \end{aligned} \tag{28}$$

where $x_g := [x \;\; \dot{x} \;\; i]^T$, u_g is a control input, v_0 and w_0 are exogenous disturbance inputs, $x(t)$ is a gap between the electromagnet and a suspended iron ball, and $i(t)$ is a current. First, let us consider the disturbances v_0 and w_0. Since v_0 mainly acts on the plant in a low frequency, and w_0 shows an uncertainty caused via unmodeled dynamics. Hence let v_0 and w_0 be of the form

$$v_0 = W_v(s) w_2 \tag{29}$$

$$W_v = \Phi C_w (sI - A_w)^{-1} B_w, \quad \Phi = [1 \quad 1]^T$$

$$w_0 = W_w w_1 \tag{30}$$

where W_w is a weighting scalar. Next we consider the variables which we want to regulate. In this study, the gap and the corresponding velocity are chosen. Then, as the error vector, let us define as follows;

$$z_g = F_g x_g, \quad F_g = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{31}$$

$$z_1 = \Theta z_g, \; \Theta = \mathrm{diag}\begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix}, \; z_2 = \rho u \tag{32}$$

where Θ is a weighting matrix on the regulated variables z_g, and ρ is an weighting scalar for the regulation of the control input $u (:= u_g)$. Finally, let $x := [\, x_g^T \quad x_w^T \,]^T$, where x_w denotes the state of $W_v(s)$, and $w := [\, w_1^T \quad w_2^T \,]^T$, $z := [\, z_1^T \quad z_2^T \,]^T$, then we can construct the generalized plant as in (33). Note that this plant does not have the orthogonality assumptions[5].

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{12} u \\ y &= C_2 x + D_{21} w \end{aligned} \tag{33}$$

$$A = \begin{bmatrix} A_g & D_g C_w \\ 0 & A_w \end{bmatrix}, \; B_1 = \begin{bmatrix} 0 & D_g D_w \\ 0 & B_w \end{bmatrix}, \; B_2 = \begin{bmatrix} B_g \\ 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} \Theta F_g & 0 \\ 0 & 0 \end{bmatrix}, \; D_{12} = \begin{bmatrix} 0 \\ \rho \end{bmatrix},$$

$$C_2 = \begin{bmatrix} C_g & 0 \end{bmatrix}, \; D_{21} = \begin{bmatrix} W_w & 0 \end{bmatrix}$$

Now our control problem setup is: find an admissible controller $K(s)$ that attenuates disturbances and initial state uncertainties to achieve DIA condition in (3).

After some control design iteration, the design parameters; $W_v(s)$, W_w, Θ and ρ are chosen appropriately, and a direct calculations yield the H_∞ DIA controller $K(s)$. The frequency response of the controller $K(s)$ is shown in Fig.1 by a solid line. And the maximum value of the weighting matrix N is $N = 2.7735 \times 10^{-2} \times I$. We designed the standard H_∞ controller denoted as K_∞ [7] via the MATLAB command `hinfsyn.m`. The frequency response of the controller K_∞ and the previous DIA controllers $K_{DIA_1}(s)$ and $K_{DIA_2}(s)$[3, 6] are shown in Fig.1 by a dotted line, a dashed line and a dash-dot line, respectively. Comparing these four controllers, $K(s)$ has a high gain at the low frequency and a good roll-off property at the high frequency, and the comprehensive frequency response looks like a modified PID controller. In the previous DIA design framework, it was difficult to let controllers $K_{DIA_1}(s)$ and $K_{DIA_2}(s)$[3, 6] get hold an integral property.

We have conducted simulations to evaluate properties of $K(s)$. To ascertain transient responses, we input a step reference signal to the system with a nonzero initial state x_0. An initial response for $x_0 = [0.0 \;\; 0.0 \;\; 0.1]^T$ is shown in Fig.2, and a time response for a step reference signal(0.0[mm] $\to$ 0.1[mm]) is shown in Fig.3, where the signal is added to the system around 1.0[s]. The K_{DIA_1} and K_{DIA_2} show relatively better performance than K for the initial state uncertainty in Fig.2, K has, however, a better transient performance than K_∞. Since our concerns are not only in the attenuation of the initial state uncertainty, but also in the basic control performance of the controllers, we then wonder whether the controller has a good performance for the step reference signal. Controller K shows better and quicker transient response than K_∞. K_{DIA_1} and K_{DIA_2} show pretty quick responses but bigger overshoots around 1.0[s] because of their high gain at the high frequency in Fig.1, however we must give careful attention for steady-state error with those controllers. K_{DIA_1} and K_{DIA_2} leave steady-state errors because of their low gain at the low frequency in Fig.1. In the previous problem setup, the degrees of freedom in the design parameters are limited, so that it is difficult to shape a good controller frequency response[6]. Considering all the factors, we reached the conclusion that K has a pretty good performance for all control requirements, and has a potential ability to be improved by using the degrees of freedom in the design parameters.

6 Conclusion

In this paper, we formulated and solved a generalized H_∞ control problem which considers a mixed attenuation of disturbance and initial-state uncertainty in the infinite-horizon case, without the orthogonality assumptions. The solution was given as a natural but complicated extension of the previous results in [3, 6]. A necessary and sufficient condition for a solution of the generalized H_∞ mixed attenuation problem to exist, together with an explicit formula of the solution, was derived. Based on the condition, a robustness property of H_∞ controls against initial-state uncertainty was discussed.

Moreover, we applied an infinite horizon disturbance and initial state uncertainty attenuation control problem without the orthogonality assumptions to the magnetic suspension system. Comparing the proposed controller with the standard H_∞ controller and the other controllers based on previous results[6], we showed the property and effectiveness of the proposed mixed attenuation controller.

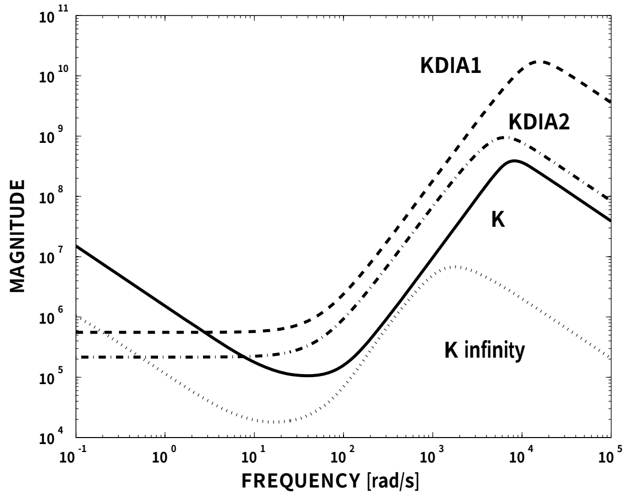

Figure 1: Frequency Response of the controller K with K_{DIA_1} , K_{DIA_2} and K_∞

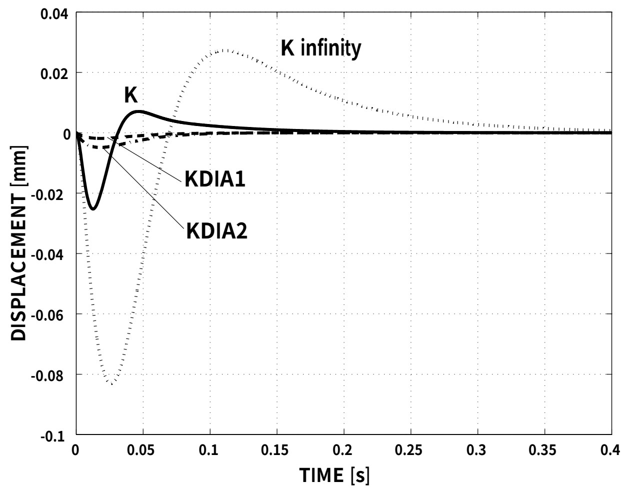

Figure 2: Initial Responses for $x_0 = [0.0 \ \ 0.0 \ \ 0.1]^T$

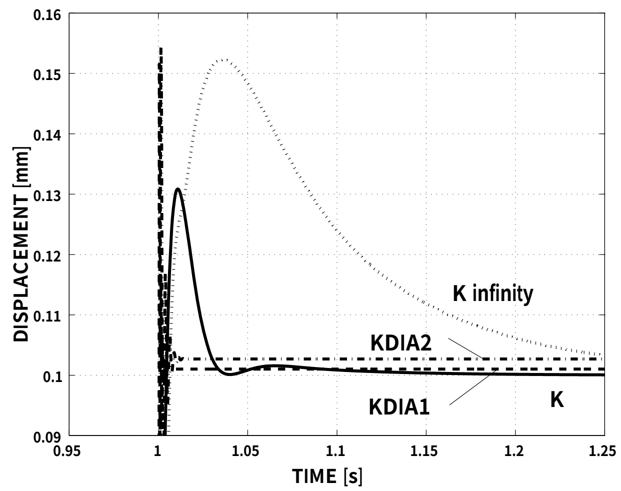

Figure 3: Step Responses

References

[1] K. Uchida and M. Fujita, "Controllers Attenuating Disturbance and Initial-Uncertainties for Time-Varying Systems," *Lecture Notes in Control and Information Sciences*, vol. 156, Springer-Verlag, pp. 187 - 196, 1991.

[2] P. P. Khargonekar, K. M. Nagpal and K. R. Poolla, "H_∞ Control with Transient," *SIAM J. Control and Optimization*, vol. 29, pp. 1373 - 1393, 1991.

[3] K. Uchida and A. Kojima and M. Fujita, "H_∞ control attenuating initial-state uncertainties," *Int. J. of Control*, vol. 66, no. 2, pp. 245 - 252, 1997.

[4] A. Kojima, M. Fujita, K. Uchida and E. Shimemura, "Linear Quadratic Differential Games and H_∞ Control - A Direct Approach Based on Completing the Square -," *Trans. SICE*, vol. 28, no. 5, pp. 570 - 577, 1992.

[5] J. C. Doyle, K. Glover, P. P. Khargonekar, and B. A. Francis, "State-Space Solutions to Standard $\mathcal{H}_\in$ and $\mathcal{H}_\infty$ Control Problems," *IEEE Trans. Automatic Control,* vol. 34, no. 8, pp. 831-847, 1989.

[6] T. Namerikawa, M. Fujita and R. S. Smith, " H_∞ Control System Design Attenuating Initial State Uncertainties : Evaluation by a Magnetic Suspension System", *Proc.of IEEE Conf. on Decision and Control*, pp. 87-92, 2001.

[7] M. Fujita, T. Namerikawa, F. Matsumura, and K. Uchida, "μ-Synthesis of an Electromagnetic Suspension System," *IEEE Trans. Automatic Control,* vol. 40, no. 3, pp. 530-536, 1995.