Four-channel force-reflecting teleoperation with impedance control

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Abstract: This paper focuses on a bilateral control of teleoperation system with four-channel force-reflection (FR) algorithm and communication time delays. In this method, we propose a new system input of impedance control that relates to FR scheme of teleoperation. The goal of this paper is to improve the tracking performance and transparency, the control-based force-reflecting teleoperation uses a force feed forward channel in comparison with a conventional method to transfer the position, velocity and human force information from the master side to the slave side. Hence, we receive a four-channel architecture of the teleoperation system. Using variable damping values, the contact stability is achieved at the time when the slave robot contacts with the environment. To analyse stability of the system, one method based on Lyapunov technique is concerned, the input-to-state stability (ISS) small gain approach is used to show the overall force-reflecting teleoperation to be input-to-state stable. Several experimental results show the effectiveness of our proposed algorithm.

Keywords: bilateral teleoperation; impedance control; four-channel force-reflection; communication delays; input-to-state stability; ISS.

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1 Introduction

Teleoperation systems allow persons to extend their intelligence and manipulation capabilities to remove place and/or hazardous environments through coordinated control of two robotic arms, i.e., a master robot controller that is used by a human operator, and a slave robot that manipulates the environment. During the last several decades, many different teleoperation systems have been developed with wide applications in different circumstances such as use in outer space, undersea, in nuclear plants, in surgical operations, in vehicle steering, in human rescues etc. and this field is being pursued by many researchers (Hokayem and Spong, 2006; Sato et al., 2008b).

In bilateral teleoperation, the master and slave robots are coupled via communication lines, where the position and/or force information are transferred. Communication delays are incurred in the transmission of data between the master and slave. It is well known that the delay in a closed-loop system may destabilise and deteriorate the tracking performance and transparency of the teleoperation system (e.g., Lawrence, 1993; Chopra et al., 2006; Polushin et al., 2007; Kawada and Namerikawa, 2008; Kawada et al., 2007).

While accurate tracking is essential for the skilful control of tasks, it is not enough to achieve the good performance on its own, since position is not the only relationship that exists between both robots. In fact, at the moment that the slave robot starts its interaction with the environment, reflecting forces appear and arise. If we do not notice, this force can be uncontrolled and can become a danger in several tasks. Consequently, the feedback of the force is very important and extremely useful, and it leads to so-called force reflection (FR) in a master-slave system. The FR scheme tries not only to achieve good tracking during unconstrained motion, but also to convey precise information of the forces between the slave robot and the environment. Therefore, the operator can actually feel these forces on the master robot (Lawrence, 1993; Alessandro and Claudio, 1998; Hashtrudi-Zaad and Salcudeam, 1999).

Up to now, many surveys concern the motion and force control problem, especially for situations in which the slave robot end-effector is in contact with the environment. On other hand, the environment may behave as a simple dynamics system undergoing small but finite deformations in response the applied forces. When contact occurs, the arising forces will be dictated by the dynamic balancing of two coupled systems, the slave robot and the environment. To assign a prescribed dynamic behaviour for the robot while its end-effector is interacting with the environment and contact force must be kept small, the impedance control is utilised. This control is to maintain a derived dynamic relationship between the end-effector and the environment, and a second order mass-spring-damper system is used to specify the target dynamics. It gives a unified approach for controlling the robot in both free space and constrained motion control.

A seminal work on impedance control was published by Hogan (1985). In this work, the complete knowledge of the dynamic model of the robot is need to implement, via computed torque technique and the desired impedance; however, the teleoperation has not been treated. Cho and Park (2005) also used impedance control based on computed torque approach and applied this method for teleoperartion system. The control objective of the control law is to make mimic a passive mechanical tool with a force-reflecting ability; and variable damping was proposed to improve the tracking performance and to satisfy the contact stability concurrently. However, the tracking position error of this method is large even if the time-varying delay has not been treated. Aliagam et al. (2004) proposed one control method-based impedance control in comparison with some controllers of a 2-DOF master-slave system; the position tracking during unconstrained motion and tracking of force in contact tasks have been concerned. One other tracking control of robots based on robust adaptive was presented by Sato et al. (2008a) with input uncertainties, nevertheless, this work has not applied for teleoperation system. A new force reflecting teleoperation methodology with adaptive impedance control was used by Love and Book (2004) to reduce operator energy requirements without sacrificing stability. However, the time delay in the communication line has not been treated.

In addition, to improve the transparency of bilateral teleoperation with communication delays, a force-reflection (FR) scheme was addressed by Polushin et al. (2007), this proposal related to one previous work of same the author in 2005, a stabilisation scheme for force reflecting teleoperation was introduced. In this new strategy, the method is based on PD control, the authors continue focusing on the problem of the stability of the force-reflecting teleoperation in the presence of time-varying possibly unbounded unknown communication delays; Using the proposed FR algorithm, they showed that the overall stability of the force-reflecting teleoperation can be achieved for an arbitrarily low damping on the master side for arbitrarily high FR gain. Following the points of this work, high damping of the master robot leads to transparency deterioration, however, it keeps the tracking performance and the contact stability to be good in case the slave robot contact with an obstacle. The simulation results of this work illustrated these points. The tracking performance seems good in the free motion, but in contact task, the tracking position error is very large. In addition, when the slave contact with the environment, this contact is unstable at the first several seconds of the contact. The force of human that exerted on the master robot also was not shown in the results of the simulation.

In this paper, we focus on transparency and tracking performance improvement of teleoperation system by using an impedance control based on inverse dynamics. The goal of this paper is to propose force feed forward which relates to a new proposed four-channel FR algorithm under time-varying delays in the communication lines to guarantee the overall stability, the convergence of position tracking to zero and small force tracking error. Beside the reflecting force from environment that transferred back to the master side (Polushin et al., 2007), our work proposes to transfer the force exerted by the human to the slave side with a system of four communication channels for teleoperation.

On the other hand, to solve the unstable contact problem of the conventional method (Polushin et al., 2007), the damping modulation method that uses distance measurement is used to achieve contact stability with good transparency and tracking performance concurrently. In our opinion, the sensation of the human operator is important; by using this proposal of FR algorithm, the human can feel alteration of the force at the end-effector of the slave robot in contact tasks. To improve the stability analysis of our previous work (Nam and Namerikawa, 2009), the input-to-state stability (ISS) small gain approach is used to show the overall FR teleoperation system to be input-to-state stable. This method is also introduced by Sontag and Wang (1995). Several experimental results show the effectiveness of our proposed algorithm.

2 Problem formulation

2.1 Dynamics of teleoperation system

In this paper, we consider a pair of robotic system coupled via communication lines with time-varying delays. Assuming the absence of friction, other disturbances and gravity term, the master and slave dynamics with $n-$ DOF are described as:

$$\begin{cases} M_m\left(q_m\right)\ddot{q}_m + C_m\left(q_m,\dot{q}_m\right)\dot{q}_m = \tau_m + J_m^T F_{op} \\ M_s\left(q_s\right)\ddot{q}_s + C_s\left(q_s,\dot{q}_s\right)\dot{q}_s = \tau_s - J_s^T F_e \end{cases} \quad (1)$$

where the subscript 'm' and 's' denote the master and slave indexes, respectively, $q_m, q_s \in R^{n\times 1}$ are the joint angle vectors, $\dot{q}_m, \dot{q}_s \in R^{n\times 1}$ are the joint velocity vectors, $\ddot{q}_m, \ddot{q}_s \in R^{n\times 1}$ are the joint acceleration vectors, $\tau_m, \tau_s \in R^{n\times 1}$ are the input torque vectors, $F_{op} \in R^{n\times 1}$ is the operational force vector applied to the master robot by the human operator, $F_e \in R^{n\times 1}$ is the environmental force vector reflected from the environment to the slave robot, $M_m, M_s \in R^{n\times n}$ are the symmetric and positive definite inertial matrices, $C_m\dot{q}_m, C_s\dot{q}_s \in R^{n\times 1}$ are the centripetal and Coriolis torque vectors, and $J_m, J_s \in R^{n\times n}$ are Jacobian matrices.

Considering that position encoders measure manipulator coordinate q_i, with $i = m, s$, Cartesian coordinate must be related to these coordinate:

$$z_i = h_i\left(q_i(t)\right)$$

where h_i are the transformed functions to converse the coordinate from joint space to task space; z_i are the positions of end-effector of robots in task space. Their derivatives through the Jacobian matrix $J_i\left(q_i\right)$ as follows:

$$\dot{z}_i = J_i\left(q_i\right)\dot{q}_i \quad (2)$$

Following the motion of the master, the slave manipulator interacts with the environment. Here, the environment is assumed to be a simple spring-damper system with constant parameter. This system is as a perturbed system described by the equations below in the form of ISS properties:

$$\begin{cases} \dot{x}_e = F_{env}\left(t, x_e, z_s, \dot{z}_s\right) + g_e\left(t, x_e, z_s, \dot{z}_s\right) \\ F_e = \Gamma_{env}\left(t, x_e, z_s, \dot{z}_s\right) \end{cases} \quad (3)$$

where $x_e \in R^z$ is a position of the environment, z_s and $\dot{z}_s \in R^{n\times 1}$ are the end-effector position vector and velocity vector of the slave robot in task space, respectively. We assume that $F_{env}\left(x_e, z_s, \dot{z}_s, t\right), \Gamma_{env}\left(t, x_e, z_s, \dot{z}_s\right)$ are piecewise continuous in t and locally Lipschitz in $x_e, z_s, \dot{z}_s$. The input $\left(z_s(t), \dot{z}_s(t)\right)$ is a piecewise continuous and essentially bounded function of t for all $t \geq 0$; $g_e\left(t, x_e, z_s, \dot{z}_s\right)$ is the perturbation term could result from modelling errors, aging, or uncertainty and disturbances which exist in any realistic problem. We assume that the operator force and environmental force satisfy following assumptions:

Assumption 1: The operator force F_{op} is bounded.

Assumption 2: The slave contacts with following spring-damper environment with constant parameter:

$$\Gamma_{env}\left(t, x_e, z_s, \dot{z}_s\right) \leq \left|x_e\right| + a\left|\dot{z}_s\right| + b\left|z_s\right| \quad (4)$$

holds for all $t \geq 0$, where a, b are constant parameters $(a, b > 0)$.

Assumption 3: Let $x_e = 0$ be a uniformly asymptotically stable equilibrium point of the nominal system (3). There exists a Lyapunov function of the nominal system such that $\alpha_{1e}\left(\left|x_e\right|\right) \leq V_e\left(x_e\right) \leq \alpha_{2e}\left(\left|x_e\right|\right)$ holds for all $x_e \in R^z$, and $V_e = 0$ while $x_e = 0$. The time derivative of V_e along trajectories of (3) satisfies:

$$\dot{V}_e(t) \leq -\alpha_{3e}\left|x_e\right|^2 + \left|\frac{\partial V_e}{\partial x}\right| g\left(t, x_e, z_s, \dot{z}_s\right) \quad (5)$$

where $\alpha_{1e}\left(\left|x_e\right|\right), \alpha_{2e}\left(\left|x_e\right|\right)$ are class $\mathcal{K}$ functions and $\alpha_{3e} > 0$. The perturbation $g_e\left(t, x_e, z_s, \dot{z}_s\right)$ in (5) satisfies the uniform bound:

$$\left|\frac{\partial V_e}{\partial x}\right|\left|g_e\left(t, x_e, z_s, \dot{z}_s\right)\right| \leq \delta\alpha_{4e}\left|x_e\right| \leq F_e^T s_e(t) \quad (6)$$

for almost all $t \geq 0$ and $\in R^z; \alpha_{4e} > 0$ and δ is a perturbation gain.

Let us define:

$$s_e(t) = \dot{z}_s(t) + \Lambda_{env} z_s(t) \tag{7}$$

where $\Lambda_{env} \in R^{n \times n}$ is a positive diagonal gain matrix.

Note the first bound of the perturbation in (6), we have:

$$\begin{aligned} \dot{V}_e &\le -\alpha_{3e}|x_e|^2 + \theta_e\alpha_{3e}|x_e|^2 - \theta_e\alpha_{3e}|x_e|^2 + \delta\alpha_{4e}|x_e| \\ &= -(1-\theta_e)\alpha_{3e}|x_e|^2 - |x_e|\left(\theta_e\alpha_{3e}|x_e| - \delta\alpha_{4e}\right) \\ &\le -(1-\theta_e)\alpha_{3e}|x_e|^2 ; \forall |x_e| \ge \frac{\delta\alpha_{4e}}{\theta_e\alpha_{3e}} \end{aligned} \tag{8}$$

where θ_e is some positive constant, $\theta_e < 1$.

Therefore, the upper bound of perturbation in (6) satisfies the time derivative of V_e as follows:

$$\dot{V}_e(t) \le -\alpha_{3e}|x_e|^2 + F_e^T s_e(t) \tag{9}$$

Remark 2.1: We consider the role of the perturbation term to the stability behaviour of the perturbation system (3). If $g(t,0) = 0, (x_e = 0, \dot{z}_s = 0, z_s = 0)$ the system (3) has an equilibrium point at the origin. In this case, we can analyse the stability behaviour of the origin as an equilibrium point of the perturbation system. Otherwise, in general case, $g(t,0) \ne 0,$ the origin will not be an equilibrium point of the perturbation system. Therefore, we can no longer concern the problem as a question of stability of equilibrium. This case will be treated later in Lemma 2 with the stability analysis of slave subsystem in Section 5.

Assumptions 1, 2 and 3 are utilised later to prove stability of the master/slave + environment subsystem results.

2.2 Control objectives

The main goals of the teleoperation system from control theoretical point of view are stability, synchronisation and transparency. Thus, we would like to design the new control algorithm for the FR teleoperation system to achieve the following three control objectives. Let us define the position tracking errors of the end-effectors as follows:

$$\begin{cases} e_m(t) = z_m(t) - z_s\left(t - T_s(t)\right) \\ e_s(t) = z_s(t) - z_m\left(t - T_m(t)\right) \end{cases} \tag{10}$$

where $T_m(t)$ and $T_s(t)$ are time varying delays in the communication lines.

Control objective 1: The overall teleoperation system with four-channel force-reflecting teleoperation is stable in the sense of master/slave position and velocity are bounded.

Control objective 2: The task-space synchronisation of teleoperation is achieved as:

$$e_i(t) \to 0 \ as \ t \to \infty, \ i = m, s \tag{11}$$

Control objective 3: The transparency is achieved with $\ddot{z}_i(t) = \dot{z}_i(t) = 0, \ i = m, s$ as:

$$F_{op}(t) \to F_e(t) \tag{12}$$

where $\ddot{z}_{m/s}$ are accelerations of the master and slave robots.

3 Control design

3.1 Impedance controller

A precise knowledge about the values of the dynamic parameters allows the implementation of an inverse dynamics algorithm as impedance controller. Here, following the proposal by Aliagam et al. (2004), the torque given by the motors can be split into two terms, the first arising from the teleoperation τ_{tel}, and the second from the impedance control τ_{inv}. The torque inputs of the system are shown as follows:

$$\tau_i = \tau_{inv_i} + \tau_{tel_i}, \ (i = m, s) \tag{13}$$

where the second term is defined as: $\tau_{tel_i} = J_i^T F_{tel_i}$. If we assume H_i and B_i to be the desired mass and desired damping and they are assumed positive definite diagonal matrices; F_{ext_i} represents the forces exerted on each robot which include reflection force information in, and F_{tel_i} represents the forces via teleoperation $(i = m, s)$.

Applying the approach of Hogan (1985), the target relationship between the movement of each robot and the force that acts on it is expressed as follows:

$$H_i\ddot{z}_i + B_i\dot{z}_i = F_{ext_i} + F_{tel_i} \tag{14}$$

Concerning (2) we get the further differentiation as:

$$\ddot{z}_i = J_i(q_i)\ddot{q}_i(t) + \dot{J}(q_i)\dot{q}_i^2 \tag{15}$$

Substituting (15) and (2) in to (14) and operating, we can calculate the acceleration of the system as follows:

$$\begin{cases} \ddot{q}_m = H_m^{-1}J_m^{-1}\left[F_{ext_m} + F_{tel_m} - B_m J_m \dot{q}_m\right] - J_m^{-1}\dot{J}_m\dot{q}_m^2 \\ \ddot{q}_s = H_s^{-1}J_s^{-1}\left[F_{ext_s} + F_{tel_s} - B_s J_s \dot{q}_s\right] - J_s^{-1}\dot{J}_s\dot{q}_s^2 \end{cases} \tag{16}$$

Here for simplicity, we assume that:

Assumption 4: The Jacobian (J_m, J_s) are invertible, i.e., they are non-singular matrices at all times in operation. They are also called pseudoinverse matrices.

Substituting (16) and (13) into dynamic system (1) and enclosing the above assumption, we get:

$$\begin{cases} \tau_{inv_m} = M_m H_m^{-1}J_m^{-1}\left[F_{ext_m} + F_{tel_m}\right] - M_m H_m^{-1}B_m\dot{q}_m \\ \qquad - M_m J_m^{-1}\dot{J}_m\dot{q}_m^2 + C_m\dot{q}_m - \left(J_m^T F_{tel_m} + J_m^T F_{op}\right) \\ \tau_{inv_s} = M_s H_s^{-1}J_s^{-1}\left[F_{ext_s} + F_{tel_s}\right] - M_s H_s^{-1}B_s\dot{q}_s \\ \qquad - M_s J_s^{-1}\dot{J}_s\dot{q}_s^2 + C_s\dot{q}_s - \left(J_s^T F_{tel_s} - J_m^T F_e\right) \end{cases} \tag{17}$$

substituting (17) into (13), we get:

$$\begin{cases} \tau_m = M_m H_m^{-1} J_m^{-1}\left[F_{ext_m} + F_{tel_m}\right] - M_m H_m^{-1} B_m \dot{q}_m \\ \quad -M_m J_m^{-1} \dot{J}_m \dot{q}_m^2 + C_m \dot{q}_m - J_m^T F_{op} \\ \tau_s = M_s H_s^{-1} J_s^{-1}\left[F_{ext_s} + F_{tel_s}\right] - M_s H_s^{-1} B_s \dot{q}_s \\ \quad -M_s J_s^{-1} \dot{J}_s \dot{q}_s^2 + C_s \dot{q}_s + J_s^T F_e \end{cases} \tag{18}$$

We receive the master slave robot dynamics with impedance controller by substituting (18) into dynamic system (1) as follows:

$$\begin{cases} M_m \ddot{q}_m + C_m \dot{q}_m = M_m H_m^{-1} J_m^{-1}\left[F_{ext_m} + F_{tel_m}\right] \\ \quad - M_m H_m^{-1} B_m \dot{q}_m - M_m J_m^{-1} \dot{J}_m \dot{q}_m^2 + C_m \dot{q}_m \\ M_s \ddot{q}_s + C_s \dot{q}_s = M_s H_s^{-1} J_s^{-1}\left[F_{ext_s} + F_{tel_s}\right] \\ \quad - M_s H_s^{-1} B_s \dot{q}_s - M_s J_s^{-1} \dot{J}_s \dot{q}_s^2 + C_s \dot{q}_s \end{cases} \tag{19}$$

From (19) we get:

$$\begin{cases} H_m\left(\dot{J}_m \dot{q}_m^2 + J_m \ddot{q}_m\right) + B_m J_m \dot{q}_m = F_{ext_m} + F_{tel_m} \\ H_s\left(\dot{J}_s \dot{q}_s^2 + J_s \ddot{q}_s\right) + B_s J_s \dot{q}_s = F_{ext_s} + F_{tel_s} \end{cases} \tag{20}$$

Considering (2) and (15) we receive the desired task space dynamics of the teleoperation system as follows:

$$\begin{cases} H_m \ddot{z}_m + B_m \dot{z}_m = F_{ext_m} + F_{tel_m} \\ H_s \ddot{z}_s + B_s \dot{z}_s = F_{ext_s} + F_{tel_s} \end{cases} \tag{21}$$

In the next section, the FR teleoepration algorithm that relates to the input of the above system control are proposed.

3.2 Four-channel FR teleoperation architecture

In this section, we consider the FR teleoperation system with four-channel FR architecture in the communication delays. The proposed control structure is shown in Figure 1.

The research of Polushin et al. (2007) introduced a FR algorithm to improve the teleoperation transparency. In this strategy, the FR scheme includes the position error, velocity error at the slave side and the actual contact force information. This FR signal is transferred from slave side to the master side. It is clear that, the force tracking performance between the operator force and the actual contact force due to the environment has not been treated in this algorithm.

Note that, since the algorithm was used only to change the FR scheme of the slave, thus when the human operator push an increasing force on master robot, the environmental force also increases, but this alteration is not felt by the human because of the force saturation in this case. However, this algorithm may prevent the teleoperation system from going into unstable mode.

Figure 1 Four-channel FR teleoperation system

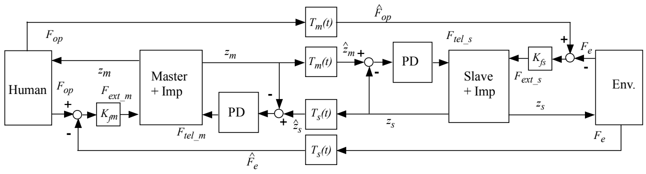
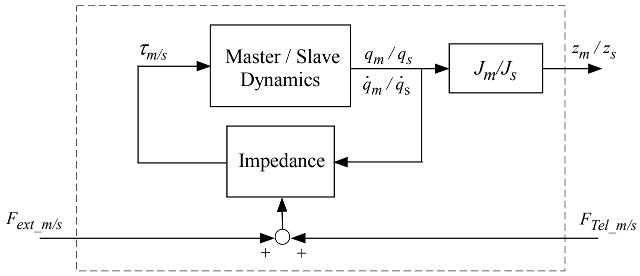


Figure 2 The master and slave robot dynamics with impedance controller



In our opinion, the sensation felt by the human operator is important as it allows the human to feel the alteration of the force exerted on the environment by the FR from the slave side. It helps the human to apply an appropriate force in the real task during teleoperation. Therefore, we propose one more communication channel to transfer the force of the master to the slave side, then, the slave robot can receive directly the force information from the human. The force tracking performance has been treated in our strategy.

To achieve the control objectives, we propose the following control law with force-reflecting on both sides of the teleoperation system. The exerted force is defined as:

$$\begin{cases} F_{ext_m}(t) = K_{f_m}\left(F_{op}(t) - F_e\left(t - T_s(t)\right)\right) \\ F_{ext_s}(t) = K_{f_s}\left(F_{op}\left(t - T_m(t)\right) - F_e(t)\right) \end{cases} \tag{22}$$

where $K_{f_i} \in R^{n\times n} (i = m, s)$ are positive definite diagonal gain matrices; $F_{op}\left(t - T_m(t)\right)$ and $F_e\left(t - T_s(t)\right)$ are reflecting forces from master and slave sides of teleoperation, reflectively.

We assume $K_{p_i}, \bar{K}_{di} \in R^{n\times n}$, $(i = m, s)$ to be positive definite diagonal gain matrices. The controller of the torque arises from teleoperation is proposed as a PD-control as follows:

$$\begin{cases} F_{tel_m}(t) = K_{p_m}\left[z_s\left(t - T_s(t)\right) - z_m(t)\right] \\ \quad + K_{d_m}\left[\dot{z}_s\left(t - T_s(t)\right)\dot{T}_s(t) - \dot{z}_m(t)\right] \\ F_{tel_s}(t) = K_{p_s}\left[z_m\left(t - T_m(t)\right) - z_s(t)\right] \\ \quad + K_{d_s}\left[\dot{z}_m\left(t - T_m(t)\right)\dot{T}_m(t) - \dot{z}_s(t)\right] \end{cases} \tag{23}$$

where K_{d_m} and K_{d_s} are defined depending on $\dot{T}_m(t)$ and $\dot{T}_s(t)$ as follows:

$$\begin{cases} K_{d_m} = \left(1-\dot{T}_s(t)\right)\bar{K}_{d_s} \\ K_{d_s} = \left(1-\dot{T}_m(t)\right)\bar{K}_{d_m} \end{cases} \tag{24}$$

We can see in Figure 1, it shows a block diagram of the control system with impedance-based four-channel FR teleoperation, Figure 2 shows a block of master/slave robot dynamics with an impedance controller.

3.3 Communication delay

Let $T_i : R \to R^+, i \in m,s$ be time-dependent time-delay in the forward $(i = m)$ and backward $(i = s)$ communication channels, respectively. If the positions and velocities of the master and slave are transmitted to each side with communication delays $T_{m/s}(\cdot)$, the following signals

$$\begin{aligned} \hat{z}_m(t) &= z_m\left(t - T_m(t)\right); \dot{\hat{z}}_m(t) = \dot{z}_m\left(t - T_m(t)\right)\dot{T}_m(t) \\ \hat{z}_s(t) &= z_s\left(t - T_s(t)\right); \dot{\hat{z}}_s(t) = \dot{z}_s\left(t - T_s(t)\right)\dot{T}_s(t) \end{aligned} \tag{25}$$

are available for the controller on both sides of teleoperation.

On the other hand, a contact force due to the environment is measured on the slave side and transmitted back to the master side. Similarly, the force exerted on the master manipulator also is measured and transmitted forward to the slave side, with communication delays $T_{m/s}(\cdot)$, i.e.:

$$\begin{cases} \hat{F}_e(t) = F_e\left(t - T_s(t)\right) \\ \hat{F}_{op}(t) = F_{op}\left(t - T_m(t)\right) \end{cases} \tag{26}$$

where $T_m(t)$ and $T_s(t)$ are assumed to be time-varying delays.

Assumption 5: Both $T_m(t)$ and $T_s(t)$ are continuously differentiable functions and possibly bounded as follows:

$$0 \le T_i(t) \le T_i^+ < \infty, \ \left|\dot{T}_i\right| < 1, \ i = m,s$$

where $T_i^+ \in R$ are upper bounds of the communication delays.

4 Damping value modulation

One of the control objectives of teleoperation systems is to achieve a good tracking performance of the slave robot during motions in free space, as well as a good contact stability during motions resulting in contact with the environment. The damping of both master and slave may be empirically constructed to provide the desired alterations. As in much previous research of force reflecting teleoperation system, when the end-effector of the slave robot is controlled to contact with a hard environment, the tracking performance is not good, especially in low damping cases. Sometimes this makes the system unstable after a short time of contact although it had a good tracking performance in free space before (Polushin et al., 2007).

The desired damping values of master and slave robots are also the parameters in the control law; they are selected depending on whether the slave is in free space or in contact with an environment. The variable damping values in these cases are assumed to be bounded for the damping modulation this method was also introduced by Cho and Park (2005).

The master and slave damping matrices are shown as below:

$$B_m = \begin{bmatrix} \tilde{b}_{v_m1}\left(z_y\right) & 0 \\ 0 & \tilde{b}_{v_m2}\left(z_y\right) \end{bmatrix}, B_s = \begin{bmatrix} \tilde{b}_{v_s1}\left(z_y\right) & 0 \\ 0 & \tilde{b}_{v_s2}\left(z_y\right) \end{bmatrix} \tag{27}$$

where $\tilde{b}_{vm1}, \tilde{b}_{vm2}, \tilde{b}_{vs1}, \tilde{b}_{vs2}$ are the variable damping values of master and slave, respectively. These values are modulated according to the distance of the end-effector of the slave robot from a staring point to the other point on the environment surface (following the y-axis). Based on the proposal by Cho and Park (2005), if we call $z_{y_{env}}$ to be y-axis position of the environment, then variable damping values are defined following two positions as:

1 $z_{y_{env}} \le 0$:

$$\tilde{b}_{v_i1/2}\left(z_y\right) = \begin{cases} \underline{b}, & z_y > 0 \\ \Gamma_3 z_y^3 + \Gamma_2 z_y^2 + \Gamma_1 z_y + \Gamma_0, & z_{y\max} \le z_y \le 0 \\ \bar{b}, & z_y < z_{y\max} \end{cases} \tag{28}$$

2 $z_{y_{env}} > 0$:

$$\tilde{b}_{v_i1/2}\left(z_y\right) = \begin{cases} \underline{b}, & z_y < 0 \\ \Gamma_3 z_y^3 + \Gamma_2 z_y^2 + \Gamma_1 z_y + \Gamma_0, & 0 \le z_y \le z_{y\max} \\ \bar{b}, & z_y > z_{y\max} \end{cases} \tag{29}$$

here, $i = m,s$ and $z_y, z_{y\max}$ are the distances from starting point to the position of the end-effector of the slave and to the contact point in the environment, respectively; $\underline{b}$ and $\bar{b}$ are lower and upper bound values of damping. The coefficients $\Gamma_i (i = 1 \square 3)$ are obtained with the constraints of: $\tilde{b}_{v_i1/2}(0) = \underline{b}$ and $\tilde{b}_{v_i1/2}\left(z_{y\max}\right) = \bar{b}$.

5 Stability analysis

This section deals with the stability of the overall teleoperation system that includes master and slave subsystems. First, we consider the master subsystem in dynamic system (1) following the below lemma.

Lemma 5.1: Consider the closed-loop master subsystem to be a piecewise continuous in t and locally Lipschitz in the state $x_M = \left(z_m^T, \dot{z}_m^T\right)^T$ and the input $u_M = \left(F_{ext_m}^{\ T}, \hat{z}_s^T, \dot{\hat{z}}_s^T\right)^T$.

There exists a continuous differentiable, positive definite, radially unbounded Lyapunov function $V_m : R^r \to R$ of the subsystem that satisfies the inequalities:

$$\alpha_{1m}\left(\left|x_M\right|\right) \le V_m \le \alpha_{2m}\left(\left|x_M\right|\right) \tag{30}$$

$$\frac{\partial V_m}{\partial t} + \frac{\partial V_m}{\partial x} f\left(t, x_M, u_M\right) \le -\alpha_{3m}\left(\left|x_M\right|\right), \quad \forall \left|x_M\right| \ge \rho_m\left(\left|u_M\right|\right) > 0 \tag{31}$$

$\forall t \ge 0, D := \left\{x_M \in R^n;\ \left|x_M\right| < r_m\right\}, D_u := \left\{u_M \in R^m;\ \left|u_M\right| < r_{mu}\right\},$

where $\alpha_{1m}\left(\left|x_M\right|\right), \alpha_{2m}\left(\left|x_M\right|\right), \alpha_{3m}\left(\left|x_M\right|\right)$ and ρ_m are class $\mathcal{K}$ functions, then the subsystem is locally input-to-state stable.

Proof: First, consider an ISS-Lyapunov function candidate:

$$V_m = \frac{1}{2}\xi_m^T H_m \xi_m + \frac{1}{2} z_m^T \Sigma_m z_m, \tag{32}$$

here ξ_m is defined as:

$$\xi_m = \dot{z}_m + \Lambda_m z_m$$

where Λ_m, Σ_m are positive diagonal gain matrices. We can easily check that the Lyapunov function V_m satisfies (30) and $V_m(0) = 0$ while $x_M = 0\left(\dot{z}_m = 0, z_m = 0\right)$. Since $\alpha_{1m}\left(x_M\right)$ is radially unbounded, so is $\alpha_{2m}\left(x_M\right)$, then V_m is said to be radially unbounded.

Calculating the time derivative of V_m along trajectories of the subsystem, we get:

$$\dot{V}_m = \xi_m^T H_m \dot{\xi}_m + z_m^T \Sigma_m \dot{z}_m \tag{33}$$

We have derivative of ξ_m as:

$$\dot{\xi}_m = \ddot{z}_m + \Lambda_m \dot{z}_m$$

Substituting $\ddot{z}_m$ from (21) into $\dot{\xi}_m$ and then replace them in (33) while noticing the formulas of F_{tel_m} in (23), we get:

$$\begin{aligned}
\dot{V}_m &= \xi_m^T \Big[F_{ext_m} + K_{p_m}\left(\hat{z}_s - z_m\right) + K_{d_m}\left(\dot{\hat{z}}_s - \dot{z}_m\right) \\
&\quad - \left(B_m - H_m \Lambda_m\right)\dot{z}_m \Big] + z_m^T \Sigma_m \dot{z}_m \\
&= \xi_m^T F_{ext_m} + \xi_m^T K_{p_m} \hat{z}_s - z_m^T K_{p_m} \dot{z}_m \\
&\quad - \dot{z}_m^T \left(B_m - H_m \Lambda_m\right)\dot{z}_m \\
&\quad + \xi_m^T K_{d_m} \dot{\hat{z}}_s - z_m^T \Lambda_m K_{p_m} z_m \\
&\quad - z_m^T \Lambda_m \left(B_m - H_m \Lambda_m\right)\dot{z}_m \\
&\quad - z_m^T \Lambda_m K_{d_m} \dot{z}_m - \dot{z}_m^T K_{d_m} \dot{z}_m + z_m^T \Sigma_m \dot{z}_m \\
&= -z_m^T \Lambda_m K_{p_m} z_m - \dot{z}_m^T \left(B_m + K_{d_m} - H_m \Lambda_m\right)\dot{z}_m \\
&\quad + \dot{z}_m^T \left(\Sigma_m - K_{p_m} - \Lambda_m \left(B_m + K_{d_m} - H_m \Lambda_m\right)\right) z_m \\
&\quad + \xi_m^T \theta_m \xi_m - \xi_m^T \theta_m \xi_m + \xi_m^T F_{ext_m} + \xi_m^T K_{p_m} \hat{z}_s \\
&\quad + \xi_m^T K_{d_m} \dot{\hat{z}}_s
\end{aligned} \tag{34}$$

where θ_m is some positive constant $\theta_m \in R^{n \times n}$. Note the definition of ξ_m, we get:

$$\begin{aligned}
\dot{V}_m &= -z_m^T \Lambda_m K_{d_m} z_m - \dot{z}_m^T \left(B_m + K_{d_m} - H_m \Lambda_m\right)\dot{z}_m \\
&\quad + \dot{z}_m^T \left(\Sigma_m - K_{p_m} - \Lambda_m \left(B_m + K_{d_m} - H_m \Lambda_m\right)\right) z_m \\
&\quad + z_m^T \theta_m \Lambda_m^2 z_m + \dot{z}_m^T \theta_m \dot{z}_m + 2\dot{z}_m^T \theta_m \Lambda_m z_m \\
&\quad - \xi_m^T \theta_m \xi_m + \xi_m^T \left(F_{ext_m} + K_{p_m} \hat{z}_s + K_{d_m} \dot{\hat{z}}_s\right) \\
&= -z_m^T \left(\Lambda_m K_{p_m} - \theta_m \Lambda_m^2\right) z_m - \dot{z}_m^T \Big(B_m + K_{d_m} \\
&\quad - H_m \Lambda_m - \theta_m\Big)\dot{z}_m + \dot{z}_m^T \Big(\Sigma_m - K_{p_m} \\
&\quad - \Lambda_m \left(B_m + K_{d_m} - H_m \Lambda_m\right) + 2\theta_m \Lambda_m\Big) z_m \\
&\quad - \xi_m^T \left(\theta_m \xi_m - F_{ext_m} - K_m \hat{z}_s - K_{d_m} \dot{\hat{z}}_s\right)
\end{aligned} \tag{35}$$

we can choose $\Lambda_m = \Lambda_m^T$ with bounded $\Lambda_m < B_m H_m^{-1}$ and choose θ_m to satisfy the first and the second term of (35) to be negative, we have:

$$\begin{cases} \theta_m < K_{p_m} \Lambda_m^{-1} \\ \theta_m < B_m + K_{d_m} - H_m \Lambda_m \end{cases} \tag{36}$$

From condition (36), we can receive:

$$0 < \theta_m < \frac{1}{2}\left(K_{p_m} \Lambda_m^{-1} + B_m + K_{d_m} - H_m \Lambda_m\right) \tag{37}$$

Concern the bound of Λ_m and θ_m, we can choose value of the gain Σ_m as:

$$\Sigma_m = K_{p_m} + \Lambda_m \left(B_m + K_{d_m} - H_m \Lambda_m\right) - 2\theta_m \Lambda_m \tag{38}$$

See condition (37), we have:

$$2\theta_m \Lambda_m < K_{p_m} + \Lambda_m \left(B_m + K_{d_m} - H_m \Lambda_m\right),$$

hence, $\Sigma_m > 0$.

Thus, the derivative of the Lyapunov function is given:

$$\begin{aligned}
\dot{V}_m &\le -z_m^T \left(\Lambda_m K_{p_m} - \theta_m \Lambda_m^2\right) z_m - \dot{z}_m^T \Big(B_m + K_{d_m} \\
&\quad - H_m \Lambda_m - \theta_m\Big)\dot{z}_m \\
&\quad - \xi_m^T \left(\theta_m \xi_m - F_{ext_m} + K_{p_m} \hat{z}_s + K_{d_m} \dot{\hat{z}}_s\right) \\
&\le -z_m^T \left(\Lambda_m K_{p_m} - \theta_m \Lambda_m^2\right) z_m \\
&\quad - \dot{z}_m^T \left(B_m + K_{d_m} - H_m \Lambda_m - \theta_m\right)\dot{z}_m
\end{aligned} \tag{39}$$

$$\forall \left|\xi_m\right| \ge \frac{\left|F_{ext_m}\right| + K_{p_m}\left|\hat{z}_s\right| + K_{d_m}\left|\dot{\hat{z}}_s\right|}{\theta_m} \left(= \rho_m\left(\left|u_M\right|\right)\right)$$

Remark 5.1: Note the Assumption 5 and the expression (25), since $\dot{T}_s(t)$ is bounded, the delay parameters of z_s

and $\dot{z}_s$ are also bounded, there exists the value of $|\xi_m|$ to guarantee above condition of (39).

Using the fact that the signals $z_m, \dot{z}_m$ are bounded. The teleoperation force, exerted force by the human are bounded by the functions of the signals $\hat{z}_m$, therefore F_{ext_m} is also bounded or the input u_M is bounded. Following the Theorem 5.2 [Khalil, (1996), p.218], we can choose a class $\mathcal{K}$ function $\gamma_m = \alpha_{1m}^{-1} \circ \alpha_{2m} \circ \rho_m$, positive constant $k_{1m} = \alpha_{1m}^{-1}\left(\alpha_{1m}\left(r_m\right)\right)$ and $k_{2m} = \rho_m^{-1}\left(\min\left\{k_{1m}, \rho_m\left(r_{mu}\right)\right\}\right)$ for any initial state $x_M\left(t_0\right)$ and any bounded input $u_M(t)$, and we can choose r_m and r_{mu} large enough that satisfies the inequalities given below:

$$\begin{aligned} &\left|x_M\left(t_0\right)\right| < \alpha_{2m}^{-1}\left(\alpha_{1m}\left(r_m\right)\right); \\ &\rho_m\left(\sup_{t \ge t_0}\left|u_M\right|\right) < \min\left\{\alpha_{2m}^{-1}\left(\alpha_{1m}\left(r_m\right)\right), \rho_m\left(r_{mu}\right)\right\} \end{aligned} \tag{40}$$

Applying the Definition 5.2 [Khalil, (1996), p.217] we have the solution $x_M(t)$ exists and satisfies:

$$\begin{aligned} &\left|x_M(t)\right| \le \beta\left(\left|x_M\left(t_0\right)\right|, t - t_0\right) + \rho_m\left(\sup_{t_0 \le \tau \le t}\left|u_M(\tau)\right|\right), \\ &\qquad \forall 0 \le t_0 \le t \end{aligned} \tag{41}$$

where β is a class $\mathcal{KL}$ function. Then the solution $x_M(t)$ only depends on $u_M(\tau)$ for $t_0 \le \tau \le t$, and the master subsystem is locally input-to-state stable. □

Now, we consider the slave-environment interconnection with the slave subsystem.

Lemma 5.2: State of the closed-loop slave subsystem is assumed as: $x_S = \left(\dot{z}_s^T, z_s^T, x_e^T\right)^T$, and input: $u_S = \left(\hat{F}_{op}^T, F_e^T, \hat{z}_m^T, \dot{\hat{z}}_m^T\right)^T$. We suppose the environment dynamics (3) satisfy Assumption 3. Then there exits a continuous differentiable, positive definite, radially unbounded Lyapunov function V_s of the subsystem that satisfies the below inequalities:

$$\alpha_{1s}\left(\left|x_S\right|\right) \le V_s \le \alpha_{2s}\left(\left|x_S\right|\right) \tag{42}$$

$$\begin{aligned} &\frac{\partial V_s}{\partial t} + \frac{\partial V_s}{\partial x} f\left(t, x_S, u_S\right) \le -\alpha_{3s}\left(\left|x_S\right|\right), \\ &\qquad \forall \left|x_S\right| \ge \rho_s\left(\left|u_S\right|\right) > 0 \end{aligned} \tag{43}$$

$$\forall t \ge 0, D := \left\{x_S \in R^n; \left|x_S\right| < r_s\right\},$$

$$D_u := \left\{u_S \in R^s; \left|u_S\right| < r_{su}\right\}$$

$\alpha_{1s}\left(\left|x_S\right|\right), \alpha_{2s}\left(\left|x_S\right|\right), \alpha_{3s}\left(\left|x_S\right|\right)$ and ρ_s are class $\mathcal{K}$ functions, then the subsystem is locally input-to-state stable.

Proof: First, consider the ISS-Lyapunov function candidate:

$$V_s = \frac{1}{2}\xi_s^T H_s \xi_s + \frac{1}{2} z_s^T \Sigma_s z_s + V_e \tag{44}$$

where V_e is introduced in Assumption 3 and ξ_s is defined as follows:

$$\xi_s = \dot{z}_s + \Lambda_s z_s \tag{45}$$

where Λ_s, Σ_s are positive diagonal gain matrices. Similar to the master subsystem, the first and the second term of the right-side in (44) are radially unbounded; note that in the Assumption 3, V_e satisfies the inequality (4) with any radially unbounded α_{1e} and α_{2e}, then V_s is also said to be radially unbounded and satisfies the inequality (42). We also can easily check that $V_s(0) = 0$ while $x_S = 0\left(\dot{z}_s = 0, z_s = 0, x_e = 0\right)$.

The derivative of V_s along the trajectories of the system as:

$$\dot{V}_s = \xi_s^T H_s \dot{\xi}_s + z_s^T \Sigma_s \dot{z}_s + \dot{V}_e \tag{46}$$

similar to the master subsystems, we get:

$$\dot{\xi}_s = \ddot{z}_s + \Lambda_s \dot{z}_s$$

Substituting $\ddot{z}_s$ from task space dynamic (21) into $\dot{\xi}_s$ and then replace them in (46) and concerning $\dot{V}_e$ in the Assumption 3, the formulas of F_{ext_s}, F_{tel_s} in (22) and (23), we obtain:

$$\begin{aligned} \dot{V}_s \le{}& \xi_s^T\Big(F_{ext_s} + K_{p_s}\left(\hat{z}_m - z_s\right) + K_{d_s}\left(\dot{\hat{z}}_m - \dot{z}_s\right) \\ &- \left(B_s - H_s\Lambda_s\right)\dot{z}_s\Big) + \dot{z}_s^T \Sigma_s z_s - \alpha_{3e}\left|x_e\right|^2 + F_e^T s_e \\ \le{}& -\dot{z}_s\left(B_s + K_{d_s} - H_s\Lambda_s\right)\dot{z}_s - z_s^T \Lambda_s K_{p_s} z_s - \alpha_{3e}\left|x_e\right|^2 \\ &- \dot{z}_s^T\left(K_{p_s} + \Lambda_s\left(B_s + K_{d_s} - H_s\Lambda_s\right) - \Sigma_s\right) z_s \\ &+ \xi_s^T K_{p_s}\hat{z}_m + \xi_s^T \hat{F}_{op} + \left(s_e - \xi_s\right)^T F_e - \xi_s^T \theta_s \xi_s \\ &+ \xi_s^T \theta_s \xi_s + \xi_s^T K_{d_s}\dot{\hat{z}}_m \end{aligned} \tag{47}$$

where θ_s is some positive constant. Using the inequality (4) of Assumption 2, the definitions of s_e (11) and ξ_s (45), we get:

$$\begin{aligned} \left|s_e - \xi_s\right|^T F_e \le{}& \left|z_s\right|^T\left(\Lambda_{env} - \Lambda_s\right)\left|x_e\right| + \left|z_s\right|^T\left(\Lambda_{env} - \Lambda_s\right) \\ & a\left|\dot{z}_s\right| + \left|z_s\right|^T\left(\Lambda_{env} - \Lambda_s\right) b\left|z_s\right| \end{aligned} \tag{48}$$

here, we choose $a = I$ and $b = \Lambda_s$.

Using Young's quadratic inequality with $\left|A^T B\right| \le (\varepsilon/2)\,|A|^2 + (1/2\varepsilon)\,|B|^2$ that holds for all $\varepsilon > 0$, therefore, we can obtain the following bound of the first term in (48):

$$\left|z_s\right|^T\left(\Lambda_{env} - \Lambda_s\right)\left|x_e\right| \le \frac{\lambda}{4}\left|z_s\right|^2 + \frac{\left(\Lambda_{env} - \Lambda_s\right)^2}{\lambda}\left|x_e\right|^2 \tag{49}$$

where λ is a small positive constant. The derivative of this Lyapunov function along trajectories of the subsystem is given as:

$$\begin{aligned}\dot{V}_s \le & -\dot{z}_s^T\left(B_s + K_{d_s} - H_s\Lambda_s\right)\dot{z}_s - z_s^T\Lambda_s K_{p_s} z_s \\ & -\alpha_{3e}\left|x_e\right|^2 - \dot{z}_s^T\left(K_{p_s} + \Lambda_s\left(B_s + K_{d_s}\right.\right. \\ & \left.\left.-H_s\Lambda_s\right) - \Sigma_s\right) z_s + \frac{\lambda}{4}\left|z_s\right|^2 + \frac{\left(\Lambda_{env} - \Lambda_s\right)^2}{\lambda}\left|x_e\right|^2 \\ & + \left|z_s\right|^T\left(\Lambda_{env} - \Lambda_s\right)\left|\dot{z}_s\right| + \left|z_s\right|^T\left(\Lambda_{env} - \Lambda_s\right)\Lambda_s\left|z_s\right| \\ & + z_s^T\theta_s\Lambda_s^2 z_s + \dot{z}_m^T\theta_s\dot{z}_s + 2\dot{z}_s^T\theta_s\Lambda_s z_s + \xi_s^T K_s\hat{z}_m \\ & + \hat{F}_{op}^T\xi_s - \theta_s\xi_s^2 + \xi_s^T K_{d_s}\dot{\hat{z}}_m\end{aligned} \tag{50}$$

We obtain:

$$\begin{aligned}\dot{V}_s \le & -\dot{z}_s^T\left(B_s + K_{d_s} - H_s\Lambda_s - \theta_s\right)\dot{z}_s \\ & - z_s^T\left(\Lambda_s K_{p_s} - \Lambda_s\left(\Lambda_{env} - \Lambda_s\right) - \frac{\lambda}{4} - \theta_s\Lambda_s^2\right) z_s \\ & - \left(\alpha_{3e} - \frac{\left(\Lambda_{env} - \Lambda_s\right)^2}{\lambda}\right)\left|x_e\right|^2 \\ & - \dot{z}_s^T\left(K_{p_s} + \Lambda_s\left(B_s + K_{d_s} - H_s\Lambda_s\right)\right. \\ & \left. - \Sigma_s - \left(\Lambda_{env} - \Lambda_s\right) - 2\theta_s\Lambda_s\right) z_s \\ & - \xi_s^T\left(\theta_s\xi_s - \hat{F}_{op} - K_{d_s}\hat{z}_m + K_{d_s}\dot{\hat{z}}_m\right)\end{aligned} \tag{51}$$

we can choose $\Lambda_s = \Lambda_s^T$ with bounded $\Lambda_s < B_s H_s^{-1}$ and choose θ_s and λ to satisfy the first three terms of (51) to be negative, we have:

$$\begin{cases}\theta_s < B_s + K_{d_s} - H_s\Lambda_s \\ \theta_s < K_{p_s}\Lambda_s^{-1} - \left(\Lambda_{env} - \Lambda_s\right)\Lambda_s^{-1} - \frac{1}{4}\lambda\Lambda_s^{-2} \\ \lambda > \frac{\left(\Lambda_{env} - \Lambda_s\right)^2}{\alpha_{3e}}\end{cases} \tag{52}$$

from condition (52) we receive:

$$\begin{aligned}0 < \theta_s < \frac{1}{2}\Big(& K_{p_s}\Lambda_s^{-1} + \left(B_s + K_{d_s} - H_s\Lambda_s\right) \\ & - \left(\Lambda_{env} - \Lambda_s\right)\Lambda_s^{-1} - \frac{1}{4}\lambda\Lambda_s^{-2}\Big)\end{aligned} \tag{53}$$

Concern the bound of Λ_s and θ_s, we can choose value of the gain Σ_s as:

$$\begin{aligned}\Sigma_s = & K_{p_s} + \Lambda_s\left(B_s + K_{d_s} - H_s\Lambda_s\right) \\ & - \left(\Lambda_{env} - \Lambda_s\right) - 2\theta_s\Lambda_s\end{aligned} \tag{54}$$

See condition (53), we also receive:

$$\begin{aligned}2\theta_s\Lambda_s < & K_{p_s} + \Lambda_s\left(B_s + K_{d_s} - H_s\Lambda_s\right) \\ & - \left(\Lambda_{env} - \Lambda_s\right) - \frac{\lambda}{4\Lambda_s}\end{aligned} \tag{55}$$

substituting condition (55) into (54), since $\lambda > 0,$ we also have $\Sigma_s > 0.$ Thus, the derivative of the Lyapunov function is given as:

$$\begin{aligned}\dot{V}_s \le & -\dot{z}_s^T\left(B_s + K_{d_s} - H_s\Lambda_s - \theta_s\right)\dot{z}_s \\ & - z_s^T\left(\Lambda_s K_{p_s} - \Lambda_s\left(\Lambda_{env} - \Lambda_s\right) - \frac{\lambda}{4} - \theta_s\Lambda_s^2\right) z_s \\ & - \left(\alpha_{3e} - \frac{\left(\Lambda_{env} - \Lambda_s\right)^2}{\lambda}\right)\left|x_e\right|^2 \\ & \forall\left|\xi_s\right| \ge \frac{\left|\hat{F}_{op}\right| + K_{p_s}\left|\hat{z}_m\right| + K_{d_s}\left|\dot{\hat{z}}_m\right|}{\theta_s}\left(= \rho_s\left(\left|u_S\right|\right)\right)\end{aligned} \tag{56}$$

Remark 5.2: Similar to the master subsystem case, note the Assumption 1, Assumption 5 and the expression (25), $\dot{T}_m(t)$ is bounded, the delay parameters of $z_m, \dot{z}_m$ and F_{op} are also bounded, there exists the value of $\left|\xi_s\right|$ to guarantee above condition of (56).

We can choose a class $\mathcal{K}$ function $\gamma_s = \alpha_{1s}^{-1} \circ \alpha_{2s} \circ \rho_s,$ positive constant $k_{1s} = \alpha_{2s}^{-1}\left(\alpha_{1s}(r)\right)$ and $k_{2s} = \rho_s^{-1}\left(\min\left\{k_{1s}, \rho_s\left(r_{su}\right)\right\}\right)$ for any initial state $x_S\left(t_0\right)$ and any bounded input $u_S(t).$ Similar to the argument in the master subsystem, we can conclude that the slave + environment subsystem is also locally input-to-state stable. □

Based on the Lemma 1 and Lemma 2, the following theorem concerning stability properties of the closed-loop system is obtained.

Theorem 1: Consider the force-reflecting teleoperation system (1), the FR algorithm (22) and (23). Suppose the environment dynamic satisfies Assumption 3, there exists $\gamma_\Lambda(\cdot) \in \mathcal{K}$ such that $\gamma_\Lambda = \gamma_m \circ \gamma_s$ implies that: for the FR algorithm, the overall teleoperation system is input-to-state stable.

Proof: We choose the state of the overall FR teleoperation as follows:

$$x_T = \left(z_m^T, \dot{z}_m^T, z_s^T, \dot{z}_s^T, x_e^T\right)^T$$

and the input as:

$$u_T = \left(\hat{z}_m^T, \dot{\hat{z}}_m^T, \hat{z}_s^T, \dot{\hat{z}}_s^T, F_{extm}^{\ \ T}, \hat{F}_{op}^T, F_e^T\right)^T$$

Now, we can combine the above presented results and the consecutive application of the ISS theorem. Indeed, denote by the ISS gain $\gamma_{m[u_M \to x_M]}(\cdot) \in \mathcal{K}$ of the closed-loop master subsystem, whole existence is guaranteed by Lemma 1.

And also, we let $\gamma_{s[u_s \to x_s]}(\cdot) \in \mathcal{K}$ be the ISS gain of the closed-loop slave + environment subsystem (3). Choose γ_Λ such that the satisfying:

$$\gamma_\Lambda = \gamma_{[u_M \to x_M]}(\cdot) \circ \gamma_{[u_s \to x_s]}(\cdot) \tag{57}$$

Applying the Definition 5.2 [Khalil, (1996), p.217], we can conclude the overall FR teleoperation system is input-to-state stable and the Control objective 1 is also achieved. The proof is completed. □

6 Evaluation by control experiments

In this section, we verify the efficacy of the proposed four-channel FR teleoperation. The experiments were carried out on a pair of 2-DOF identical direct-drive planar robots with two links revolute-joints. The first DOF is the revolute-joint of link 1 and second DOF is the revolute-joint of link 2. The inertia matrices and the Coriolis matrices are identified as:

$$M_i = \begin{bmatrix} M_{i1} + 2R_i \cos(q_{i2}) & M_{i2} + R_i \cos(q_{i2}) \\ M_{i2} + R_i \cos(q_{i2}) & M_{i2} \end{bmatrix},$$

$$C_i = \begin{bmatrix} -R_{i1} \sin(q_{i2}) \dot{q}_{i2} & -R_{i1} \sin(q_{i2})(\dot{q}_{i1} + \dot{q}_{i2}) \\ R_i \sin(q_{i2}) \dot{q}_{i1} & 0 \end{bmatrix}$$

where

$$M_{i1} = 0.366\ kgm^2, M_{i2} = 0.0291\ kgm^2,$$
$$R_i = 0.0227\ kgm^2; l_{i1} = l_{i2} = 0.2\ m,$$

with $i = m, s.$ The remote environment on the slave side is a hard aluminium wall covered by hard rubber as shown in Figure 3. The contact forces between the end-effector of the slave robot with the environment are shown in Figure 4. We also receive joint angle values from encoders in each joint of the robots, and measure the operational and environment reflecting forces by using the force sensors at the end-effectors of the robots (F_{Sx}, F_{Sy}). For implementation of the controllers and communication lines, we utilise a dSPACE digital control system (dSPACE Inc.). All experiments have been done with the artificial time varying communication delays as:

$$T_m(t) = 0.5 \sin 0.3t + 0.6[s]$$
$$T_s(t) = 0.5 \sin 0.3t + 0.6[s]$$

We can see the above communication delays also satisfy Assumption 5 with $T_{m/s}(\cdot): R \to R^+$. Here, the slave is controlled to contact the surface of environment in (x_1, y_1) from initial position (x_0, y_0). The initial joint angles of the robots are chosen to satisfy Assumption 4, then we set $q_1 = 45^\circ, q_2 = -90^\circ$ and they are equivalent in task space with $x_0 = 0.2828\ m,\ y_0 = 0.0\ m.$ The contact position is set as: $x_1 = 0.2828\ m,\ y_1 = -0.08\ m.$ The controller parameters are selected as follows:

$$H_m = H_s = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix};$$

$$K_{p_m} = \begin{bmatrix} 40 & 0 \\ 0 & 40 \end{bmatrix}, K_{p_s} = \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix};$$

$$\bar{K}_{d_m} = \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix}, \bar{K}_{d_s} = \begin{bmatrix} 35 & 0 \\ 0 & 35 \end{bmatrix};$$

$$K_{f_m} = \begin{bmatrix} 0.03 & 0 \\ 0 & 0.03 \end{bmatrix}, K_{f_s} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

In this control task, the varying damping values are received from (28) with following parameters:

$$z_{y\max} = 0.07,\ \bar{b} = 20,\ \underline{b} = 5$$

Two kinds of experimental conditions are given as:

Case 1 The slave moves without any contact.

Case 2 The slave moves in contact with the environment.

Figure 5–Figure 10 show the results with two cases of experimental conditions. Figure 5–Figure 7 show the results of Case 1. We can see from Figure 5, the free movement of slave robot is achieved accurately the movement of the master robot. In this case, there is only the force exerted on the master by the human operator. Since the end-effector of the slave robot does not contact with the environment, obviously there is not the reflecting force from there.

Figure 8–Figure 10 show the results of the second case. From Figure 8, we can see that after moving in free space (0–28 sec), the slave robot contacts with the environment (28–51 sec), the reflecting force appears and increases while the human pushes an increasing force on the master robot. As shown in Figure 9, this contact force is faithfully reflected to the master side. The human operator can perceive the environment through the reflection force; however, in this case the position error is larger than the error in free movement case of slave robot. When the slave robot departs from the environment and the human does not exert more force on the master (51–70 sec), the position error becomes smaller.

Figure 3 Experimental setup

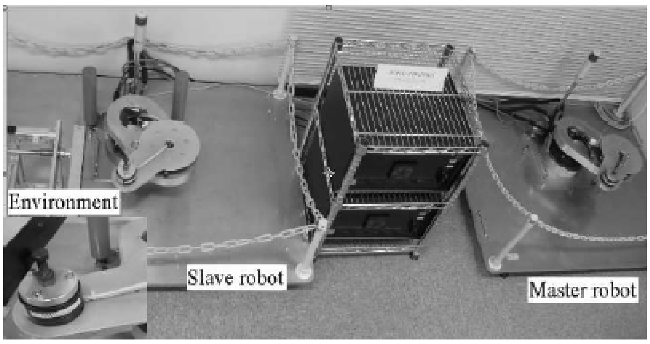


Figure 4 Force in the contact task

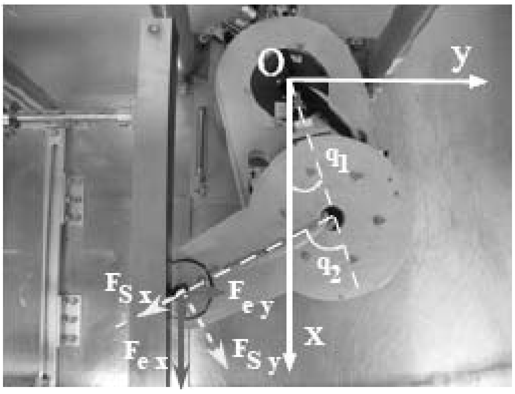


Figure 5 Position data in free space (Case 1)

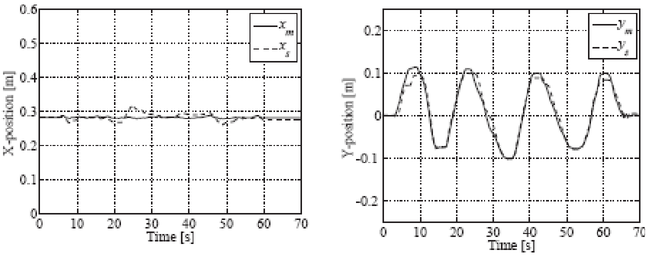


Figure 6 Force data in free space (Case 1)

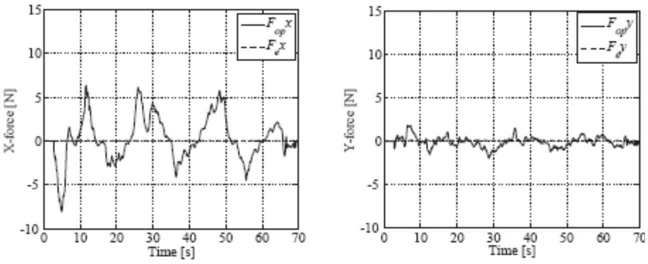
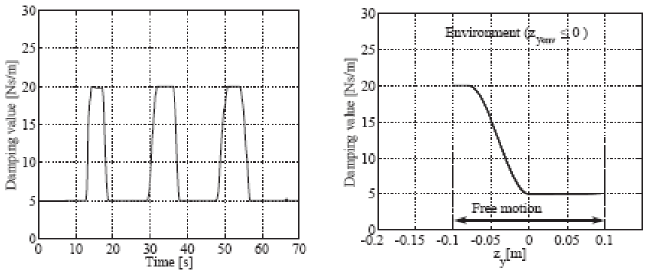


Figure 7 Varying damping values in free space (Case 1)



In Figure 7 and Figure 10, the varying damping values of the master and the slave robots are shown in two cases; the environment position is set with $z_{y_{env}} \leq 0$; these values depend on the distance (following the y-axis) from the starting point (x_0, y_0) to the current position of the end-effector of the slave robot. We can set the upper and lower bound values of damping, they depend on the distance from the starting point of end-effector of slave robot to the surface of the environment. The overall system guarantees the ISS and achieves contact stability and also good transparency while the damping values satisfy the conditions from (28) and (29). In the experimental task, when the end-effector of the slave robot contacts with the surface of the environment, the damping achieves the upper bound value (see Figure 10) to keep the contact stability. When it departs from the surface, this value reduces to lower bound to keep the tracking performance.

Figure 8 Position data in contact with environment (Case 2)

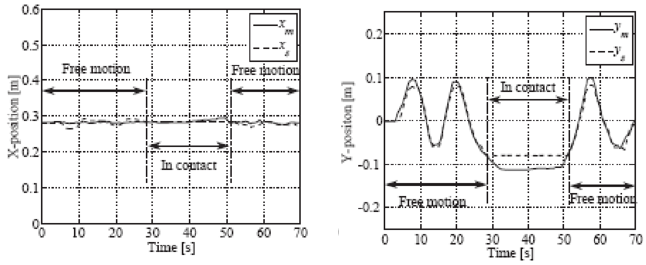


Figure 9 Force data in contact with environment (Case 2)

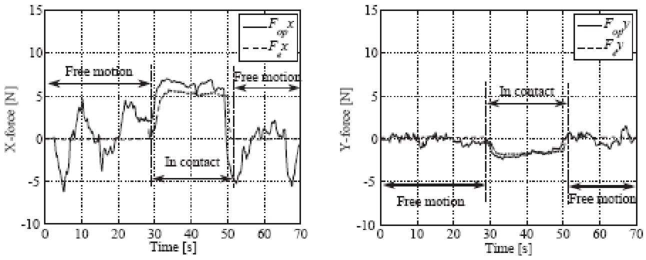
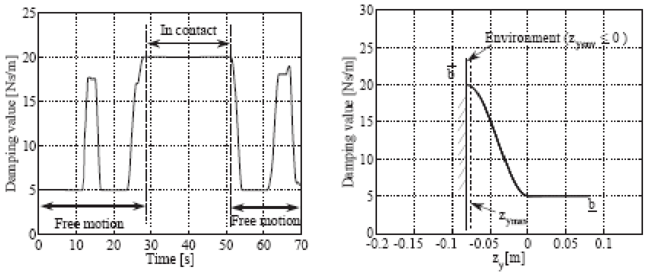


Figure 10 Varying damping values in contact with environment (Case 2)



One method to improve the tracking performance of teleoperation is to use low damping of the robots. However, it is only effective for position tracking control. In this paper, if we use the low damping for the force tracking performance, the error will be larger sometimes the contact between the slave and the environment is unstable. Therefore, the high damping was used to solve this problem when the interaction occurs. It means that, a reflecting force appears and to be transferred back to the human side, this force must be same value with the exerted force by the human on the master. Concerning these points, variable damping was proposed to treat these problems in our developed approach, also in Nam and Namerikawa (2009).

In teleoperation system, time delay makes the system destabilise and deteriorate the tracking performance and transparency, the position and force information may lost in the transmission of communication lines. The time delay is assumed to be a constant or variable value. If it is a constant, we have a simpler case of the stability, tracking performance achievement (Kawada et al., 2007). In our proposal, the time varying delay was used in the communication lines. We proposed one more delay channel to transfer the reflecting force from the environment to the operator. The contact was stable, and then this force was safely feedback to the master side. Therefore, the human can sense accurately this force; in addition, s/he can exert an appropriate value on the master robot. Although the effect of the time delays, the experimental results have shown the achievement of the contact stability. Nevertheless, when the time varying delay becomes larger, the position and force tracking errors may be also larger.

The stiffness of the environment also effect to the contact stability. For example, in the case of high stiffness environment, while the end-effector of slave interacts with the environment surface, some impulses appear and increase quickly in a very short time (Polushin et al., 2007), because these impulses may not exterminate thoroughly by themselves in this case. For treating different level of stiffness of the environments, especially high stiffness case, we proposed an impedance control law based on a force reflecting algorithm and variable damping of master and slave robots, and then the contact stability was achieved, correspondingly the force tracking error was also small.

7 Conclusions

In this paper, we proposed new control method with four-channel FR algorithm for bilateral teleoperation based an impedance control. In this proposed strategy, besides using the new proposed FR algorithm, we used the varying damping to improve contact stability and transparency of teleoperation with the effective tracking performance in comparison with the previous research. To analyse stability, the ISS small gain theorem was used to show the overall FR teleoperation system to be input-to-state stable. Finally, several experimental results showed the effectiveness of the proposed method. In future work, this proposal need be developed more to over some shortcomings and extend to apply for a larger time varying delay in the communication lines, and for variable stiffness level of the environment to evaluate more the effectiveness of this proposal. A different configuration of the master and slave robots with power scaling will be considered.

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