Optimal Coordination and Control of Connected and Automated Vehicles at Intersections via Mixed Integer Linear Programming

Mohamad Hafizulazwan Mohamad Nor * and Toru Namerkawa **

Abstract: This paper addresses the crossing order problem for connected and automated vehicles at an intersection where the problem can lead to higher traffic congestion, especially in high traffic density. First, we formulated the problem with mixed integer linear programming where the solution yields optimal crossing time for each vehicle to cross the intersection. Then, we present an optimization framework to drive each vehicle towards the assigned crossing time while conserving the fuel consumption. Finally, we simulate the intersection scenario with the proposed solution, where it is shown that both congestion and fuel consumption can be reduced significantly.

Key Words: connected and automated vehicles, cooperative driving, intersection control, mixed integer linear programming.

1. Introduction

Currently, we are witnessing an alarming state of traffic congestion and accidents caused by a broad utilization of vehicles on the roadway every year. It has been reported in [1] that the driver response to various disturbances, for example, a single driver all of a sudden hitting the brake too hard can cause congestion. Even so, a study has found that the primary sources of bottlenecks for traffic congestion are intersections and merging roadways [2]. According to [3] and [4], traffic congestion can bring a huge negative impact on health, economic, etc especially for those who are living in urban areas. Therefore, many solutions have been proposed to overcome the problem, and one of the ideas is by employing connected and automated vehicles (CAVs) technology without any physical traffic light. The congestion problem can be well solved by CAVs technology because they have much faster response times compared to human-driven vehicles and are capable of controlling their own speed precisely. Moreover, these features from the technologies can potentially reduce the percentage of traffic accidents too while reducing energy consumption and greenhouse gas emissions and increasing passengers comfort.

Many research efforts have been conducted on CAVs, particularly on the intersection control problem. In this field of research, the earliest challenge is to formulate the problem to achieve smooth traffic flow (to alleviate congestion) and at the same time guaranteeing the safety of passengers (collision-free passage through the intersection). Through analyzing the literature, some approaches are the identified [5]–[12] and basically, they can be categorized into heuristic [5],[6] and optimization [7]–[12] based approaches. In [5] and [6], a novel reservation-based approach has been proposed where each vehicle is required to reserve a space-time block in an intersection via an autonomous intersection manager or also known as a coordinator. There are two drawbacks for utilizing this approach where the first one is a heavy communication requirement and another one is the possible occurrence of deadlock. A different approach has been utilized in [7], which is based on minimizing the overlap of vehicles’ trajectories in the intersection area. However, this approach has resulted in a complex optimization problem formulation, i.e., the objective function and constraints are nonlinear, and this factor makes the solution hard to be obtained. Another interesting approach to formulate the aforementioned problem is based on scheduling technique. Several research efforts in this direction have been obtained for example considering the optimal time interval that each vehicle needs to spend in an intersection [8],[9], considering the optimal time for each vehicle to arrive at an intersection [10],[11], and job-shop scheduling based formulation [12].

The next challenge is to incorporate energy efficiency into the problem formulation, i.e., smoothing the traffic flow, reducing the energy consumption, and guaranteeing the safety of passengers. One way to achieve these goals is by avoiding severe stop-and-go driving. Several research efforts have been reported in this direction [13]–[16]. In 2015, Rios-Torres and Malikopoulos [13] have formulated the problem based on the optimization approach and proposed a centralized, closed-form solution using Hamiltonian analysis to derive the optimal input (acceleration and deceleration) for each vehicle to achieve the goals. The detailed work is then provided in [14]. Even though the studies in [13] and [14] were focusing on a merging roadway, it is actually can be adapted into an intersection easily [17]. Therefore, [15] and [16] have utilized the same scheme as [13] and [14] for intersections but in a decentralized manner. All the results reported in [13]–[16] have shown significant improvement for both energy minimization and traffic flow maximization. However, the passing order of vehicles to merge into the roadway or to cross the intersection in these studies follow first-in-first-out (FIFO) order. According to [18], the FIFO order will lead to a local optimal solution in many situations. In addition, the per-
formance of traffic flow also will become worse in high traffic demand if we utilize the FIFO order.

The contribution of this paper is to address the crossing order problem in [15],[16] at an intersection environment. By introducing mixed integer linear programming (MILP) into the framework, we can obtain the optimal crossing order of vehicles at the intersection. As a result, we will show in our simulation that the traffic flow and the energy consumption can be further improved as compared to the FIFO order utilized in [15],[16]. Note that the idea to find the optimal passing order is adopted from [10] and [11] because of the simplicity in their formulation. However, there is no energy minimization problem provided in [10] and [11]. The content of this paper has been partially presented in our prior conference publication [19]. In this paper, we improve and provide a detailed formulation of the prior publication.

This paper is organized as follows. In Section 2, we explain the modeling framework, formulate the optimal crossing order and energy minimization problems, and then provide an analytical solution for the energy minimization problem. In Section 3, we provide simulation results to show the effectiveness of our approach in terms of reducing travel time and fuel consumption under the hard constraint of collision avoidance. Finally, concluding remarks are provided in Section 4.

2. Problem Formulation

We consider a single intersection with two phases, i.e., Phase X (\(\phi_X = [X' = \text{southbound}, X'' = \text{northbound}]\)) and Phase O (\(\phi_O = [O' = \text{westbound}, O'' = \text{eastbound}]\)) as shown in Fig. 1. As can be seen from the left-hand side of the figure, the intersection has a merging zone (MZ) which we assume to be a square of size S at its central region shaded in grey. Note that this region is where the lateral collision between vehicles from different phases (\(\phi_X \neq \phi_O\)) can potentially happen. The intersection also has a coordinator (centralized controller) that coordinates the CAVs movements, \(M = [X', X'', O', O'']\) inside the control zone (CZ). This CZ (depicted as a circle) is the region where the coordinator can reach the CAVs via communication or sensing devices. The distance from the entry of the CZ to the entry of the MZ is denoted as L, and we assume that this distance is the same for all entry points of the CZ. As the first stage of this research, we do not consider any lane changing and turning of the vehicles in order to reduce the complexity of our problem formulation.

Let \(i = 1, 2, ..., N(t)\) be the identification (ID) of each CAV and \(N(t) \in \mathbb{N}\) be the total number of all CAVs at time \(t \in \mathbb{R}^+\) from the entry of CZ until the exit of MZ. The ID can be assigned by the coordinator according to the CAVs distance to the MZ. For example, the nearest CAV to the MZ will have the ID, \(i = 1\), and the furthest (might be the new reaching CAV) will have the ID, \(i = N(t)\), as illustrated in Fig. 1. However, in some cases, there will be two or more CAVs which have the same distance to the MZ. If this happened, the coordinator would assign the ID based on the priority of movement of each CAV, \(m_i \in M\), where this priority can be assigned easily. In this paper, we assign the most priority movement in an anti-clockwise direction starting from \(O''\) and ending at \(X''\). As an example, if one is \(m_i = O''\) and another is \(m_i = X''\) having the same distance to the entry of MZ, then the CAV with \(m_i = O''\) will be numbered first. Finally, the ID will be eliminated once the CAV cleared the MZ and the CAV may change to human-driving or car-following (using adaptive cruise control or cooperative adaptive cruise control) modes.

Table 1 summarizes the list of abbreviations used in this paper. The readers might need to refer to the table to avoid confusion about the abbreviations.

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
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<tbody>
<tr>
<td>CAVs</td>
<td>Connected and Automated Vehicles</td>
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<tr>
<td>FIFO</td>
<td>First-In-First-Out</td>
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<tr>
<td>MILP</td>
<td>Mixed Integer Linear Programming</td>
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<tr>
<td>CZ</td>
<td>Control Zone</td>
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<tr>
<td>MZ</td>
<td>Merging Zone</td>
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<td>ID</td>
<td>Identification</td>
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2.1 Vehicle Model, Constraints, and Assumption

For simplicity, we only consider the dynamics of each CAV \(i\) to be second order dynamics

\[
\begin{align*}
\dot{p}_i &= v_i(t), \quad p_i(t_0) \text{ given}, \\
\dot{v}_i &= u_i(t), \quad v_i(t_0) \text{ given},
\end{align*}
\]

(1)

where \(t_0\) is the current time, and \(p_i(t) \in P_i\), \(v_i(t) \in V_i\), \(u_i(t) \in U_i\) are the position, speed, and acceleration/deceleration (control input) of CAV \(i\) respectively. The sets \(P_i\), \(V_i\), and \(U_i\) are closed bounded subsets of \(\mathbb{R}\). The state space \(P_i \times V_i\) for each CAV \(i\) is closed with respect to the induced topology.

For the system in (1), we need to ensure that the control input and vehicle speed are within a given admissible range. This can be done by imposing the following constraints:

\[
\begin{align*}
0 \leq v_{\min} &\leq v_i(t) \leq v_{\max} \quad \forall t \in [t_0, t_i^m], \\
0 \leq u_{\min} &\leq u_i(t) \leq u_{\max},
\end{align*}
\]

(2)

where \(u_{\min}\) and \(u_{\max}\) are the minimum deceleration and maximum acceleration for each CAV \(i \in N(t)\), and \(v_{\min}\) and \(v_{\max}\) are the minimum and maximum speed limits, respectively, and \(t_i^m\) is the time that CAV \(i\) reaches the MZ. In this paper, we do not consider any diversity of the CAVs and thus we can set \(u_{\min} = u_{\min}\) and \(u_{\max} = u_{\max}\).

Rear-end collision can potentially happen if two consecutive CAVs are traveling on the same movement (e.g., westbound).
Therefore, to ensure the absence of the rear-end collision, we impose the following rear-end safety constraint on each CAV $i$:

$$s_i(t) = p_i(t) - p_i(t) \geq \delta$$

$$\forall t \in [t_i^1, t_i^f] \quad \forall m_i, m_k \in M, \quad m_i = m_k,$$  \hspace{1cm} (3)

where $k = \max \{j : m_i = m_j, j = 1, \ldots, i - 1\}$ and $t_i^f$ is the time that CAV $i$ exits the MZ, and $\delta$ is a predefined safety distance and the value can be chosen to be $\delta < S$ so that the value is not ridiculously large when the square side of MZ is large. In simpler word, CAV $k$ is the vehicle which is immediately ahead of CAV $i$, for example, CAV 1 is the immediate vehicle of CAV 7 in Fig. 1. In the next section, we will show that we can satisfy the constraint (3) for all $t \in [t^a_i, t^f_i]$ only. This is because satisfying the constraint for all $t \in [t^a_i, t^f_i]$ is difficult (because of the inequality) and this problem will be addressed in future research.

As mentioned previously, a lateral collision can potentially occur inside the MZ between two CAVs traveling from different phases (see CAV 2 and CAV 3 in Fig. 1). Before explaining the strategy on how this collision can be avoided, first, we provide the following definition:

**Definition 1.** For each CAV $i \in N(t)$, we define the set $\Gamma_i$ that includes all the time instants when a lateral collision involving CAV $i$ is possible:

$$\Gamma_i = \{t \mid t \in [t_i^a, t_i^f]\}.$$  \hspace{1cm} (4)

From the above definition, we see that one of the strategies to avoid the lateral collision is to prevent the two CAVs $i, j \in N(t), i \neq j$ to be inside the MZ at the same time. This strategy can be formalized as the following constraint

$$\Gamma_i \cap \Gamma_j = \emptyset$$

$$\forall \phi_i, \phi_j \in \Phi, \quad \phi_i \neq \phi_j, (5)$$

where $\emptyset$ is the empty set.

Next, we impose the following assumptions into the modeling framework described above:

**Assumption 1.** At current time $t_0$, none of the constraints provided in (2)-(3) are active.

**Assumption 2.** The CAVs speed inside the MZ is constant, i.e., $v_i(t) = v_i(t^a_i) = v_i(t^f_i), \forall t \in [t_i^a, t_i^f]$. This implies that

$$t_i^f = t_i^a + \frac{S}{v_i(t^a_i)}.$$  \hspace{1cm} (6)

Note that the purpose of this assumption is to increase safety awareness.

**Assumption 3.** The coordinator can obtain the CAVs states inside the CZ by using communication or sensing devices such as cameras (see [20]) without errors or delays.

For simplicity of notation in the remainder of this paper, we will write $p_i(t_0) \equiv p_i^1, v_i(t_0) \equiv v_i^1, v_i(t^a_i) \equiv v_i^a, v_i(t^f_i) \equiv v_i^f$.

### 2.2 Proposed Crossing Order Formulation

In the previous studies [15],[16], the crossing order problem was formulated based on the time that CAV $i$ reaches the MZ, i.e., $t_i^m$ and each CAV must enter the MZ by following a strict FIFO order structure. This means that they must enter the MZ the same order they entered the CZ to satisfy

$$t_i^m \geq t_{i+1}^m \quad \forall i \in N(t), i > 1.$$  \hspace{1cm} (7)

As pointed out earlier, the FIFO order cannot guarantee optimal coordination, i.e., the optimal solution of $t_i^m$ would cause the performance of traffic flow to become worse, especially in high traffic demand. To solve the problem, we propose to find an optimal crossing order for all CAVs inside the CZ to cross the intersection replacing the FIFO order where the condition provided in (7) can be ignored.

We begin the optimization problem to obtain the optimal crossing order by defining the following objective function:

$$\min_{t_i^m, \forall i} \sum_{i=1}^{N(t)} t_i^m.$$  \hspace{1cm} (8)

From the objective function above, we try to minimize the position’s gap between the CAVs when merging into the MZ so that the road can be fully exploited and thus, maximizing the traffic flow. One way to do that is to minimize the total merging time between all CAVs. We realize that the above objective function only allows the CAVs to cross the intersection as soon as possible, i.e., maximize the traffic flow but, it cannot provide the trade-off between energy minimization and maximizing the traffic flow.

**Remark 1.** Another terms can be added to the objective function to provide the aforementioned trade-off. For example, $\Sigma_{i \neq j}^{N(t)} (t_i^m - \delta_{i,j})$ where $\delta_{i,j}$ is the desired time for each CAV $i$ to cross the intersection without having to accelerate much can technically reduce the fuel consumption. However, adding the new terms to the objective function will make the problem more complicated, and we still investigate it in the subject of ongoing research.

As stated in (8), we can see that no constraints are imposed on $t_i^m$. Since the dynamics of each CAV $i$ are imposed with the constraints (2), sometimes it would be infeasible for the CAVs to reach the assigned $t_i^m$. To avoid such a scenario, we impose the following constraint on $t_i^m$ in (8):

$$t_i^m \geq t_{i_{\text{min}}}^m.$$  \hspace{1cm} (9)

where $t_{i_{\text{min}}}^m$ is the minimum time for each CAV to enter the MZ and can be calculated (the illustration is provided in Fig. 2) as follows:

$$t_i^m = \min \left\{ \frac{v_{\text{max}} - v_i^a}{u_{\text{max}}} - \frac{v_i^m - v_i^a}{u_{\text{max}}} \right\}$$

$$= \frac{1}{v_{\text{max}}} \left( v_{\text{max}}^2 - 2v_{\text{max}} v_i^m + v_i^m \right),$$

$$t_i^m = \left\{ \frac{d_i}{v_{\text{max}}} - \frac{(v_{\text{max}} - v_i^a)^2}{4d_i v_{\text{max}}}, 0 \right\},$$

$$t_{i_{\text{min}}}^m = t_0 + t_i^f + \delta_i.$$  \hspace{1cm} (10)

Referring to Fig. 2, there are two possibilities for CAV $i$ to reach the MZ from $t_0$. The first one is that it will accelerate with $u_{\text{max}}$ until it reaches $v_{\text{max}}$ and then cruise at this speed until it arrives at the MZ. Another one is that it will accelerate with $u_{\text{max}}$ but arrive at the MZ without reaching the $v_{\text{max}}$. These two possibilities are represented in both $t_i^f$ and $\delta_i$ in (10).

We can also notice from (8) that if CAV $i$ follows the assigned $t_i^m$ to enter the MZ, there is no guarantee it will enter
Please note that the constraint above is different from (3) where (3) is to guarantee the absence of rear-end collision when CAV i moves to $t_0$ until $t^m_i$ while the constraint (11) is to separate the CAVs at $t^m_i$ only.

As described in (5), for two CAVs with different phases, we need to ensure that only one CAV is inside the MZ at a time. To do that, we impose the following constraint for $t^m_i$ in (8):

$$t^m_i - t^m_j \geq S/v^m_i$$

or

$$t^m_i - t^m_i \geq S/v^m_i$$

$$\forall \phi_i, \phi_j \in \phi, \phi_i \neq \phi_j.$$  \tag{12}

As we plan to solve the problem defined in (8) with linear programming, we need to remove the discontinuity and nonlinearity caused by the or logic operator in the above constraint. This can be done by converting the or logic operator into an and logic combination of the inequalities. To do the conversion, we utilize the big-M method, which would require an additional decision variable $B_i (1 \leq i \leq \text{no. of constraints})$ and a constant $M_{big}$ [11]. Then, we can obtain the and combination of the constraint (12) as follows:

$$t^m_i - t^m_i + M_{big}B_i \geq S/v^m_i$$

and

$$t^m_i - t^m_i + M_{big}(1 - B_i) \geq S/v^m_i$$

$$\forall \phi_i, \phi_j \in \phi, \phi_i \neq \phi_j, \quad B_i \text{ binary variable.}$$  \tag{13}

Here, we provide a brief explanation of how the and combination in (13) works. If $B_i = 0$, both of the equations hold but only the first equation is true while the second one is redundant if $M_{big}$ is big enough. This will allow CAV $i$ to enter the MZ once CAV $j$ exits the MZ. If $B_i = 1$, both of the equations also hold but this time the second equation is true while the first one is redundant if $M_{big}$ is big enough. In this case, CAV $j$ will enter the MZ without having rear-end collision with another CAV on the same movement. Therefore, the following constraint is imposed on $t^m_i$ to separate the vehicles on the same movement for some distance (recall Assumption 2 that the speed inside the MZ, $v_i^m$ is the same as $v_i$):

$$t^m_i - t^m_i \geq \delta/v^m_i$$

$$\forall m_i, m_2 \in M, \quad m_i = m_2.$$  \tag{11}

Please note that the constraint above is different from (3) where (3) is to guarantee the absence of rear-end collision when CAV $i$ moves from $t_0$ until $t^m_i$ while the constraint (11) is to separate the CAVs at $t^m_i$ only.

The problem above can be solved using various kind of available MILP solvers. In our research, we use intlinprog function from Matlab optimization toolbox to solve the problem. Note that, we denote the solution of the problem as $t^m_i = [t^m_i, t^m_2, \ldots, t^m_N]$ where this solution may be used in the energy minimization problem in the next subsection to specify the terminal time.

2.3 Energy Minimization Problem Formulation

According to [14], we can achieve minimal energy consumption (this paper focus on fuel consumption) by minimizing the acceleration of each vehicle, $+u_i(t)$. The deceleration, $-u_i(t)$, however, is not the main concern since only braking is applied while the fuel injection is terminated. Therefore, we can have a direct benefit on fuel consumption by minimizing the following objective function:

$$\min_{u_i} \frac{1}{2} \sum_{t \in N(t)} \int_{t_i}^{t^m_i} u_i^2(t)dt$$

s.t. (1)-(3), (5), $p(t^m_i) = L,$

and given $t_0, p^0_i, v^0_i, t^m_i.$  \tag{14}

From the objective function above, $t_0, p^0_i,$ and $v^0_i$ are known when a new CAV $i$ enters the CZ and terminal time $t^m_i$ can be obtained by solving (14). Note that, the solution $u_i$ cannot be the trivial solution $u_i = 0$ because CAVs need to satisfy the boundary conditions at $t^m_i$, e.g., $p(t^m_i) = L$.

For the solution and online implementation of energy minimization problem (15), we apply Hamiltonian analysis under Assumption 1 where none of the constraints (2)-(3) are active at $t_0$. This means that the optimal solution that we are going to obtain may not satisfy the limits in (2) $\forall t \in [t_0, t^m_i]$. Even though we already provided the minimum merging time in (10), it still cannot guarantee that the CAVs will travel within limits. For the constrained problem, one may refer to [16].

From (15) and the state equations (1), for each CAV $i \in N(t)$ we can formulate the Hamiltonian function as follows:

$$H_i(t, p(t), v(t), u(t)) = \frac{1}{2} u_i^2 + \lambda^0_i \cdot v_i$$

$$+ \lambda^1_i \cdot u_i.$$  \tag{16}

where $\lambda^0_i$ and $\lambda^1_i$ are the co-states. The necessary condition for optimality is

$$\frac{\partial H_i}{\partial u_i} = u_i + \lambda^1_i = 0.$$  \tag{17}
From (17), the optimal control is given by
\[ u_i + \lambda_i^* = 0, \quad i \in \mathbb{N}(t) \] (18)
and the Euler-Lagrange equations yield
\[ \dot{\lambda}_i^* = -\frac{\partial H_i}{\partial p_i} = 0, \] (19)
\[ \dot{\lambda}_i^* = -\frac{\partial H_i}{\partial v_i} = -\lambda_i^*. \] (20)

From (19) we have \( \lambda_i^* = a_i t + b_i \), and (20) implies \( \lambda_i^* = -a_i t - b_i \), where \( a_i \) and \( b_i \) are integration constants. Substituting the previously obtained \( \lambda_i^* \) back to (18) gives us the optimal control input (acceleration/deceleration) as a function of time as follows:
\[ u_i^*(t) = a_i t + b_i. \] (21)

Next, by substituting (21) into the vehicle dynamics (1) we can find the optimal speed and position for each vehicle, namely
\[ v_i^*(t) = \frac{1}{2} a_i t^2 + b_i t + c_i, \] (22)
\[ p_i^*(t) = \frac{1}{6} a_i t^3 + \frac{1}{2} b_i t^2 + c_i t + d_i, \] (23)
where \( c_i \) and \( d_i \) are integration constants. These four constants can be computed by using the initial and final conditions of the problem (15), i.e., \( v_i^0, p_i^0, v_i^m, \) and \( p_i^m = L \). Then, we can form the system of four equations of the form \( \mathbf{b}_i, \mathbf{q}_i \):
\[
\begin{bmatrix}
\frac{1}{2}(t_i^0)^2 & \frac{1}{2}(t_i^0) & t_i^0 & 1 & [a_i] & [p_i(t_i^0)] \\
1 & 0 & 0 & b_i & v_i(t_i^0) \\
\frac{1}{2}(t_i^m)^2 & \frac{1}{2}(t_i^m) & t_i^m & 1 & c_i & p_i(t_i^m) \\
1 & 0 & 0 & d_i & v_i(t_i^m)
\end{bmatrix}
\] (24)
where \( t_i^m \) is obtained by solving (14). Note that since (24) can be computed online, the controller may re-evaluate the four constants in the form \( a_i(t, p_i, v_i), b_i(t, p_i, v_i), c_i(t, p_i, v_i), \) and \( d_i(t, p_i, v_i) \) at any time \( t > t_0 \) to get
\[ \mathbf{b}_i(t, p_i(t), v_i(t)) = (\mathbf{T}_i)^{-1} \cdot \mathbf{q}_i(t, p_i(t), v_i(t)), \] (25)
and thus, the optimal control input in (21) also is updated as
\[ u_i^*(t, p_i(t), v_i(t)) = a_i(t, p_i(t), v_i(t)) + b_i(t, p_i(t), v_i(t)). \] (26)

As we can observe from the above equation, a sense of feedback can be indirectly provided through the recalculation of the vector \( \mathbf{b}_i(t, p_i(t), v_i(t)) \) in (25).

Recall that we have imposed the safety constraints in (3) and (4) to avoid rear-end and lateral collisions, respectively. Even though the constraint (4) is not implicitly included in a solution of (15), but it can be satisfied through the selection of \( t_i^m \) in (14) because it only affects (15) at \( t = t_i^m \). On the other hand, we have stated previously that we cannot satisfy (3) for all \( t \in [t_i^0, t_i^m] \). However, since we only consider the unconstrained problem, two conditions may satisfy (3) for all \( t \in [t_i^0, t_i^m] \). First, all CAVs must satisfy (3) at \( t_i^0 \) (if CAV exists). Second, all CAVs must have the same speed at \( t_i^0 \). Note that, we adopt these conditions in performing our simulations in the next section.

3. Simulation Results

To validate the effectiveness of the proposed solution, we simulated the crossing scenario presented previously in Matlab. In the simulation, we set the length of the CZ, L, and MZ, S, to be 400 m and 30 m, respectively. The safety distance between the CAVs on the same movement, \( \delta \), is set to be 10 m. For the maximum and minimum speed limits, we set them as 13 m/s and 0 m/s while for the acceleration and deceleration limits, we set them as 3 m/s² and -3 m/s², respectively. Previously we mentioned that we need to choose the constant \( M_{big} \) to be sufficiently large. By following the provided guide, we chose the \( M_{big} \) to be 2000. Finally, we assumed that the current time, \( t_0 \), is 0 s and the speed of all CAVs at the current time, \( v_i^0 \), is 11.1 m/s, and this speed also is assumed to be the same when the CAVs enter the MZ.

There are two case studies that we considered: (1) coordination of 8 CAVs, 2 for each movement (2) coordination of 30 CAVs with random current positions, \( p_i^0 \), and movement, \( m_i \). For the first case study, we want to see how the controller assigns the optimal merging time, \( t_i^m \), to each CAV and to see how the CAVs reaction to achieve the assigned merging time. For the second one, we want to compare our proposed solution with FIFO passing order regarding time delay and fuel consumption.

3.1 Case Study 1: Coordination of 8 Vehicles

In this case study, we simulated 8 CAVs with the current position, \( p_i^0 \), and movement, \( m_i \), given as [30 m, 30 m, 20 m, 20 m, 15 m, 15 m, 1 m, 1 m] and \( [O', X', O', X', O', X', O', X'] \) respectively. The illustration of these positions and movement are provided in Fig. 3. As can be seen from the figure, the CAVs might have a collision between each other if no appropriate merging time and control input are assigned to the CAVs.

By solving the problem (14), we obtained \( t_i^m \) for each CAV as shown in Fig. 4. As shown in the figure, the controller tends to assign \( t_i^m \) by grouping, i.e., all Phase O CAVs merge first and then followed by Phase X CAVs, which can logically maximize the traffic flow. The obtained \( t_i^m \) is used as the terminal time for the problem (15) to obtain the control input as in Fig. 5. These inputs drove the CAVs to cross the intersection. As can be seen from Fig. 6, the CAVs do not come to a full stop when cross-
ing the intersection. This means less acceleration is required for each CAV, and therefore, the fuel consumption can be preserved. To see either the CAVs collide with each other or not, we can analyze it from Fig. 7, where the upper part (vertical axis \( \geq 0 \)) of the figure is phase O CAVs and the down part (vertical axis \( \leq 0 \)) is phase X CAVs. As can be seen from the figure, only the vehicles with the same phase but different movement (e.g., CAV 2 and CAV 4) share the merging time while the others maintain the rear-end (e.g., CAV 3 and CAV 7) and lateral (e.g., CAV 5 and CAV 6) safety distances. This shows that no collision happens inside the CZ.

### 3.2 Case Study 2: Coordination of 30 Vehicles

In this case study, we considered the coordination of 30 CAVs with random \( p_i^0 \) and compared its impact on time delay and fuel consumption with FIFO crossing order. For the time delay, it can be computed as

\[
t_d^i = t_{i_{\text{opt}}} - t_{i_{\text{min}}},
\]

while to quantify the impact on the fuel consumption, we utilized the polynomial metamodel proposed in [21]. As shown in Fig. 8, the average time delay improvement of the proposed solution greatly outperformed the FIFO crossing order. On the other hand, the proposed solution also has shown some improvements in terms of fuel consumption as can be seen in Fig. 9. Note that, we cannot reduce the fuel consumption much better because, in the FIFO strategy, the CAVs also cross the intersection without stop and go driving. This means that less acceleration is required for the CAVs. Because of this factor, the CAVs in the FIFO strategy also consumed less fuel. To ensure no collision issue under the conditions mentioned previously, i.e., at \( t_{0} \) all CAVs must satisfy (3) and also have the same \( v_i^0 \), we provide Fig. 10 which illustrates the movement of vehicles inside the CZ. As shown in the figure, no CAVs from the same movement as well as CAVs from different phases share the same \( t_{i_{\text{opt}}} \). Thus, this shows that no collision occurs at the intersection.

### 4. Conclusion

This paper has addressed the optimal crossing order problem for CAVs to cross an intersection without any physical traffic light. The problem was formulated with MILP, whose solution yielded the optimal time for each vehicle to cross the intersection. The optimal crossing time then became a terminal time for the energy minimization problem to obtain optimal acceleration and deceleration that drive the vehicles to reach the time while conserving fuel consumption. Next, we validated the proposed solution through simulation, and it was shown that it could reduce both time delay and fuel consumption significantly. This
means that we can allow the vehicles to cross the intersection with minimal traffic congestion and at the same time reducing fuel consumption. In future research, we are going to extend the current work by considering the turning of vehicles (right or left) as well as considering multi-intersections.

Acknowledgments

This work was supported by JSPS KAKENHI Grant Number JP17H03283.

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