Bi-level Control for Connected and Automated Vehicles at Signal-free Intersections: A Performance Analysis

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Abstract This paper deals with the coordination problem for connected and automated vehicles (CAVs) to cross an intersection. First, we formulate the problem with Mixed Integer Linear Programming (MILP) which the solution yields the appropriate crossing time for each CAV. Then, we formulate the optimal energy problem to obtain the required acceleration or deceleration to drive to accurately achieve the assigned crossing time while minimizing fuel consumption. Through the analysis from our simulation, more work need to be done to reduce the computation burden in solving the MILP problem and the optimal trajectory formulation should be revised to further reduce the fuel consumption.

Key Words: connected and automated vehicles (CAVs), autonomous intersection, bi-level control

1 Introduction

We are witnessing the alarming state of traffic accident and congestion due to the current transportation systems. Some studies suggested that driver response to various disturbances at the roadways can cause the congestion but in fact, the intersection and merging roadways are the primary bottlenecks [1]. The traffic congestion can cause many negative impacts to the people especially to those who are living in the urban areas. Because of this, many solutions have been proposed to overcome the traffic congestion. As we all aware, the current approach to solve the traffic congestion especially at intersections is by using traffic light. Many researches have been done to optimize the traffic light signal which also include the vehicle-to-infrastructure (V2I) and infrastructure-to-infrastructure (I2I) communications. However, the results still did not provide significant improvement in reducing the percentage of congestion especially in high traffic density. With the recent advancement of connected and automated vehicle (CAV) technology, it drives the researchers to consider a whole new method to solve the congestion problem. The new method is to coordinate and control the CAVs at the intersection without needing the traffic light and this method can be called as autonomous intersection.

From the literature, we can categorize the coordination strategy for the autonomous intersection into two main categories which are resource (space and time slots) reservation and trajectory planning. For the space reservation strategy, the CAV needs to reserve a space-time block in the conflict area so that it can cross the conflict area without colliding with each other. To the best of our knowledge, one of the earliest works that proposed the space reservation strategy was done in [2] and the recent variation of the work can be found in [3]. On the other hand, the time slots reservation is to assign the optimal crossing time for each CAV to cross the conflict area. Some mathematical optimization tools such as linear programming [4], mixed integer linear programming (MILP) [5], mixed integer quadratic programming (MIQP) [6], and dynamic programming [7] were utilized to formulate the optimal crossing time problem where the problem is also known as a scheduling problem. The advantage of utilizing the mathematical optimization tools is that it can reduce the travel time for each vehicle but it is usually requires high computational effort. Because of this, several works formulated the scheduling problem based on first-in-first-out (FIFO) order [8], [9] and with dynamic resequencing [10]. To reach the crossing time precisely, the trajectory planning problem can be formulated with several additional objectives for example avoiding rear-end collision, minimizing fuel consumption, and increasing passenger comfort. Some existing strategies in solving or formulating the optimal trajectory planning problem are Hamiltonian analysis [8],[10], model predictive control (MPC) [11], and discrete optimization [12]. Note that the trajectory planning problem also can be independent from the crossing time.

In our previous work [13], we proposed a bi-level control for CAVs to cross a conflict area at signal-free intersection. However, no detailed analysis was conducted for the computation time in solving the MILP problem and we are not sure that how to reduce the fuel consumption further.

2 Problem Formulation

Fig. 1 shows a two-phase and four-movement intersection that we consider in this paper. The abbreviation and symbols used are tabulated in Table 1. The intersection has a coordinator to assign crossing time for all vehicles inside the control zone.

Before further explaining, we provide some assumptions that are commonly used in this field of research with some additional assumptions: (1) all vehicles are CAVs, (2) the dynamics of all vehicles are the same, (3) the speed inside the MZ is constant, (4) all vehicles can follow exactly the assigned crossing time,
2.1 Vehicle Model and Constraints

In this paper, we model each vehicle \( i = 1, 2, ..., N(t) \) as second order dynamics as follows:

\[
\begin{bmatrix}
\dot{p}_i \\
\dot{v}_i
\end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p_i(t) \\ v_i(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i(t),
\]

where \( N(t) \) is the total vehicles subscribed (subscription process will be discussed later) to the coordinator at time \( t \), \( p_i(t) \in [0, L + S] \) is the position, \( v_i(t) \) is the speed, and \( u_i(t) \) is the control input (acceleration/deceleration). Note that at every current time, \( t_0 \) the position and speed, i.e., \( p_i(t_0) \) and \( v_i(t_0) \) are given.

In the vehicle model described above, we include the constraints on the input and speed so that the vehicle moves within a given admissible range. Mathematically, the constraints can be defined as

\[
\begin{align*}
\forall i \in N(t), \quad & u_{i,\text{min}} \leq u_i(t) \leq u_{i,\text{max}}, \quad \text{and} \\
& 0 \leq v_{\text{min}} \leq v_i(t) \leq v_{\text{max}}, \quad \forall t \in [t_0, t_i^f],
\end{align*}
\]

where \( u_{i,\text{min}}, u_{i,\text{max}} \) are the minimum and maximum inputs for each vehicle \( i \), and \( v_{\text{min}}, v_{\text{max}} \) are the minimum and maximum speed limits respectively, and \( t_i^f \) is the time that vehicle \( i \) leaves the MZ (the time also can be called as final time). Since we assume that all vehicles are the same, therefore, we can set \( u_{i,\text{min}} = u_{\text{min}} \) and \( u_{i,\text{max}} = u_{\text{max}} \).

Rear-end collision can potentially happen if two consecutive CAVs are traveling on the same movement (e.g., CAV 4 and CAV 5 in Fig. 1). Therefore, to ensure the absence of the rear-end collision, we can define a safety distance between the two CAVs:

\[
s_i(t) = p_k(t) - p_i(t) \geq h_R v_i(t), \quad \forall t \in [t_0, t_i^f], \quad \forall m_i, m_k \in M, \quad m_i = m_k,
\]

where \( m_i \in M \) is the movement of CAV \( i \), \( k \) is the preceding vehicle of CAV \( i \) and \( h_R \) is the safety headway time to avoid rear-end collision between CAV \( i \) and CAV \( k \).

On the other hand, to avoid the lateral collision, two vehicles from different movement and phases, for example vehicle 1 and vehicle 2 in Fig. 1, have to enter the MZ one at a time. Mathematically, the strategy can be described as follows:

\[
\forall \phi \in \Phi, \quad \forall t \in [t_i^m, t_i^f], \quad \forall \phi_i, \phi_j \in \Phi, \quad \phi_i \neq \phi_j,
\]

where \( \Gamma_i \) is the set of time for CAV \( i \) from \( t_i^m \) until \( t_i^f \) and \( \emptyset \) is the empty set.

Before explaining the optimal crossing time and trajectory planning formulations, we provide an illustration of the proposed bi-level control structure in Fig. 2. Once the vehicle subscribed to the coordinator, i.e., reaches the CZ, the vehicle will be assigned an ID and it is numbered as a sequence of the vehicle entering the CZ. Then, the vehicle will start to share a set of information, \( Y_i = \{ m_i, p_i, v_i \} \) with the coordinator at every execution time, \( t_e \). The sharing of \( Y_i \) provides a sense of feedback to the bi-level control structure. The information set is used by the coordinator to compute the crossing time, \( t_i^m \) for each vehicle. Incorporating the assigned crossing time with the current position and speed, the vehicle can obtain the required control input, \( u_i \) to reach the assigned crossing time through the embedded merging controller. The process described above is repeated until the vehicle unsubscribed from the coordinator, i.e., leaving the MZ.

2.2 Optimal Crossing Time Formulation

The main objective of obtaining the optimal crossing time is to minimize the total travel time of all vehicles which theoretically can reduce the congestion at the intersection. In this problem, we denote \( t_i^m \) as the crossing time that we wish to obtain and by utilizing MILP, we can formulate the optimal crossing
time problem as follows:

$$\begin{align*}
\min_{T, B} & \sum_{i=1}^{N(t)} t_i^m + \omega_2 \sum_{i=1}^{N(t)} t_i^{abs}, \\
\text{s.t.} & \quad t_i^m \geq t_{i, \text{min}}, \\
& \quad t_i^m - t_i^B \geq h_R, \\
& \quad t_i^m - t_j^m + M_{big} b_{i,j} \geq h_L, \\
& \quad t_j^m - t_i^m + M_{big} (1 - b_{i,j}) \geq h_L, \\
& \quad t_i^{abs} \geq (v_i^m - v_i^s), \\
& \quad t_j^{abs} \geq -(t_j^m - t_i^B),
\end{align*}$$

(5)

where $T$ is the set of all $t_i^m$, i.e., $T := \{t_1^m, t_2^m, \ldots, t_{N(t)}^m\}$, $B$ is the set of all binary variables $b_{i,j}, i \neq j$, and $\phi_i, i \neq \phi_j$, i.e., $B := [b_{1,2}, b_{1,3}, \ldots, b_{N(t)-1, N(t)}]$, $\omega_1$ and $\omega_2$ are the weights, and $t_i^{abs}$ is the absolute time difference between the newly assigned crossing time $t_i^m$ and previously assigned crossing time $t_i^B$, i.e., $t_i^{abs} = |t_i^m - t_i^B|$. The first terms of (5) is to minimize the total travel time of all vehicles while the second terms is added to minimize the difference between $t_i^m$ and $t_i^B$ which may increase passenger comfort and reducing energy consumption [9].

Constraint (6) ensures that the obtained $t_i^m$ is within the earliest crossing time, $t_{i, \text{min}}$, and the latest crossing time, $t_{i, \text{max}}$. To achieve the earliest crossing time, the vehicle has to accelerate to $v_{\text{max}}$ as fast as possible. This $t_{i, \text{min}}^m$ can be calculated as follows:

$$
t_i^m = \min \left\{ \frac{v_{\text{max}} - v_i^s}{u_{\text{max}}}, \frac{v_{\text{max}} - v_i^m - v_i^s}{u_{\text{max}}} \right\},
$$

(12)

$$
t_i^m = \max \left\{ \frac{D_i}{v_{\text{max}}}, \frac{v_{\text{max}} - v_i^2}{2u_{\text{max}}v_{\text{max}}}, 0 \right\},
$$

(13)

$$
t_i^{m, \text{min}} = t_0 + t_i^1 + t_i^2,
$$

(14)

where $D_i = L - p_i^0$, is the distance of each vehicle $i$ to the MZ, and $v_{i, \text{max}} = \sqrt{(v_i^m)^2 + 2u_{\text{max}}D_i}$ is the maximum merging speed that the vehicle can reach before entering the MZ.

Constraint (7) guarantees the absence of rear-end collision for two vehicles in the same movement when reaching the entry of the MZ which also satisfies (3). Constraints (8) and (9) guarantee the absence of lateral collision for two vehicles from different movement and phases. As can be noticed from (8) and (9), an arbitrarily large constant number, $M_{big}$ and auxiliary binary variables, $b_{i,j}$ are introduced. These constant number and auxiliary binary variables are necessary to model the disjunction constraints (8) and (9). In addition, by deciding the binary variables will decide which vehicle crosses first. For example, if $b_{i,j} = 0$ vehicle $j$ will cross first and if $b_{i,j} = 1$ vehicle $i$ will cross first. $t_{i}^m$ is the time that the vehicle spends inside the MZ.

Finally, constraints (10) and (11) are introduced so that the slack variable $t_i^{abs}$ in (5) is equivalent to $|t_i^m - t_i^B|$. Note that to solve the MILP problem above, we utilized intlinprog function from Matlab. We denote the solution of $t_i^m$ in problem (5) as $v_i^m = \{t_1^m, t_2^m, \ldots, t_{N(t)}^m\}$ where this solution will be used as terminal time for optimal energy problem formulation in the next section.

### 2.3 Optimal Trajectory Planning Formulation

In this subsection, we explain the optimal trajectory planning formulation for the vehicle to reach the optimal crossing time, $t_i^{m*}$ assigned by the coordinator while minimizing fuel consumption. The problem of minimizing the control input in particular the acceleration, is equivalent to the problem of minimizing fuel consumption [12] and thus, the optimal trajectory planning problem can be formulated as below:

$$
\begin{align*}
\min_{u_i} \frac{1}{2} \int_{t_0}^{t_{i, \text{m}}} u_i^2(t)dt, \\
\text{subject to: } & \ (1), (2), \ p_i(t_i^{m*}) = L, \\
& \ \text{and given } t_0, p_i^0, v_i^0, t_i^{m*}.
\end{align*}
$$

(15)

In this paper, we solve the problem above using an analytical solution, i.e., Hamiltonian analysis which can provide a very fast computation time in obtaining the solution. Due to space limitations, all the solution steps are omitted but can be found in [13],[7].

### 3 Simulation Results

For analyzing the performance of the proposed bi-level control strategy comprehensively, we have developed three testbeds which are testbed A, testbed B and testbed C by using Matlab as shown in Fig. 3. In testbed A, fixed time traffic light signals are utilized to coordinate the movement of the human-driven vehicles where we modeled the human-driven vehicle as Gipps car-following model. In testbed B, the bi-level control strategy is utilized but for the coordinator level, the crossing time for all vehicles is solved based on FIFO order [13]. Note that, these testbed A and testbed B were developed for benchmarking purpose only. For testbed C, the bi-level control strategy
that we have discussed in the previous section is utilized.

In general, we run all the simulation testbeds on Windows 10 PC with Intel Core i7@3.60 GHz and 16 GB of RAM. For all testbeds, the length of the CZ, L (for testbed A, the length of CZ is the same as the length of the intersection legs) and the size of the MZ, S are set as 300 m and 20 m respectively. The vehicles in the same movement are separated with safety headway $h_R = 1$ s while the vehicles from different movements and phases are separated with safety headway $h_x = 2.2$ s. We assumed the arrival rate of the vehicles are given by a Poisson process with $\lambda = 450$ veh/h and $\lambda = 850$ veh/h for low and medium traffic environments respectively. The arrival speed is assumed to be the same for all vehicles which are 11.1 m/s. The minimum and maximum speeds limits are 0 m/s and 13 m/s while the minimum and maximum inputs for each vehicle are 2 m/s and $-2$ m/s, respectively.

In choosing the value of $M_{big}$, in particular for testbed C, we followed the provided guide from [9] and we chose the value to be $M_{big} = 3000$. Since vehicles are kept arriving into the CZ, the coordinator has to recompute the crossing time frequently. For testbed B, the coordinator recomputes the crossing time when a new vehicle entered the CZ while for testbed C, the crossing time is recomputed at every execution time $t_e = 2$ s.

The results for the performance analysis for the bi-level control are tabulated in Table 2 and Table 3 for low and high traffic environments respectively. As can be seen from Table 2, the maximum computation time of testbed C is kinda feasible to implement the bi-level control strategy in real-time. For the average travel time, the performance of testbed C does not vary much from testbed B but is way better than testbed A. On the other hand, testbed C improves a lot in terms of travel time as compared to testbed A and B for medium traffic environment (see Table 3). However the computation is quite high in order to implement the bi-level control strategy in real-time (below 0.1 s is better). In addition, the cumulative fuel consumption also improved a little as compared to testbed B. As can be can be concluded from the literature, the formulation of the trajectory planning problem in (15) should be revisited to further improve the fuel consumption.

### Table 2: Performance evaluation in medium traffic environment.

<table>
<thead>
<tr>
<th>Performance evaluation</th>
<th>Testbed A</th>
<th>Testbed B</th>
<th>Testbed C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test duration [s]</td>
<td>600</td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td>Max computation time [s]</td>
<td>-</td>
<td>0.005</td>
<td>0.14</td>
</tr>
<tr>
<td>Average travel time per vehicle [s]</td>
<td>54.39</td>
<td>25.94</td>
<td>25.07</td>
</tr>
<tr>
<td>Cumulative fuel consumption [l]</td>
<td>8.69</td>
<td>3.78</td>
<td>3.88</td>
</tr>
</tbody>
</table>

### Table 3: Performance evaluation in medium traffic environment.

<table>
<thead>
<tr>
<th>Performance evaluation</th>
<th>Testbed A</th>
<th>Testbed B</th>
<th>Testbed C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test duration [s]</td>
<td>600</td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td>Max computation time [s]</td>
<td>-</td>
<td>0.001</td>
<td>0.52</td>
</tr>
<tr>
<td>Average travel time per vehicle [s]</td>
<td>79.40</td>
<td>42.22</td>
<td>26.18</td>
</tr>
<tr>
<td>Cumulative fuel consumption [l]</td>
<td>15.32</td>
<td>7.47</td>
<td>6.90</td>
</tr>
</tbody>
</table>

4 Conclusion

This paper has addressed the coordination problem for CAVs to cross an intersection with bi-level control. At first, the problem was formulated with MILP...
which the solution yielded the appropriate crossing
time for each CAV. Then, the optimal control input
was obtained in the second level to drive each CAV
towards the assigned crossing time with minimum fuel
consumption. The simulation results have shown that
the proposed solution can reduce the travel time sig-
ificantly but the computation time to solve the MILP
problem is quite high. Therefore in the future re-
search, more work need to be done to reduce the com-
putation burden in solving the MILP problem and the
optimal trajectory formulation should be revised to
further reduce the fuel consumption.

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