Traffic Signal Control Considering Switching Timing via Distributed Model Predictive Control

S. Sasaki* and T. Namerikawa
Department of System Design Engineering, Keio University, Yokohama, Japan (shuns@nl.sd.keio.ac.jp, namerikawa@sd.keio.ac.jp)

Abstract: In this paper, we aim to alleviate traffic congestion on each link online by traffic signal control for local road transport network to be controlled. We first propose a transportation model with time delay system considering the travel time of the vehicle and calculate the optimal signal phases with variable cycle length and offset using distributed model predictive control for that model to reduce congestion rate on local roads. We then explain algorithms for distributed control that each intersection solves the optimization problem in parallel sharing information only with adjacent intersection. Finally, the effectiveness of the proposed method is confirmed by numerical simulation.

Keywords: Traffic Signal Control, Intelligent Transport Systems, Distributed Model Predictive Control

1. INTRODUCTION

In recent years, the increase in traffic volume and traffic congestion have become problems. Traffic congestion not only hinders economic activities, but also leads to environmental deterioration due to increased energy consumption and an increase in traffic accidents. Various solutions have been proposed for general roads, such as speed control[1] and parking management[2], but the traffic signal control is the most effective countermeasure against the traffic congestion because the traffic is easy to be concentrated at the intersection on the general road. In addition, traffic information can be acquired in real time by the spread of vehicle detectors, and research on traffic signal control is still actively conducted.

Performance indicators for traffic signal control include: vehicle delay time [3], queue[4], number of stops of vehicle[5], expected travel time[6], fuel consumption[7]. In this paper, the queue is used as an evaluation index, and the queue near the intersection is predicted by model predictive control, and the optimal green light time for each route and the timing to switch the signal are calculated online to alleviate traffic congestion on ordinary roads.

Although various control methods have been proposed for signal control, many are based on macroscopic traffic flow models[8], [9] which do not take into consideration the travel time of the vehicle. On the other hand, micro-viewpoint models[10] and movement time probability distribution models[11] considering movement of individual vehicles are also proposed, but the computational load increases due to complexity. And there is a problem that it is difficult to take account of turning left and right. In this paper, we consider the travel time of the vehicle as the time delay until the input of the traffic signal affects the queue, and propose a new traffic flow model with a dead time system.

Also, although it is common to set the cycle length to be constant with the sampling time with the split of the green light time as the control input, and solve the optimization problem without considering the offset, a method of using signal indication as 0-1 variable input[12] is also proposed. The latter advantage is that the cycle length and offset can also be varied and controlled, and the appropriate timing to switch the signal indication can also be found. Taking this into consideration, in this paper, the system displays a signal as a binary variable of 0-1 and uses it as input to make a system that determines whether each signal should be switched at each sampling time while performing traffic signal control by varying the three signal control parameters of cycle length, split and offset.

Furthermore, this paper proposes a traffic signal control algorithm for distributed model predictive control. In centralized traffic signal control, one controller solves the optimization problem for the target traffic network, whereas in the proposed distributed traffic signal control, controllers are distributed at each intersection. Decentralized control is considered to be practical because it has various merits such as reduction of calculation load and resistance to failure of equipment.

This paper is constructed as follows. Chapter 2 describes the problem setting of the traffic model to be targeted, and then proposes the traffic signal control method of distributed control in Chapter 3. Chapter 4 confirms the effectiveness of the proposed method by numerical simulation, and Chapter 5 gives a summary of this paper.

2. PROBLEM SETTING

The traffic network shown in Fig.1 is to be controlled, and there are 6 traffic intersections and 14 links. It is a bi-directional network like an arrow. The road extending to the left and right under the network is two lanes to assume the case where traffic congestion is likely to occur. In this paper, it is assumed that the total number of vehicles in the traffic network is almost constant. Control the signal so that the vehicle is not concentrated on the link, and reduce congestion. In addition, inflow and outflow of vehicles with the outside network are performed through the intersection 2, 3, 5 and 6.

The traffic flow model is shown in Fig.2. It is divided into vehicles in the queue and vehicles in motion. The
Fig. 1 Traffic network

Table 1 Definition of parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_z$</td>
<td>Number of vehicles in queue</td>
</tr>
<tr>
<td>$u_z$</td>
<td>Signal phase (0:red 1:green)</td>
</tr>
<tr>
<td>$\tau_{w,z}$</td>
<td>Inflow ratio (Turning Rate)</td>
</tr>
<tr>
<td>$s_z$</td>
<td>Number of vehicles passing the stop line per unit time</td>
</tr>
<tr>
<td>$d_z$</td>
<td>Travel time of vehicles</td>
</tr>
<tr>
<td>$I_j$</td>
<td>The set of inflow roads to the intersection $j$</td>
</tr>
<tr>
<td>$T$</td>
<td>Sampling time</td>
</tr>
</tbody>
</table>

The definition of each parameter is shown in Table 1. Only the number of vehicles in the queue is $x$. Then, consider controlling this so that it does not bias to a specific path.

Here, the following equation holds for $x$ on the path $z$ at time $t$.

$$ \dot{x}_z(t) = -s_z(t)u_z(t) + \sum_{w \in I_j} \tau_{w,z} s_w(t) - d_z u_w(t - d_z) + e_z(t) $$

(1)

$I_j$ is a set of inflows for a certain intersection $j$. Also, $\tau$ is the rate of flowing from path $w$ to path $z$ (Turning Rate), and $s_z$ is the number of vehicles passing the stop line per unit time of path $z$.

And $u_z$ is a signal indication in the outflow direction of path $z$, and is an input added to the control target. As mentioned above, this input is 0-1 variables, where 0 corresponds to red light and 1 corresponds to green light. $d_z$ is the travel time for vehicles from $w$ to $z$ to enter the queue, $e_z(t)$, is a disturbance that represents the difference between the number of vehicles flowing into the link $z$ minus the number of vehicles flowing out of the link $z$.

Converting Eq. (1) into a discrete-time model to use model predictive control:

$$ x_z(k+1) = x_z(k) - s_z(k)T u_z(k) + \sum_{w \in I_j} \tau_{w,z} s_w(k) - d_z u_w(k - d_z) + e_z(k) $$

(2)

$T$ is the sampling time, and is set $T \leq d_z, \frac{d_z}{T} \in Z$ for all paths. ($Z$ is the set of integers.)

The number of vehicles passing the stop line per unit time $s$ is usually set as the saturated traffic flow rate. Saturated traffic flow rate refers to the maximum number of vehicles that can pass the stop line per hour of effective blue hour on a certain route. However, on real roads, queues do not always exist on the route. Therefore, by setting $s$ according to $x$ of each time, we considered to be a more realistic model.

$$ s_z(k) = \min(s_z^{max}, x_z(k)/T) $$

(3)

In Eq. (3), $s_z^{max}$ is the saturated traffic flow rate.

3. CONGESTION RATE MITIGATION

3.1 Optimization problem

As mentioned at the beginning, the congestion is alleviated by decentralized control in which each intersection solves the optimization problem in parallel. Each intersection controls the inflow path as shown in Fig. 3. (For example, in the case of intersection 1, Link1, Link9, Link12) Here, the following assumptions are made.

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Assumption

Each intersection can share $u$ and $x$ information only with adjacent intersections.

From the fact that the third term of the right side of Eq. (2) represents the number of vehicles inflowing, and the input $u$ (signal expression) is included, and from Eq. (3) $s$ depends on the state quantity $x$ (the number of vehicles in the queue), the above assumption means that each intersection can know the number of vehicles coming in from adjacent intersections as shown in Fig. 3 (For example, intersection 1 can share information with intersections 2, 4 and 6).

The evaluation function to calculate the optimal input for a certain intersection $j$ was set as follows.

$$
\min_{u_j} \sum_{i=1}^{H_p} x_j^T(k+i)Q_jx_j(k+i) + \sum_{i=0}^{H_p-1} [u_j(k+i) - u_j(k+i+1)]^T R_j [u_j(k+i) - u_j(k+i+1)]
$$

(4)

$x_j$ and $u_j$ are matrices summarizing the state quantities and inputs for the set of inflow roads to the intersection $j$. (For example, in the case of intersection 1, $x_j = [x_1 x_0 x_0^T]$, $u_j = [u_1 u_0 u_1^T]$) $H_p$ is the prediction horizon in model predictive control. The first term on the right side of Eq. (4) is the term related to the state quantity (number of vehicles) in the traffic network of the forecast section, and it prevents the vehicle from concentrating on a specific link for the purpose of alleviating traffic congestion, which is the purpose of this research.

The second term is a term related to the amount of change of the input (signaling indication) in the prediction interval, and is for suppressing frequent fluctuations of the input.

That is, the first term and the second term are in a trade-off relationship with each other. $Q_j$ and $R_j$ are weight matrices.

The constraints are as follows.

$$
x_z(k+1) = x_z(k) - s_z(k)Tu_z(k) + \sum_{w \in \ell_j} \tau_{w,z}s_w(k) - d_z(T)Tu_w(k - d_z(T)) + e_z(k)
$$

(5)

$$
Tu_z(k) \in \{0, 1\} \quad \forall z
$$

(6)

$$
u_{j,a}(k) + u_{j,b}(k) = 1 \quad \forall j
$$

(7)

Eq. (7) is a constraint on the input for each intersection, and means for roads with two directions, horizontal $a$ and vertical $b$, if one is blue, the other must be red at a certain intersection $j$ as in Fig. 4.

Furthermore, in addition to Eq. (5) ~ (7), the constraint of the following equation also holds in a bidirectional network.

$$
\begin{align*}
u_{j,a}(k) = u_{j,-a}(k) & \quad \forall j
\end{align*}
$$

(8)

This means that the signal indication is the same for roads in the opposite lane relationship among the inflow roads to the intersection $j$.

3.2 Distributed Traffic Signal Control Algorithm

The merit of performing distributed control is that the computational load is reduced. For example, if we tried to solve Eq. (4) with $H_p = 3$ centralized control the input is $2^{18}$ cases in all, it is necessary to find the best solution among them, but in the distributed control in this study, it can be divided into several simple subproblems, only $2^3$ cases. Thus, it can be said that distributed control is highly effective for the larger target network.

On the other hand, in the centralized model predictive control, it is necessary to solve the optimization problem only once at each time, but in the distributed type in model predictive control, it is necessary to solve the optimization problem again considering the future number of vehicles coming in from the adjacent intersection while the future state quantities and inputs predicted by calculating the optimization problem are shared between adjacent intersections. It is because the number of vehicles inflowing two or three steps ahead is unknown when solving the optimization problem first at each time. (Note that the number of vehicles inflowing one step ahead is known because of travel time.)

Therefore, in the first calculation at each time, the state quantity prediction ahead of $i$ step is calculated as follows.

$$
x_z(k+i) = x_z(k+i-1) - s_z(k+i-1)Tu_z(k+i-1) + \sum_{w \in \ell_j} \tau_{w,z}s_w(k+i-1 - \frac{d_z}{T})Tu_w(k+i-1 - \frac{d_z}{T}) + e_z(k+i-1 - \frac{d_z}{T})
$$

(9)

This means that the state quantity prediction of the link $z$ ahead of the $\frac{d_z}{T}$ step is not taken into consideration because the number of vehicles entering can not be predicted. After solving the first optimization problem at each time, share information on future state quantities...
and inputs of adjacent intersections, and for the second
and subsequent optimization problems, complementing
the number of vehicles inflowing that were unknown at
the first time and solving Eq. (4). In addition, the fluctua-
tion range of the solution is restricted so that the optimal
solution converges at each iteration. To summarize the
above, the distributed traffic signal control algorithm is
as follows.

Distributed Traffic Signal Control Algorithm

Step 1: Each intersection shares $u$ and $x$ in-
formation up to the current intersection $k$ and
solve the optimization problem under the con-
straints Eq. (6)~(9).

Step 2: Information on the predicted values $\hat{u}$
and $\hat{x}$ obtained by solving the optimization
problem is shared between adjacent intersec-
tions, and solve the optimization problem un-
der constraints (5) to (8) using number predic-
tion obtained of the inflowing vehicle.

Step 3: Repeat Step 2 a predetermined number
of times, and each intersection corrects its op-
timal solution. However, in proportion to the
number of iterations $l$ as $\|u_j^{l+1} - u_j^l\| \leq a$
decrease the value of $a$.

Step 4: Add control input obtained by calcula-
tion to your system.

Step 5: Return to Step 1 with $k = k + 1$ as the
time.

4. SIMULATION VERIFICATION

4.1 Simulation conditions

The sampling time was 15 [s], and the simulation was
performed in 48 steps. The prediction horizon is $H_p = 3$.

Saturated traffic flow rate for each route $s_{j\max}$, travel
time $d_i$ are set as in Table 2. The saturated traffic flow
rate of a two-lane road is doubled for one lane.

The number of vehicles in the initial state and the ini-
tial input are as shown in Table 3. The number of vehicles inflowing that were unknown at
the first time and solving Eq. (4). In addition, the fluctua-
tion range of the solution is restricted so that the optimal
solution converges at each iteration. To summarize the
above, the distributed traffic signal control algorithm is
as follows.

<table>
<thead>
<tr>
<th>link $z$</th>
<th>saturation flow $s_{j\max}$ [veh]</th>
<th>travel time $d_i$ [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>2400</td>
<td>30</td>
</tr>
<tr>
<td>$x_2$</td>
<td>2400</td>
<td>30</td>
</tr>
<tr>
<td>$x_3$</td>
<td>1200</td>
<td>15</td>
</tr>
<tr>
<td>$x_4$</td>
<td>1200</td>
<td>15</td>
</tr>
<tr>
<td>$x_5$</td>
<td>1200</td>
<td>30</td>
</tr>
<tr>
<td>$x_6$</td>
<td>1200</td>
<td>30</td>
</tr>
<tr>
<td>$x_7$</td>
<td>1200</td>
<td>15</td>
</tr>
<tr>
<td>$x_8$</td>
<td>2400</td>
<td>30</td>
</tr>
<tr>
<td>$x_9$</td>
<td>2400</td>
<td>30</td>
</tr>
<tr>
<td>$x_{10}$</td>
<td>1200</td>
<td>15</td>
</tr>
<tr>
<td>$x_{11}$</td>
<td>1200</td>
<td>30</td>
</tr>
<tr>
<td>$x_{12}$</td>
<td>1200</td>
<td>15</td>
</tr>
<tr>
<td>$x_{13}$</td>
<td>1200</td>
<td>30</td>
</tr>
<tr>
<td>$x_{14}$</td>
<td>1200</td>
<td>15</td>
</tr>
</tbody>
</table>

Table 3 Initial conditions

<table>
<thead>
<tr>
<th>$x_i$ [veh]</th>
<th>$u_\max$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>30</td>
<td>1</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
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<tr>
<td>20</td>
<td>0</td>
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<tr>
<td>30</td>
<td>1</td>
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<tr>
<td>20</td>
<td>1</td>
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<td>20</td>
<td>0</td>
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<tr>
<td>10</td>
<td>1</td>
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<tr>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
</tr>
</tbody>
</table>

The comparison target was the control method used in
[8] or [9]. This is a control method in which only the
green light time is controlled with a fixed cycle length
and without consideration of time delay and offset. Here,
the cycle length is 120 seconds.

Table 4 Inflow from the outside of the traffic network

<table>
<thead>
<tr>
<th>intersection</th>
<th>inflow</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>[3,5]</td>
</tr>
<tr>
<td>3</td>
<td>[2,5]</td>
</tr>
<tr>
<td>5</td>
<td>[2,5]</td>
</tr>
<tr>
<td>6</td>
<td>[3,6]</td>
</tr>
</tbody>
</table>

4.2 Simulation result

The congestion rates on the route are shown below for
each of the six intersections. The solid line represents
the proposed method, and the broken line represents the
case of comparison. The congestion rate is expressed as
a ratio when the number of vehicles in the queue on the
route is divided by the number of lanes and 60 vehicles
are regarded as 100%.

As can be seen from Fig. 9, at Link 7 at intersection 5,
in the case of the comparison target, the congestion rate
has risen to 70%. On the other hand, it was confirmed
that the congestion rate of any route was suppressed to
less than 60% in the proposed method of this paper.
5. CONCLUSION

In this paper, traffic signal control is performed by distributed model predictive control, using a dead-time traffic model considering the travel time of the vehicle. In addition, by using the indication as a control input, the cycle length, split, and offset were made variable, and it was possible to calculate the optimal green signal time for each path and the timing to switch the signal. Finally, the effectiveness of the proposed method was confirmed by numerical simulation.

One of the future works is to equalize the congestion rate in the traffic network by coordinating the intersections in performing distributed control. Although they share information with adjacent intersections, each intersection solves an optimization problem to alleviate only the congestion rate on the links around itself. Therefore, it is thought that the accuracy of control can be improved more by performing signal control in cooperation with each intersection.

REFERENCES

