Parking Lot Allocation and Dynamic Parking Fee System
Based on a Mechanism Design Approach

Hiroaki Nakanishi and Toru Namerikawa

Abstract—In this paper, we present a discussion on smart parking systems in urban traffic networks. Reduced vehicle speed when drivers search for parking lots contributes to increased traffic jams in recent urban traffic networks. We aim to shorten the searching time for parking lots, which is one of the causes of traffic jams, by allocating available parking lots to drivers. Furthermore, we design a dynamic parking fee system and redistribute parking lots to equalize the profits earned by managers of multiple parking lots in traffic-congested areas. We then finally confirm the effectiveness of the proposed algorithm through numerical simulations.

I. INTRODUCTION

Traffic congestion because of increasing traffic volume or underdeveloped road infrastructure is a recent global phenomenon. The problem is even more severe in particular urban areas with heavy concentrations of traffic. 30 percent of the traffic congestion in urban areas can be attributed to the deceleration of vehicles while searching for parking lots and parking spaces[1]. In addition, it takes an average of 7.8 minutes for drivers to find a desired parking space[2]. Such traffic jams cause environmental pollution and economic loss to society, requiring urgent solutions. As a solution to these problems, intelligent transportation systems (ITS) have been recently developed. These systems integrates people, road infrastructure, and vehicles, ensuring cooperation based on real-time traffic information obtained through information communication technologies (ICT), and are generally considered for addressing problems related to traffic management, advanced serviceability, and other forms of transportation.

One of the ITS technologies that has recently gained considerable attention is the “Smart Parking System.” The smart parking system is a system that collects information related to drivers searching for parking lots, including real-time parking lot usage information, and suppresses traffic congestions using such information. By allocating and guiding drivers to the appropriate parking lot or by dynamically changing the parking fee based on the degree of congestion, smart parking systems can alleviate traffic congestion.

Several studies on the smart parking system[3] have been conducted. The paper[4] suggests that parking lot search times can be shortened by presenting available parking lot information to the driver searching for a parking lot.

As conventional research on parking lot allocation, the paper[5] presented the basic concept and structure of a smart parking system and defined the optimization problem associated with parking lot allocation for shortening parking lot search times, which contribute to traffic congestion. However, when a parking lot is allocated, only the demand and interests of the driver searching for the parking lot is considered, failing to include provisions for the parking lot manager.

On the other hand, the design of a dynamic parking fee system, presented in paper[6], prevents the concentration of a specific parking lot by changing the parking fee according to the full occupancy rate of the parking lot. The paper[7] focused on minimizing the monetary cost borne by the driver and maximizing the utilization rate of the parking lot through the design of an appropriate dynamic parking fee according to the vacant or full ratio of each parking lot. However, a theoretical analysis was not performed to validate the design of the algorithm for a dynamic parking fee system in the paper cited above.

Consequently, we propose a parking lot allocation algorithm that considers the benefits for the driver searching for a parking lot and the manager of a parking lot based on matching theory. In addition, by proposing a redistribution algorithm for parking lots based on a mechanism design approach, we develop a system for parking lot allocation that is theoretically desirable according to the mechanism design approach, and we attempt to equalize and maximize the profit of the parking lot manager in the traffic network.

II. PROBLEM FORMULATION

Fig. 1. Traffic network around Ginza station
Fig. 1 illustrates the area surrounding Ginza station in Japan. The length of the longer side of the area, surrounded by a thick line, is approximately 1.0[km], while the length of the shorter side of the area is approximately 0.5[km]. In the transportation network, square markers denote parking lots and triangle markers denote the final destinations that each driver travels toward. Each driver searches and parks in the area parking lot driving for five destinations.

### TABLE I

<table>
<thead>
<tr>
<th>index j</th>
<th>Parking fee [Yen/min]</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500/30</td>
<td>26</td>
</tr>
<tr>
<td>2</td>
<td>600/10</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>300/30</td>
<td>54</td>
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<tr>
<td>4</td>
<td>300/30</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>500/30</td>
<td>29</td>
</tr>
<tr>
<td>6</td>
<td>500/15</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>400/15</td>
<td>19</td>
</tr>
<tr>
<td>8</td>
<td>400/15</td>
<td>15</td>
</tr>
<tr>
<td>9</td>
<td>400/10</td>
<td>50</td>
</tr>
<tr>
<td>10</td>
<td>500/20</td>
<td>2</td>
</tr>
<tr>
<td>11</td>
<td>400/20</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>400/30</td>
<td>56</td>
</tr>
<tr>
<td>13</td>
<td>300/30</td>
<td>30</td>
</tr>
<tr>
<td>14</td>
<td>200/10</td>
<td>2</td>
</tr>
</tbody>
</table>

Details of the parking lots in the transportation network are presented in the following Table I. In Table I, parking lot manager $h = 1$ manages parking lots $j = 1, 2, 4, 6, 8, 9, 11$; parking lot manager $h = 2$ manages parking lots $j = 3, 7, 10, 12, 13, 14$; and parking lot manager $h = 3$ manages parking lot $j = 5$. In other words, there are three parking lot managers in the area, and each parking lot manager manages one or more parking lots.

Let $P$ be the set of parking lots in the transportation network, and $|P| = 14$. The set of parking lots managed by the parking lot manager $h$ is denoted by $P_h \subset P$. The position coordinates of the parking lot $j \in P$ are $(y_{j,x}, y_{j,y})$ and the fee structure is incremented by $f_j$ yen every $\tau_j$ minutes in the parking lot $j$. The number of parking spaces in the parking lot $j$ is assumed to be $N_j$, and the number of empty parking spaces $e_j$ in the parking lot $j$ is expressed as follows.

$$ e_j(k + 1) = e_j(k) - \left[ \mu_k(j) \right] + n_j^{\text{out}}(k) $$

where $n_j^{\text{out}}(k)$ is the number of vehicles leaving a parking lot $j$ at time $k$, and $[\mu_k(j)]$ is the number of drivers who make parking reservations for parking lot $j$ at time $k$. $V_n(k)$ denotes the set of drivers entering the traffic network. Driver $i$’s destination is denoted by $d_i$, and the coordinate of $d_i$ is $(d_{i,x}, d_{i,y})$. The position coordinates of the current location of the driver $i$ at time $k$ is denoted by $(z_{i,x}, z_{i,y})$. We assume the following assumption related to the traffic network.

**Assumption 1:** The number of drivers entering the network at time $k$ is less than or equal to the total number of empty spaces in the area at time $k$; that is (2) holds.

$$ |V_n(k)| \leq \sum_{j \in P} e_j(k) $$

### III. PARKING LOT ALLOCATION BASED ON MATCHING THEORY

In order to determine the allocation of parking lots that satisfies the demand of both drivers and parking lot managers, we use matching theory.

#### A. Preference Determination

As a discussion on matching theory, the driver searching for a parking lot has preferences for all parking lots in the area, and the parking lot managers have preferences for all the drivers entering the traffic network at time $k$. We define the optimization problems for drivers and parking lot managers to decide preference, respectively.

1) **Drivers’ Preferences:** Each driver entering the network has a clear preference $P_i$ for all parking lots in the area. Each driver searches for the desired parking lot by considering the following conditions:

- Distance from parking lot to destination
- Distance from current location to parking lot
- Parking fee

In terms of the first condition, the distance $D_{ij}$ from $j$ to the destination of the driver $i$ is expressed as follows.

$$ D_{ij} = |d_{i,x} - y_{j,x}| + |d_{i,y} - y_{j,y}| $$

For the second condition, the distance from the current location of driver $i$ at time $k$ to parking lot $j$ $R_{ij}(k)$ is expressed as follows.

$$ R_{ij}(k) = |z_{i,x}(k) - y_{j,x}| + |z_{i,y}(k) - y_{j,y}| $$

For the third condition, we consider a parking fee $F_{ij}$ if driver $i$ parks in parking lot $j$ as follows.

$$ F_{ij} = \left[ \frac{o_i}{\tau_j} \right] f_j $$

Therefore, we express the performance index $J_{ij}^d$ of the parking lot $j$ used by driver $i$ as follows.

$$ J_{ij}^d = \frac{D_{ij}}{D_i} + \frac{R_{ij}(k)}{R_i} + \frac{F_{ij}}{F_i} $$

In other words, the performance index (6) indicates the costs associated with searching and parking in parking lots, and we consider $J_{ij}^d$ the evaluation value that driver $i$ has for parking lot $j$. In addition, $D_i$, $R_i$, and $F_i$ are the parameters that are uniquely determined by driver $i$ and assume a nonnegative real value. These values assume the role of a weighting factor for each element of the search condition, and the smaller these values are, the larger the value of the corresponding element becomes, such that the driver searches for a parking lot with an emphasis on that element. We also introduce binary variable $x_{ij}^d$, which is defined below.

$$ x_{ij}^d = \begin{cases} 1, & \text{if user } i \text{ is assigned to resource } j \\ 0, & \text{otherwise} \end{cases} $$

Therefore, driver $i$ solves the following optimization problem, such that the most desirable parking lot for driver $i$ can be determined.

$$ \min_{x_{ij}^d} \sum_{j \in P} J_{ij}^d x_{ij}^d $$
subject to
\[ J_{ij}^d = \frac{D_{ij}}{D_i} + \frac{R_{ij}(k)}{R_i} + \frac{F_{ij}}{F_i} \]  
(9)
\[ \sum_{j \in P} x_{ij}^d = 1 \]  
(10)
\[ x_{ij}^d \in \{0, 1\} \]  
(11)

\( x_{ij}^d \) is obtained by solving the above optimization problem, parking lot \( j \) which is \( x_{ij}^d = 1 \) of value \( J_{ij}^d \) is the smallest, which is the most desirable parking lot for driver \( i \).

Furthermore, we consider determining the preference \( P_i \) of driver \( i \) for all parking lots in the network by iteratively solving the above optimization problem. Let the number of iterations be \( \tau_d(1-|P|) \), and the optimal solution obtained through the optimization problem solved for the \( \tau_d \)th time be \( x_{ij}^{d, \tau_d} \). Every time \( x_{ij}^{d, \tau_d} \) is obtained, we add the following constraint and repeatedly solve the optimization problem.

\[ x_{ij}^{d, \tau_d} = 0 \]  
(12)

By repeating the above operation for \( |P| \) times, driver \( i \) determines the preferences for all parking lots in network \( P_i = \{x_{ij}^{d,1}, x_{ij}^{d,2}, \ldots, x_{ij}^{d,|P|}, \ldots, x_{ij}^{d,|P|^2}\} \).

2) Parking Managers’ Preferences: Parking lot managers have a clear preference \( P_j(k) \) for all drivers entering the network at time \( k \), including the driver searching for a parking lot. Parking lot managers’ profits are based on the following two points:

- Financial benefit from parking fees
- Average utilization rate

In terms of the first point, it is denoted by (5). In terms of the second point, it is the average utilization rate of a parking lot managed by the parking manager and is expressed as follows.

\[ U_{ij} = \frac{e_j(k)}{N_j} \left(1 - \int_0^{o_i} \frac{1}{\lambda} e^{-\frac{x}{\lambda}} dx\right) \]  
(13)

\( \lambda \) in the equation above denotes the average number of vehicles entering the network within the sampling time. The first term denotes the ratio of empty parking spaces to total parking spaces in the parking lot \( j \), and the second term denotes the probability that driver \( i \) with a parking time that is longer than the scheduled parking time \( o_i \) will enter the network in the future. In other words, the expression above corresponds to the evaluation value that indicates whether it is appropriate to accept driver \( i \) at time \( k \). Therefore, the gain function of parking managers can be expressed as follows.

\[ J_{ij}^m = F_{ij} + w_m U_{ij} \]  
(14)

\( w_m \) is a weighting factor and introduces the binary variable \( x_{ij}^m \), which is defined as follows.

\[ x_{ij}^m = \begin{cases} 
1, & \text{if user } i \text{ is assigned to resource } j \\
0, & \text{otherwise} 
\end{cases} \]  
(15)

Based on the equation above, parking lot manager \( h \) solves the following optimization problem and determines the driver with the highest gain for parking lot \( j \in P_h \).

\[ \max_{x_{ij}^m} \sum_{i \in \mathcal{V}_i(k)} J_{ij}^m x_{ij}^m \]  
(16)

subject to
\[ J_{ij}^m = F_{ij} + w_m U_{ij} \]  
(17)
\[ \sum_{i \in \mathcal{V}_i(k)} x_{ij}^m = 1 \]  
(18)
\[ x_{ij}^m \in \{0, 1\} \]  
(19)

Furthermore, as in the previous section, we repeatedly solve the optimization problem to determine the preference \( P_j(k) \). The number of iterations is \( t_m(1-|\mathcal{V}_i(k)|) \) if \( \mathcal{V}_i(k) = \emptyset \) and \( t_m = 0 \), and let \( x_{ij}^{m,*} \) be the optimal solution of the optimization problem solved for \( t_m \)th time. Every time \( x_{ij}^{m,*} \) is obtained, parking managers solve the optimization problem (16)–(19) by adding the following constraints.

\[ x_{ij}^{m,*} = 0 \]  
(20)

By repeating this operation \( |\mathcal{V}_i(k)| \) times, parking lot \( j \) determines the preference \( P_j(k) = \{x_{ij}^{m,*}, x_{ij}^{m,*}, \ldots, x_{ij}^{m,*}, \ldots, x_{ij}^{m,*} \}_{i \in \mathcal{V}_i(k)} \} \) for drivers arriving at time \( k \) in the network.

B. Parking Lot Allocation Decision Algorithm Based on Matching Theory

Based on the preferences of the driver and the parking manager, determined in the previous section, we determine the allocation of parking lots based on matching theory.

The upper limit of the number of drivers entering parking lot \( j \) at each time is different, depending on the empty space \( e_j(k) \) or the inbound vehicle \( |\mathcal{V}_i(k)| \). The upper limit of the number of drivers that parking lot \( j \) can accept at time \( k \) \( c_j(k) \) can be expressed as follows.

\[ c_j(k) = \begin{cases} 
|\mathcal{V}_i(k)|, & \text{if } |\mathcal{V}_i(k)| < e_j(k) \\
e_j(k), & \text{if } 0 < e_j(k) \leq |\mathcal{V}_i(k)| \\
0, & \text{if } e_j(k) = 0 
\end{cases} \]  
(21)

Based on the equation above, the algorithm that determines the allocation of a parking lot using matching theory is expressed as follows. In the following algorithm, the set of temporary matching at time \( k \) is denoted by \( X_k^t \). We update \( X_k \) to determine the final matching \( X_k \) in each step. In addition, in \( X_k^t \), the driver with the lowest preference under \( P_j(k) \) of parking lot \( j \) is denoted by \( X_k^t(j)_{\succ \succ j} \).

Parking Lot Allocation Algorithm

Step 1 : Preference Determination
Each driver searching for a parking lot enters the network at time \( k \) and determines preference \( P_i \) for all parking lots based on (8)–(12). In addition, each parking lot determines preference \( P_j(k) \) for all drivers searching for a parking lot when entering a network at time \( k \) according to (16)–(20).

Step 2 : Parking Request according to \( P_i \)
Driver \( i \) searches for a parking lot with the current highest preference based on his own preference \( P_i \).

Step 3 : Accepting or Rejecting Parking Request
Parking lot \( j \) receives parking requests and either
approves a request while establishing a temporary pair of drivers based on driver preference $P_j(k)$ and acceptance upper limit $c_j(k)$ or rejects the request. Consequently, the following three situations may arise.

- **if** $|X'_k(j)| < c_j(k)$
  
  Parking lot $j$ accepts the parking request of driver $i$ and establishes a temporary pair of drivers.

  $$i \in X'_k(j) \quad (22)$$

  $i \rightarrow i + 1$, and return to **Step 2**.

- **else** $|X'_k(j)| = c_j(k)$ and $i \succ_j X'_k(j)$
  
  Parking lot $j$ removes the temporary pair of drivers based on the lowest preference among provisional pairs and accepts the request of driver $i$.

  $$i \in X'_k(j) \quad (23)$$

  $$X'_k(j) \succ_j \notin X'_k(j) \quad (24)$$

  $i \rightarrow X'_k(j) \succ_j$, and return to **Step 2**.

- **else** $|X'_k(j)| = c_j(k)$ and $i \prec_j X'_k(j)$
  
  Return to **Step 2**.

**Step 4**: Matching Determination

When temporary pairs among drivers searching for parking lots in the network are established, $X'_k$ denotes the set of driver pairs and parking lots to be matched at time $k$, and the algorithm is run at time $k$.

$$X'_k = X_k \quad (25)$$

**Step 5**: Repetition

$k \rightarrow k + 1$ and return to **Step 1**.

**IV. DYNAMIC PARKING FEE SYSTEM DESIGN AND PARKING LOT REDISTRIBUTION**

In the previous section, we presented a discussion on the algorithm that decides on parking lot allocation based on matching theory, but there are problems associated with imbalances in parking managers’ profits. Therefore, in this section, we redistribute drivers to parking lot managers earning less profit to maximize and equalize the profits of all parking managers in the network. We propose a dynamic parking fee system design and driver redistribution algorithm that do not violate the property of stable matching while satisfying individual rationality and efficiency, which is a desirable property in mechanism design.

**A. Dynamic parking fee design**

We then assume the following assumption 2.

**Assumption 2**: Smart parking systems can obtain a driver’s performance index (6) and each driver’s scheduled parking time $o_i$, in [min].

In terms of the performance index (6) for determining the preferences of drivers searching for parking lots, parking lot managers can only design $f_j$. Parking lot managers earning less profit will change preference $P_i$ by reducing the cost associated with parking fees and draw drivers to their own parking lots to increase profits.

Let the dynamic parking fee at parking lot $j$ be $\rho_j(k)$ [Yen], and we set the range of parking fees being designed as follows.

$$0 < \rho_j(k) \leq f_j \quad (26)$$

A parking lot manager who does not have driver pairs for matching $X_k$ is assumed to be $h'$, and the parking lots managed by $h'$ is denoted by $P_{h'}$. Therefore, $j' \in P_{h'}$ designs a dynamic parking fee $\rho_{j'}$ within the range of (26) so that a set of drivers $D_{j'}(k)$ with a preference for parking lot $j'$ can have higher priority compared to the pair $X_k(i)$, which is denoted as follows.

$$D_{j'}(k) = \{ i : \frac{D_{kX}(i)}{D_i} + \frac{R_{ij'}(k)}{R_i} < \frac{d_i}{o_i} f_{X_k(i)} \} \quad (27)$$

Therefore, when the distribution after redistribution with dynamic parking fee $\rho_{j'}(k)$ is $\mu_k$, the following theorem 1 holds.

**Theorem 1**: For matching $X_k$ determined by the parking lot allocation algorithm, assumption 2 holds and a dynamic parking fee $\rho_{j'}$ designed for driver $i \in D_{j'}(k)$ satisfies the condition $\mu_k$ with dynamic parking fee $\rho_{j'}(k)$, which indicates the individual rationality of drivers searching for parking space.

$$0 < \rho_{j'}(k) \leq \frac{F_j}{o_{j'}} \left( \frac{D_{kX}(i)}{D_i} \right) + \frac{R_{ij'}(k)}{R_i} \left( \frac{\frac{d_i}{o_i} f_{X_k(i)}}{o_{j'}} \right). \quad (28)$$

According to theorem 1, if condition (28) is satisfied, driver $i$ prefers parking lot $j'$ over parking lot $X_k(i)$ and has no disadvantage because of redistribution. However, in this section, a smart parking system will redistribute drivers to less profitable parking lots for profit equalization within the network. Parking lot managers paired with a driver in matching $X_k$ will be disadvantaged by redistribution. In the next section, parking lot which paired with driver newly by redistribution pays monetary incentive to parking lot which driver originally paired to compensate loss of profits.

**B. Fair Redistribution with Monetary Incentive**

In the context of parking lot $j$ managed by parking lot manager $h$, denoting the monetary incentive that parking lot $j$ receives with the redistribution of driver $i$ as $l_{ij}$(Yen), parking lot $j \in P_{h}$ that benefits from the redistribution of driver $i$ can be expressed as follows.

$$l_{ij} = \frac{o_i}{o_{j'}} f_j a_{ij} + t_{ij}. \quad (29)$$

$a_{ij}$ in (29) is defined as follows.

$$a_{ij} = \begin{cases} 1, & \text{if user } i \text{ is allocated to resource } j \\ 0, & \text{otherwise} \end{cases} \quad (30)$$
In addition, incentive \( t_{ij} \) holds as follows.

\[
\sum_{j \in P} t_{ij} = 0 \quad (31)
\]

If \( t_{ij} > 0 \), this means that the parking lot manager who manages parking lot \( j \) receives incentives and if \( t_{ij} < 0 \), the parking lot manager pays the incentive. Let us assume that set of drivers redistributed from \( D_i^w(k) \) to parking lot \( j' \) is \( D_i^w(k) \). Then, the following theorem holds.

**Theorem 2:** When assumption 2 holds for matching \( X_k \) derived by the parking lot allocation algorithm and incentive \( t_{ij} \) satisfies the following condition, the redistribution allocation \( \mu_k \) with dynamic parking fee \( \rho_j(k) \) and incentive \( t_{ij} \) satisfies the individual rational of the parking manager.

\[
\left[ \frac{\alpha_i}{\tau_{X_k(i)}} \right] f_X(i) \leq t_{iX_k(i)} \leq \left[ \frac{\alpha_i}{\tau_{j'}} \right] \rho_{j'}(k), \quad \forall i \in D_i^w(k) \quad (32)
\]

According to theorem 2, if a smart parking system includes a dynamic parking fee design and an incentive for satisfying (32), it can be guaranteed that redistribution presents no disadvantage to parking lot managers. If more than one driver belongs to \( D_i^w(k) \), \( \rho_j(k) \) is designed to be smaller by parking lot \( j' \). The number of drivers who change their preferences will increase, and so the benefit from the parking fee will increase. However, the incentive to pay will increase, decreasing gross profit. It is necessary to appropriately design a dynamic parking fee \( \rho_j(k) \) by considering the relationship of this trade-off. The purpose of a smart parking system in redistribution is the maximization and equalization of the parking lot manager’s total profit so that the smart parking system can execute an efficient redistribution \( \mu_k \). Then the following theorem holds.

**Theorem 3:** Assumption 2 holds for matching \( X_k \) derived by the parking lot allocation algorithm with a dynamic parking fee \( \rho_j(k) \), and the incentive \( t_{ij} \) is designed as in (34), (35). The distribution after the redistribution \( \mu_k \) with them is performed denotes the individual rational of drivers searching for parking space and parking lot managers.

\[
\left[ \frac{\alpha_i}{\tau_{X_k(i)}} \right] \rho_{j'}(k) \geq \left[ \frac{\alpha_i}{\tau_{X_k(i)}} \right] f_X(i), \quad \forall i \in D_i^w(k) \quad (33)
\]

\[
\rho_{j'}(k) = \max_{i \in D_i^w(k)} \left\{ \frac{F_i}{\tau_{j'}} \left( \frac{D_iX_k(i) - D_{ij'}}{D_i} \right) \right. \\
+ \frac{R_iX_k(i) - R_{ij'}(k)}{R_i} \left. \right\} \quad (34)
\]

\[
t_{iX_k(i)} = \left[ \frac{\alpha_i}{\tau_{X_k(i)}} \right] f_X(i), \quad \forall i \in D_i^w(k) \quad (35)
\]

According to theorem 3, by designing a dynamic parking fee and an incentive as denoted in (34) and (35), redistribution indicates an efficient individual rational. In other words, there is no profit loss for drivers searching for parking space and parking lot managers, and the desired redistribution can be achieved.

### C. Parking Lot Distribution Algorithm

A driver’s personal information is necessary for parking lot redistribution, and it is practically impossible for a parking lot manager to acquire and use this information. Therefore, in this paper, as shown in assumption 2, it is assumed that this information is acquired by the smart parking system, and the smart parking system collectively performs redistribution using such information. The parking lot redistribution algorithm based on a mechanism design approach is summarized follows.

**Algorithm:** Redistribution of parking lot algorithm at time \( k \)

1. **Initialization**

2. **Set** \( X_k \)

3. **Set** \( D_i^w(k) = \{ i : \frac{D_{ij'}(k)}{D_i} + \frac{R_{ij'}(k)}{R_i} < \sum_{j \in P} J_{ij'}x_{ij'} \} \)

4. **if** \( D_i^w(k) = \emptyset \)

5. **\mu_k = X_k \)

6. **else**

7. **Set** \( \rho_{j'}(k) = \max_{i \in D_i^w(k)} \left\{ \frac{F_i}{\tau_{j'}} \left( \frac{D_iX_k(i) - D_{ij'}}{D_i} \right) \right. \\
+ \frac{R_iX_k(i) - R_{ij'}(k)}{R_i} \left. \right\} \quad (32) \)

8. **if** \( \left[ \frac{\alpha_i}{\tau_{j'}} \right] \rho_{j'}(k) \geq \left[ \frac{\alpha_i}{\tau_{X_k(i)}} \right] f_X(i), \quad i \in D_i^w(k) \)

9. **\( t_{iX_k(i)} = \left[ \frac{\alpha_i}{\tau_{X_k(i)}} \right] f_X(i), \quad \forall i \in D_i^w(k) \)

10. **end if**

11. **end if**

12. **end if**

### V. Simulation Verification

We verify the effectiveness of the proposed algorithm through numerical simulations. The targeted transportation network is presented in Fig. 1, and the conditions for a driver’s parking lot search are determined in terms of the shortest route based on coordinates, routes, and the traffic rules in the network. The parking time \( o_i \) of each driver and the number of drivers entering the network at each time follow an exponential distribution and a Poisson distribution, and each variable is randomly determined without being influenced by other drivers or previous time. The number of vehicles entering at each time interval is shown in Fig. 2.

For comparison, we use the results of the paper [5] and the General Parking (GP). The results of the paper [5] comprise the allocation of parking lots by solving an optimization problem for all drivers searching for parking lots that enter the network at each time expressed by (36)–(40) by considering only the drivers’ request.

\[
\min_{x_{ij}} \sum_{i \in V} \sum_{j \in P} J_{ij} x_{ij} \quad (36)
\]

subject to

\[
J_{ij} = D_{ij} + R_{ij}(k) + F_{ij} \quad (37)
\]
In the context of GP, each driver searches for parking lots in descending order based on the driver’s preference $P_i$ and parks if an empty parking lot is found. The driver searches for the next favorite parking lot if no parking space is available in GP. Table 2 presents the design parameters simulated.

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<tr>
<th>Parameters</th>
<th>Symbols</th>
<th>Value</th>
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<tr>
<td>Sampling rate [min]</td>
<td>$T$</td>
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</tr>
<tr>
<td>Number of users</td>
<td>$i$</td>
<td>1–339</td>
</tr>
<tr>
<td>Number of parking spaces</td>
<td>$N$</td>
<td>361</td>
</tr>
<tr>
<td>Mean value of driver’s parking time [min]</td>
<td>$\mu$</td>
<td>60</td>
</tr>
<tr>
<td>Mean value of inflow interval</td>
<td>$\lambda$</td>
<td>6</td>
</tr>
</tbody>
</table>

Weight value for distance between parking lot and destination

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<thead>
<tr>
<th>Distance between parking lot and destination</th>
<th>$D_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 60</td>
<td>50 100</td>
</tr>
<tr>
<td>200 300</td>
<td>500 700</td>
</tr>
<tr>
<td>1000 2000</td>
<td></td>
</tr>
</tbody>
</table>

Weight value for parking fee

<table>
<thead>
<tr>
<th>Parking fee</th>
<th>$F_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 200</td>
<td>300 500</td>
</tr>
<tr>
<td>700 1000</td>
<td>1500 2000</td>
</tr>
<tr>
<td>3000 5000</td>
<td></td>
</tr>
</tbody>
</table>

Weight value for distance between current location and parking lot

<table>
<thead>
<tr>
<th>Distance between current location and parking lot</th>
<th>$R_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 150</td>
<td>200 300</td>
</tr>
<tr>
<td>400 500</td>
<td>700 1000</td>
</tr>
<tr>
<td>1500 2000</td>
<td></td>
</tr>
</tbody>
</table>

Simulation results are presented below. Fig. 3 depicts the total distance from a parking lot to a driver’s destination, which is the search condition for drivers searching for parking space, and Fig. 4 depicts the total parking fee. From Fig. 3, the proposed method can reduce the distance from the parking lot to the destination in the same manner as conventional research [5] and GP do. It can be confirmed that parking lot allocation is performed by considering the drivers’ request. In addition, in Fig. 4, comparisons consider parking fee a drivers’ request. On the other hand, the proposed method also considers the benefits for parking lot managers, and so it is a parking fee that balances both. It can be confirmed that the parking lot manager’s profit is also considered. Increasing parking lot managers’ profits means that drivers’ parking fees increase. In other words, drivers’ monetary costs increase. However, because stable matching that corresponds to parking lot allocation based on matching theory satisfies individual rationality, by considering other search conditions, parking lot allocation that does not dissatisfy the driver can be achieved.

The total parking lot search time is presented in Fig. 5. GP searches a parking lot regardless of the parking lot being empty or full. Therefore, it can be observed that the parking lot search time is extremely large. In the proposed and conventional methods, it can be verified that search time can be shortened. Furthermore, the parking fee at parking lot ($j = 5$) managed by the parking lot manager earning the least profit is illustrated in Fig. 6. In addition, Fig. 7 exhibits the monetary incentive that each parking manager pays or receives based on redistribution. The broken line in Fig. 6 indicates a fixed parking fee before a dynamic parking fee system was designed, and the solid line denotes the parking fee at each time interval. As can be observed from Fig. 6, to change the preferences of drivers paired with other parking lots, it can be confirmed that a lower parking fee can be dynamically designed compared to a fixed price.
parking lot manager’s profit earned from parking fees before redistribution. In addition, in fig. 9 the dark blue frame denotes the monetary benefits of parking fees after redistribution, and the yellow frame depicts the incentive that each parking lot manager receives. Although parking lot managers’ income from parking fees are lower than before redistribution is performed, the fall in profits because of redistribution is compensated by incentives. Moreover, in Fig. 10, the broken line indicates the monetary benefits from parking fees after redistribution, and the dark-blue frame denotes the actual benefit of deducting incentive payments after redistribution. As can be observed in 10, even though the incentive of paying other parking lot managers is considered, the proposed method improves the profit of parking lot manager $h = 3$ through redistribution. Consequently, we can confirm the effectiveness of the proposed method.

VI. Conclusion

In this paper, we focused on parking lot allocation performed by a smart parking system to alleviate traffic congestion because of decreasing vehicle speeds when drivers search for parking space, which can be problematic in urban transportation networks. To realize the allocation of parking lots, we proposed a parking lot allocation decision algorithm based on the GS algorithm, one of several matching algorithms, to accommodate the interests of drivers searching for parking space and parking lot managers. Furthermore, to address any imbalances in the earnings of parking lot managers, which can be problematic in parking lot allocation based on matching theory, we proposed the redistribution algorithm, including a dynamic parking fee and incentive system design, that satisfy the desirable characteristics of mechanism design approaches. Finally, we confirmed the effectiveness of the proposed algorithm through numerical simulation.

References


In Figs. 9 and 10, the outer red frame indicates each