Cooperative transport of quad-rotor by consensus algorithm

Kento Kotani and Toru Namerikawa

Abstract—At present, UAV is common to use as a single unit, but by operating as multi-agent system, it is possible to possibly be able to carry out more sophisticated tasks. For example, it may be possible to realize long-distance transportation with heavy object which impossible by a single drone. Therefore, we propose a control method to achieve cooperative transport so that the payload follows the virtual leader. We assume the transporting problem connected payload and multiple quad-rotors with cable. In this research, we treated quad-rotor as a model regarded as a linear secondary system and used consensus algorithm to achieve the purpose. Finally we verified its usefulness by simulation.

I. INTRODUCTION

In these years, the development of drone is remarkable, and it is utilized in several fields. It is used for searching, surveillance, photography, transport, etc. It seems that the spread to society will advance in the future. At present, drone is common to use as a single unit, but by operating as multi-agent system, it is possible to possibly be able to carry out more sophisticated tasks. For example, it may be possible to realize long-distance transportation with heavy object which impossible by a single drone. Although research on coordinated transport by multiple vehicles has been conducted for a long time, there is still little research on methods using UAVs. Originally the name of drone is a common name for UAV (unmanned aerial vehicles) in general, but in this research we will study about quad-rotors.

The actual quad-rotor is nonlinear system in which the motion of each axis affects the motion of the other axis, but many linearized models have been devised[1]. Research using a graph theory and a consensus algorithm to make multiple quad-rotor follow the leader has been made[1], [2], [3], [4], but we cannot find the case of focusing on cooperative transport. Moreover, in addition to the research to transport by one quad-rotor alone[5], some study to model cooperative transport it is necessary to arrange the problems. Therefore in this research, we assume the problem of agents through payload, so in order to apply this method to cooperative transport is necessary to arrange the problems.

In this paper, we describe motion model and network structure in Chapter 2. Chapter 3 is described cooperative transport control based on consensus algorithm, and chapter 4 is analyzed the results of simulation experiments. In Chapter 5, we confirms the results by of simulation experiments.

II. MODELING

In this control method, we consider n quad-rotors. It follow virtual leader and cooperatively transports the payload suspended by the cable (Fig. 1).

In this chapter, we explain the model of quad-rotor and payload and the network structure.

A. Quad-rotor model

The quad-rotor moves in the horizontal direction by adjusting the thrust of the four actuators and tilting the attitude. Hence, the typical quad-rotor motion model is a complex nonlinear model in which the motion of one axis affects the other motion of the other axis, and it is generally expressed by a horizontal quaternary system, a vertical secondary system[1], [8], [9]. However, quad-rotor, whose have become widespread in recent years, can be controlled independently each axis by highly developed flight controller, and artificially realize linear input of velocity or acceleration.

Therefore, in this research, we regard quad-rotor as linearized secondary system. We discret the state equations in the horizontal and vertical directions into equations (1) and (2).

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Fig. 1. Outline drawing
\[
\dot{h} = (I_2 \otimes A_h)h + (I_2 \otimes B_h)(M + \tau) \quad \text{(Horizontal)} (1)
\]
\[
\dot{r} = A_r + B_r(J + w) \quad \text{(Vertical)} (2)
\]
\begin{equation}
\begin{aligned}
\text{Input: } M \in \mathbb{R}^2; \ J \in \mathbb{R} \quad \text{(Control input)}
\end{aligned}
\end{equation}
\begin{equation}
\begin{aligned}
\text{disturbance: } \tau \in \mathbb{R}^2, \ w \in \mathbb{R} \quad \text{(Disturbance)}
\end{aligned}
\end{equation}

\( \otimes \) is the Kronecker product. The expression (1) summarizes the \( x, y \) components of \( h = [h_x, h_y]^T \), \( M = [M_x, M_y]^T \), \( \tau = [\tau_x, \tau_y]^T \).

\[
\begin{aligned}
\dot{h}_x &= [x \ y]^T, \ h_y = [y \ z]^T, \ r = [z \ \dot{z}]^T (3)
\end{aligned}
\]
\[
\begin{aligned}
A_h = A_r = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \ B_h = B_r = [0 \ 1]^T
\end{aligned}
\]

This model shall be used under the following conditions.

**Assumption 1:** The quad-rotor shall fly at a sufficiently low speed with sufficient margin for thrust, the rotor responds fast enough, and the response delay until the rotor actually generates thrust from the input command shall be negligible.

**B. Suspended payload dynamics**

The payload is suspended by a cable, and it is necessary to consider the dynamical model to swing. Dynamic model of payload was derived. The symbols and coordinate systems are defined in Fig.2 and Table I.

\[
SO(3) \text{ denotes the 3-dimensional rotation group (special orthogonal group), and it is } [R \in \mathbb{R}^{3 \times 3}] R^T R = I, \det(R) = 1. \text{ In order to consider the payload rotation element } T'_i, \text{ we introduce } n \text{ installation position coordinate systems that handle tension with rotating element components (roll, pitch, yaw) at each mounting position. }
\]

1) **Payload’s equation of motion:** Using the Lagrange equation of motion, we derive the equation of motion of translation and rotation of payload.

\[
\begin{aligned}
d \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \dot{Q}'_i, \quad L = T - U (4)
\end{aligned}
\]

The translational motion equation of payload in the reference coordinate system is derived from physical energy

\[
T = \frac{1}{2} M_0 \dot{X}_0^2, \text{ potential energy } U = M_0 g \dot{X}_0 \cdot [0 \ 0 \ 1]^T \text{ and external force } \ddot{Q}' = \sum_{i=1}^n T_i \text{ to the following equation.}
\]

\[
\begin{aligned}
\ddot{X}_p = \frac{1}{M_p} \sum_{i=1}^n T_i - g [0 \ 0 \ 1]^T
\end{aligned}
\]

The rotational motion equation in the payload coordinate system is derived from physical energy \( T = \frac{1}{2} I_0 \dot{\theta}^2 \), potential energy \( U = 0 \) and external force \( \ddot{Q}' = \sum_{i=1}^n l_i T'_i \) to the following equation. The tension \( T_i \) is rotation component affecting translational motion, but it needs to transrate to the rotation component \( T'_i \) which contributes to rotation of payload. The posture angle in the reference coordinate system is \( \begin{bmatrix} \hat{\phi} \ \hat{\theta} \ \hat{\psi} \end{bmatrix} = R' \begin{bmatrix} \dot{p} \ \dot{q} \ \dot{r} \end{bmatrix}. \ \circ \text{ is Hadamard product.}
\]

\[
I_0 \circ [\dot{p} \ \dot{q} \ \dot{r}] = \sum_{i=1}^n c_i T'_i
\]

2) **Derivation of tension:** Derive the tension \( T_i \) of the next step from the state of quad-rotor and payload. The symbol \( b_i \in [0 \ 1] \) determines the presence or absence of tension.

\[
\begin{aligned}
b_i = \begin{cases} 
1 & ||C_i|| > C_i \\
0 & \text{else}
\end{cases}
\end{aligned}
\]

**TABLE I**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_p ) [kg]</td>
<td>Payload mass</td>
</tr>
<tr>
<td>( X' \in \mathbb{R}^3 ) [m]</td>
<td>Payload position in body coordinate system</td>
</tr>
<tr>
<td>( \Omega_p \in \mathbb{R}^3 ) [rad/s]</td>
<td>Payload attitude angle velocity (roll, pitch, yaw)</td>
</tr>
<tr>
<td>( \Phi_p \in \mathbb{R}^3 ) [rad]</td>
<td>Payload attitude angle (roll, pitch, yaw)</td>
</tr>
<tr>
<td>( \alpha_i \in (0, 1) )</td>
<td>Cable vector</td>
</tr>
<tr>
<td>( \beta_i \in \mathbb{R} )</td>
<td>Cable joint point in body coordinate system</td>
</tr>
<tr>
<td>( \gamma_i \in \mathbb{R} )</td>
<td>Cable joint point distance</td>
</tr>
<tr>
<td>( R \in SO(3) )</td>
<td>Rotation matrix ground to body coordinate system</td>
</tr>
<tr>
<td>( T' \in \mathbb{R}^{3 \times 3} ) [N]</td>
<td>Rotation matrix body to ground coordinate system</td>
</tr>
<tr>
<td>( C \in \mathbb{R}^3 ) [m]</td>
<td>Cable length</td>
</tr>
<tr>
<td>( \alpha_i \in \mathbb{R} )</td>
<td>Cable vector</td>
</tr>
<tr>
<td>( T \in \mathbb{R}^{3 \times 3} ) [N]</td>
<td>Tension in ground coordinate system</td>
</tr>
<tr>
<td>( T' \in \mathbb{R}^{3 \times 3} ) [N]</td>
<td>Tension for rotate payload</td>
</tr>
<tr>
<td>( b \in \mathbb{R} )</td>
<td>Determination of tension ((1 \ 0))</td>
</tr>
<tr>
<td>( F_{G_i} )</td>
<td>Payload load ( M_{0g} ) exerts on joint ( i )</td>
</tr>
<tr>
<td>( F_{N_i} )</td>
<td>The force exerts on joint ( i ) by translational motion of Payload</td>
</tr>
<tr>
<td>( F_{R_i} )</td>
<td>The force exerts on joint ( i ) by rotational motion of Payload</td>
</tr>
<tr>
<td>( F_{Q_i} )</td>
<td>The force that ( UAV ) exerts on joint ( i )</td>
</tr>
</tbody>
</table>

Fig. 2. Coordinate system of payload

\[
S_0(3) \text{ denotes the 3-dimensional rotation group (special orthogonal group).}
\]

\[
\begin{array}{c|c|c}
\text{Parameter} & \text{Description} \\
\hline
n & \text{Number of UAV} \\
X_p \in \mathbb{R}^3 & \text{Position in ground coordinate system} \\
M_p \in \mathbb{R}^3 & \text{Payload mass} \\
X' \in \mathbb{R}^3 & \text{Payload position in body coordinate system} \\
\Omega_p \in \mathbb{R}^3 & \text{Payload attitude angle velocity (roll, pitch, yaw)} \\
\Phi_p \in \mathbb{R}^3 & \text{Payload attitude angle (roll, pitch, yaw)} \\
\alpha_i \in (0, 1) & \text{Cable vector} \\
\beta_i \in \mathbb{R} & \text{Cable joint point in body coordinate system} \\
\gamma_i \in \mathbb{R} & \text{Cable joint point distance} \\
R \in SO(3) & \text{Rotation matrix ground to body coordinate system} \\
T' \in \mathbb{R}^{3 \times 3} & \text{Rotation matrix body to ground coordinate system} \\
C \in \mathbb{R}^3 & \text{Cable length} \\
\alpha_i \in \mathbb{R} & \text{Cable vector} \\
T \in \mathbb{R}^{3 \times 3} & \text{Tension in ground coordinate system} \\
T' \in \mathbb{R}^{3 \times 3} & \text{Tension for rotate payload} \\
b \in \mathbb{R} & \text{Determina}
\[ T_i = F_{Gi} - F_{Ti} - F_{Ri} + F_{Qi} \]  
\[ F_{Gi} = \sum_{i=1}^{n} b_i M_{P}\frac{C_i}{\|C_i\|} P_{g} \]  
\[ F_{Ti} = \frac{b_i}{\sum_{i=1}^{n} b_i} M_{P}\dot{X}_P \]  
\[ c_i \circ F'_{Ri} = \frac{b_i}{\sum_{i=1}^{n} b_i} I_{P}^T P_P \]  
\[ F_{Qi} = b_i M_{I_i} \dot{X}_i \]

\( F_{Ri} \) is converted from the installation position coordinate system to the payload coordinate system and further converted to \( F_{Ri} \) in the reference coordinate system.

3) Cable restraint condition: When the distance between quad-rotor and cable installation position exceeds the length of cable, it needs to correct the position of quad-rotor \( i \) with \( X'_i \) (Fig.3).

\[ X'_i = X_P + R_i X_i + C_i \frac{C_i}{\|C_i\|} (IF \ b_i = 1) \]  

In the future, it is necessary to confirm the validity of the quad-rotor and payload model in actual experiments.

C. Network structure

Explain the representation of the network structure. In the consensus problem used by all agents to make formation, we treat the network structure which agent can acquire information of other agent by using graph theory[10].

When the agent \( i \) can obtain information from the agent \( j \) ( \( i, j = 1 \ldots n, i \neq j \) ), the element \( a_{ij} \) of the adjacency matrix \( A = [a_{ij}] \in \mathbb{R}^{n \times 2n+2} \) is defined by the following expression and indicates the presence or absence of connection.

\[ a_{ij} = \begin{cases} 1 & \text{if agent } i \text{ is connected to agent } j \\ 0 & \text{else} \end{cases} \]  

Our past study, we assigned 1 to \( n \) units of quad-rotors and \( n+1 \) is as virtual leader. In this study, \( n+2 \) is assigned as payload. An example of the network structure will be described in Fig.4.

The acquisition of the information of the virtual leader or the other agents is to acquire the position and velocity information using communication or sensor. We assume the virtual leader, the system that sends a reference command value from the server to the agent through communication.

III. PROPOSED CONTROL METHOD

We consider the control method that \( n \) units of quad-rotor group follow virtual leader and cooperatively transports the payload.

Purpose 1: Make payload follow virtual leader.

However, when each quad-rotor individually controls transportation, there is a danger of crashing due to collisions and accident of cable entanglements, that’s why we make a formation flight.

Purpose 2: The quad-rotors make formation.

A method has been proposed in which makes formation using consensus algorithm and follow the virtual leader[1], [2], [3]. In these researches, the condition for convergence which satisfy the assumption of the next network structure was derived.

Assumption 2: The network between agents can always acquire information in both directions.

Assumption 3: The virtual Leader connects to all followers global reachable.

Based on this method, this research proposes the method to make a payload follow a virtual leader. Depending on whether or not it fulfills the next assumption to be newly added, the control method for achieving the purpose 1 is different.

Assumption 4: All agents can acquire information on payload and virtual leader.

For formation, the following variables are defined.

\[ h_{fi} = h_i - d_{f}, \quad r_{fi} = r_i - d_{r} \]  
\[ d_{fi} \in \mathbb{R}^2, \quad d_{r} \in \mathbb{R} \] is a vector specifying the position of the agent from the reference point of the formation. Also, let \( h_{f} = h_{f_1}, r_{f} = r_{f_1}, h_{r} = h_{r_1} \) and \( r_{r} = r_{r_1} \) are the virtual leader and payload respectively.

We also think of keeping the posture of the conveyed payload constant is also important in terms of practical application.

Purpose 3: Maintain the posture of the conveyed object horizontally.

The analysis of this problem is difficult, because of the effect of the quad-rotor acts on the payload via the cable and effect that the inertia force and the load due by the
payload act as a disturbance to the quad-rotor as the tension via the cable. However, in this study, the slack of cables and suppression of vibration of payload are not considered.

A. Situation all agent can acquire the target information

The proposed control laws for the horizontal direction and vertical direction of the agent \( i \in \{1, 2, ..., n\} \) when the assumption 4 is satisfied is described below.

\[
M_i = - \sum_{j=1}^{n} a_{ij} \left[ \delta_0(h_{fi} - h_{fj}) + \delta_1(h_{fi} - h_{fj}) \right] \tag{16}
\]

\[
J_i = - \sum_{j=1}^{n} a_{ij} \left[ y_0(r_{fi} - r_{fj}) + \gamma_1(r_{fi} - r_{fj}) \right] \tag{17}
\]

These expressions are composed of terms for making formation among agents and terms for making payload follow virtual reader. \( \delta_0, \delta_1, y_0, \gamma_1(> 0) \) are the control constants.

\( \hat{\tau}_i \) and \( \hat{\omega}_i \) term that cancel the tensions \( \tau_i \) and \( \omega_i \) that are added to the agent, but concrete methods how to decide value are out of subject in this research. By matching with \( \hat{\tau}_i \rightarrow \tau_i, \hat{\omega}_i \rightarrow \omega_i \), the purpose 1 is satisfied.

B. Situation not all agent can acquire the target information

If we do not satisfy assumption 4, when decide one or more formation leaders capable of acquiring information on payload and virtual leader and the following proposed control method, achieve to purpose 1. However, the formation leader needs to satisfy the following assumption.

**Assumption 5:** Formation leaders are not affected by other agents. (The influence of tension is not defined in this assumption)

The control method of the agent \( k < n \) set in the formation leader is described below.

\[
M_k = - \sum_{j=1}^{n-k} a_{kj} \left[ \delta_0(h_{pi} - h_{pj}) + \delta_1(h_{pi} - h_{pj}) \right] \tag{18}
\]

\[
J_k = - \sum_{j=1}^{n-k} a_{kj} \left[ y_0(r_{pi} - r_{pj}) + \gamma_1(r_{pi} - r_{pj}) \right] \tag{19}
\]

The control method of the agent \( i \in \{1, 2, ..., n-k\} \) other than the formation leader is described below.

\[
M_i = - \sum_{j=1}^{n-k} a_{ij} \left[ \delta_0(h_{fi} - h_{fj}) + \delta_1(h_{fi} - h_{fj}) \right] \tag{20}
\]

\[
J_i = - \sum_{j=1}^{n-k} a_{ij} \left[ y_0(r_{fi} - r_{fj}) + \gamma_1(r_{fi} - r_{fj}) \right] \tag{21}
\]

C. Maintain attitude

For the purpose 3, maintains the posture of the payload horizontally, control variably the position vector \( d_i \) of the vertical formation. Consensus between agents can not be achieved with the method of controlling thrust vector \( J \) instead of \( d_i \). Replace \( d_i \) of the expression(15)(17)(21) with the following \( d_i' \).

\[
r_{fi}' = r_i - d_i' - a_i P \left[ \kappa_p((h_{ix} - x_i)\theta + (h_{iy} - y_i)\psi) \right]
\]

\[
- \kappa_i \left[ \int_0^t (\theta + \psi)dt \right] \tag{22}
\]

\( \delta_r, \delta_i \) is the PI control gain, \( \theta \) is the Roll angle of the payload around the \( x \) axis, and \( \psi \) is the Pitch angle around the \( y \) axis. Apply \( y_0 \times d_i \) to the expression(19).

IV. SIMULATION

The model that suspends payload with multiple quad-rotors is a nonlinear dynamics model that mutually affects each other complicatedly. We simulated different conditions and examined problems before constructing the system with actual quad-rotor.

The position and speed of the virtual leader are had to be specified. Virtual leader move forward/stop in the \( x \) axis direction \( lm \) in 3.5 seconds and move it \( lm \) in next 3.5 seconds in the \( y \) axis direction. Implement the proposed method with different conditions and check whether the payload can follow the virtual leader.

A. All agent can acquire the target information

1) All agent can acquire the target information: When the assumption 4 is satisfied, apply the control law of the expression (16)(17). We set situation and parameters assuming actual machine experiments.

<table>
<thead>
<tr>
<th>Simulation parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation time / step [sec]</td>
<td>10 / 0.05</td>
</tr>
<tr>
<td>Formation(x y z)[m]</td>
<td>(0.3, 0.3, 0.2), (0.3, -0.3, 0.2), (-0.4, 0, -0.1)</td>
</tr>
<tr>
<td>Set position(x y z)[m]</td>
<td>(0.03, 0.2, 0.02), (0.03, -0.2, 0.02), (-0.05, 0, 0.05)</td>
</tr>
<tr>
<td>Cable length [m]</td>
<td>0.4, 0.5, 0.45</td>
</tr>
<tr>
<td>Payload weight [kg]</td>
<td>0.03</td>
</tr>
<tr>
<td>Consensus algorithm gain (Horizontal, Vertical)</td>
<td>(6.0, 2.5), (6.0, 2.5)</td>
</tr>
<tr>
<td>Attitude gain (( \delta_r, \delta_i ))</td>
<td>(5.0, 0.01)</td>
</tr>
</tbody>
</table>
| Adjacency matrix         | \[
|                          | \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \) |

It is understood that the payload follows the virtual leader, and the attitude of payload is also controlled horizontally.
2) Case that does not satisfy assumption 4: In cases where 2nd and 3rd agents cannot acquire information on payload and virtual leader, deviation occurs between payload and virtual leader. Agents that cannot acquire the state of payload cannot contribute to the attitude control of payload.

3) Case that cannot correct the influence of cable tension: If the tension applied to the agent cannot be corrected, the payload’s attitude control function, although a slight deviation occurs between the payload and the virtual leader as shown in Fig. 7 below.

The trajectory of 2.5 seconds interval of each case is shown in Fig. 8.

The red circle represent the virtual leader, the rectangular solid represent the payload, and the disk represent quadrotor. It can be said that the control accuracy is better as the red circle and the rectangular solid match, and the more horizontal the disc is, the more stable it is.

B. Not all agent can acquire the target information

1) Not all agent can acquire the target information: Verify the case that the assumption 4 is not satisfied. Treat 1st and 2nd agents as the formation leaders that can acquire information on payload and virtual leader, and apply the control method of formula (18)(19). Apply the control measure of equation (20)(21) to 3rd agent.

The structure of the information network is shown in Table III and Fig. 9.

<table>
<thead>
<tr>
<th>Adjacency matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0 1 1</td>
</tr>
<tr>
<td>0 0 0 1 1</td>
</tr>
<tr>
<td>1 1 0 0 0</td>
</tr>
<tr>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>0 0 0 0 0</td>
</tr>
</tbody>
</table>

Although agent 3 does not contribute to attitude control of payload, payload follows virtual reader.

2) Case with one formation leader: Behavior became unstable when formation leader set only 1st agent. It is necessary to verify the conditions of the number of formation leaders required for stable cooperative transport.
3) Case that cannot correct the influence of cable tension:
If the tension applied to the agent cannot be corrected, a deviation occurs between the payload and the virtual leader as shown Fig. 12, and attitude control has not been achieved either. Formation leaders are under condition affected by tension from cable.

The trajectory of 2.5 seconds interval of each cases is shown in Fig. 13.

Based on the above results, in order to properly control the position and attitude of payload, it is confirmed that all quad-rotors need functions to correct tension and disturbance, and unstable unless they cooperate with each other it was.

V. CONCLUSION

In this paper, we considered about problems of transporting payload by suspending multiple quad-rotor system in order to realize heavy payload’s long-distance transportation. The method of transporting payload by combining multiple quad-rotors may be realize long-distance transfer of heavy objects and also ensuring redundancy.

In this problem, the dynamics model of payload becomes complicated, depending on slackness of cable and constraint condition. We proposed some control method to achieve cooperative transport so that payload follows virtual reader. The utility was verified by several simulation experiment. In order to properly control the position and posture of payload, it was confirmed that unless providing all the quad-rotors with functions to correct tension and disturbance and they cooperate with each other, it becomes unstable.

Moreover, move heavy payload at high speed, in order to be influenced by inertia, the behavior becomes unstable. However, this is contrary to the purpose of heavy payload transportation. Also, in actual machine experiments, if slight behavior disorder occurs in the quad-rotor due to some cause, it will be propagated to other quad-rotors by consensus algorithm, and also the action of payload will be destabilized through the cable.

In this paper, concrete means for disturbance compensation and confirmation by actual experiments are not described. Out of the scope of this paper, we had already studied about methods to specifically compensate for tension disturbance. And currently, an apparatus for controlling three quad-rotors has been completed. So the next paper will be reported the method to compensate the influence of disturbance and its results of actual experiments.

In addition, it is necessary to consider about derivation of stability condition, consideration of looseness of cable and vibration suppression of payload to stabilize.

REFERENCES


