Robust prediction and MPC-based optimal energy management for HVAC System

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Abstract: The purpose of this paper is to achieve a cut in peak electricity demand and lower electricity costs through the optimal management of a heating ventilation and air conditioning (HVAC) system. In addition, robust solar radiation prediction is required for optimal HVAC management. First, solar radiation prediction is applied using a clustering technique based on the k-means method, classifying all data on similar types of solar radiation. Second, an $H_\infty$ filter, which is a robust prediction method for outliers, is applied. Next, an optimal HVAC management system is considered for operating the air conditioning, such as the cooling and heating of each room. Model predictive control (MPC), which is used to predict and control the future changes in the temperature of the room and the electrical charge from solar radiation, is applied. Finally, we confirmed the validity of this research through numerical simulations.

Keywords: BEMS, Solar radiation prediction, $H_\infty$ filter, HVAC, MPC

1. INTRODUCTION

Technology enabling the efficient use of energy is gaining attention against the backdrop of such social problems as global warming from excess CO2, the depletion of energy resources, and an increase in power demand owing to the development of cities and advanced information utilization. Studies on energy management systems (EMSs) for the efficient use of energy have been undertaken. An EMS refers to a system that optimizes the energy supply and demand in factories and in residential and commercial buildings, and in this regard, building energy management systems (BEMSs) for commercial buildings are gaining attention in the USA. S. Koehler, F. Borrelli. (2013) because about 40% of the energy consumption there comes from such buildings. Moreover, advanced features required in a next-generation BEMS include such functions as a prediction of uncertain factors of renewable energy, the use of energy storage elements such as storage batteries or thermal energy storage, and control of equipment for maintaining a comfortable environment such as lighting, air-conditioning, and elevators. Accordingly, the purpose of this study is to propose measures to achieve a comfortable indoor environment, saving in lower electricity costs, and cuts in the peak electricity demand through the optimal management of heating ventilation and air conditioning (HVAC) systems.

Previous studies A. Parisio, et al. (2013); Y. Ma, et al. (2012); H. Hao, et al. (2014); J. Shi, et al. (2014) have reported the operation of HVAC (air-conditioning) systems that achieve both environmental comfort and energy saving. One study A. Parisio, et al. (2013) on such an operation considered not only comfortable indoor temperatures but also comfortable indoor ventilation (CO2 levels), and another study Y. Ma, et al. (2012) was performed on an HVAC operation in order to achieve load leveling when using thermal energy storage systems. In recent times, HVAC has also attracted attention as a solution for handling instability in the electricity supply and demand from a demand-response perspective, as reported in H. Hao, et al. (2014). The studies in A. Parisio, et al. (2013) and Y. Ma, et al. (2012) adopt central air-conditioning HVAC system models. However, a problem in a central air-conditioning HVAC system model is that environment control under conflicting local conditions, such as heating in one room and cooling in another, cannot be accomplished. Although studies A. Parisio, et al. (2013) and Y. Ma, et al. (2012) attempted to achieve the optimal management of an HVAC operation using model predictive control (MPC), no prediction of external disturbances (e.g., solar radiation through windows, or human body temperature) affecting the indoor temperature was considered, and pre-set values were used instead of predicted disturbances. Accordingly, the present study focuses on systems that take into consideration contradicting air-conditioning requirements (such as heating in one room, and cooling in another), and uses the prediction of external disturbances affecting the indoor temperature. With respect to external disturbances affecting the indoor temperature, only solar radiation is considered as the main source of disturbance in this study, and disturbances from a human body temperature is assumed to be negligible.

In a recent study, the problem of determining the size of battery storage used in grid-connected photovoltaic (PV) systems is discussed. T. Mukai and T. Namerikawa. (2013). Also T. Namerikawa and S. Igari. (2016) propose a method that uses model predictive control (MPC) to predict photovoltaic (PV) power generation, plan for the electricity demand in a building using the predicted value, and apply it online to correct the prediction error. Q. Hu et al. (2016) and D. P. Zhou et al. (2017) identify a
low-dimensional data-driven model and a high-dimensional physics-based model for the same system at different spatial granularities and temporal seasons using experimental data collected from an entire floor of an office building. Y. Lin et al. (2015) illustrates the power grid value with the first experimental demonstration of frequency regulation from commercial building heating ventilation and air conditioning (HVAC) systems. Y. Wasa et al. (2018) investigates a heating, ventilation, and air-conditioning (HVAC) system in a data center equipped with a previously developed super-multipoint temperature sensing system.

The prediction of photovoltaic solar power generation and/or solar radiation was the subject of a previous study J. Shi, et al. (2012), which used support vector machines (SVMs) with a high discrimination capability for unseen data to classify the photovoltaic power generation data into clusters of similar data, and apply forecasting using models developed for each data cluster. However, outliers from factors such as weather forecast errors were not considered. Therefore, the present study uses a robust prediction method with an $H_\infty$ filter for handling outliers during the prediction.

The purpose of this study is to achieve an optimal operation of an HVAC system that can maintain a comfortable indoor temperature, while realizing lower electricity bills and a cut in peak electricity demand. Moreover, solar radiation prediction has not been considered in previous studies A. Parisio, et al. (2013), Y. Ma, et al. (2012). The authors believe that lower electricity bills and a cut in peak electricity demand may be realized through air-conditioning using an HVAC system, where future indoor conditions are assumed, and that MPC can be used to control the operation by predicting the changes in indoor temperature and electricity bills from future solar radiation. In addition, the current study introduces systems enabling the management of conflicting air-conditioning requirements for each room, such as heating in one room and cooling in another, which was not considered in the previous studies A. Parisio, et al. (2013), Y. Ma, et al. (2012). In predicting the solar radiation, k-means clustering is first applied to classify the radiation data into groups with similar values, and the outliers are then handled by applying $H_\infty$ filter robust prediction methods. The outliers in this study represent the values resulting in disparate solar radiation characteristics owing to such factors as errors in weather forecasting, and the purpose of applying an $H_\infty$ filter is to improve the accuracy of a prediction involving outliers.

2. SOLAR RADIATION PREDICTION

In this section, the characteristics of solar radiation and the procedure used for the prediction method are discussed, followed by further details of the prediction method.

2.1 Characteristics of solar radiation

Figure 1 shows observed solar radiation values under different types of weather, and Figure 2 shows the observed solar radiation values for a ten-day period. Moreover, the red dots in Figure 2 indicate the solar radiation at noon, and a radiation distribution for the daytime that is close to a normal distribution with the peak occurring at noon can be seen.

Characteristic 1: The solar radiation pattern for each weather type is different.

Characteristic 2: There is a correlation between the values at the same time of day (periodicity in a 24 h cycle).

2.2 Estimation procedure

The estimation procedure is shown in Figure 3.

According to characteristic 1, with solar radiation prediction, separate models for each group of similar radiation data are required because the radiation pattern for each weather type is different. Accordingly, clustering of the radiation data is required for groups of similar data. In this study, the k-means clustering method is used to cluster the data into groups of similar data. After clustering, based on the weather prediction for the next day, the appropriate cluster is selected for modeling. For example, if the weather prediction for the next day is sunny, then based on the
clustering results, the solar radiation data grouping (past data) for “sunny” days is selected. Next, for each group of data (with a similar solar radiation pattern) selected, a model is generated. In this solar radiation prediction model design, characteristic 2 regarding the correlation between values at the same time of the day (24 h periodicity) is taken into consideration. The model designed for solar radiation prediction is expressed in terms of the state and space, and the unknown parameters are estimated using an $H_\infty$ filter. Lastly, substituting the unknown parameters in the prediction model with these estimated values, the solar radiation level is predicted. The following sections describe the estimating method in greater detail.

2.3 k-means clustering

In the present study, clustering is achieved using the k-means method. The k-means clustering method is a non-hierarchical method suitable for the classification of large data sets. It was chosen for the present study because of the large volume of data used. Six groups were identified, as the weather data concerning solar radiation were grouped into patterns of similar solar radiation using past data on solar radiation and weather. Separate prediction models were designed for each of these groups, as described in the next section. The following describes the algorithm used in k-means clustering. Let us assume $y_1, y_2, \ldots, y_L$ as the daily average solar radiation (based on past data) for the last $L$ days, $Y_L = \{y_1, y_2, \ldots, y_L\}$ as the set of their values, $Y_0 = \{Y_0, Y_0, \ldots, Y_0\}$ as the initial clusters, $\mu_k = \{\mu_0, \mu_5, \ldots, \mu_1\}$ as the centers of the initial $Y_0$ clusters, and $Y = \{Y_6, Y_5, \ldots, Y_1\}$ as the final set of clusters.

The k-means method

Step 1: Sort $Y_L$, the past data on solar radiation.
Step 2: Divide the sorted data into six groups based on the data value. This creates the initial clusters $Y_0$. The values used in the initial group assignment are $l_1, \ldots, l_6$ where $l_1 = \max(Y_L) \times 1/6$, $\ldots$, $l_6 = \max(Y_L) \times 6/6$.
Step 3: Calculate the centers $\mu_k$ of the initial clusters $Y_0$.
Step 4: For each element in the solar radiation data $Y_L$ and the cluster centers $\mu_k$, calculate the degree of dissimilarity.

$$d(Y_L, \mu_k) = \sum_{i=1}^{L} \sum_{j=1}^{6} \| y_j - \mu_i \|$$ (1)

Step 5: Finalize clusters $Y$ such that the degree of dissimilarity is minimized.
Step 6: If $Y_0$ and $Y$ are not the same (i.e., a data relocation has occurred between clusters) then go back to Step 3 and repeat the process. End if $Y_0$ and $Y$ are the same.

2.4 Solar radiation prediction model

The solar radiation prediction model given by equation (2) takes into account the correlation between the values at the same time of day, described as characteristic 2 D. Labarre et al. (2007).

$$\hat{y}(t) = a(t - 24)y^2(t - 24) + \cdots + a(t - 24L)y^2(t - 24L)$$ \hspace{1cm} \text{where } \hat{y}(t) \text{ is the predicted solar radiation at time } t, a(t - 24), \ldots, a(t - 24L) \text{ are the unknown parameters, } y^2 \text{ is the time-series ordered grouping of past solar radiation data based on the clusters obtained using k-means clustering, and } L \text{ is the model order, representing the number of days of past data. To calculate the unknown parameters, if the solar radiation time series model given by equation (2) is discretized in terms of the state and space, the resulting state and observation models are given by the following,}

$$x_{k+1} = x_k + w_k$$ \hspace{1cm} (3)

$$y_k = C_k x_k + v_k$$ \hspace{1cm} (4)

where $w_k \in \mathbb{R}^{L \times 1}$ is assumed to be white noise with covariance matrix $W > 0$ (process noise with a normal distribution) and mean $= 0$, and $v_k \in \mathbb{R}^{1 \times L}$ is assumed to be white noise with covariance matrix $V > 0$ (process noise with a normal distribution) and mean $= 0$. The following describes the assumptions regarding the noise components.

Assumptions regarding noise

- $E[w_k w_k^T] = E[v_k v_k^T] = 0$ ($k \neq s$): Zero except for the autocorrelation at the current time.
- $E[w_k v_k] = 0$: Assuming white noise, no correlation between system noise and observation noise exists.
- $E[w_k w_k^T] = W > 0, E[v_k v_k^T] = V > 0$: The level of noise is assumed to be known a-priori.

2.5 Prediction using $H_\infty$ filtering

The $H_\infty$ filter robust prediction method is applied to handle outliers in the solar radiation patterns resulting from errors in the weather prediction. The $H_\infty$ filter algorithm is described below.

$H_\infty$ filter algorithm

Step 1: Prediction

$$\hat{x}_{k+1|k} = x_k$$ \hspace{1cm} (5)

$$\hat{y}_{k+1|k} = C_k \hat{x}_{k+1|k}$$ \hspace{1cm} (6)

$$P_{k+1|k} = P_k + W$$ \hspace{1cm} (7)

Step 2: Observation

$$S_{k+1} = cov\{y_{k+1} - \hat{y}_{k+1|k}\}$$ \hspace{1cm} (8)

Step 3: Update

$$K_{k+1} = P_{k+1|k} C_k^T (S_{k+1})^{-1}$$ \hspace{1cm} (9)

$$P_{k+1|k+1} = (P_{k+1|k} - C_k K_{k+1}) V^{-1} C_k + \gamma^2 I$$ \hspace{1cm} (10)

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}(y_{k+1} - \hat{y}_{k+1|k})$$ \hspace{1cm} (11)

where $x_{k+1}$ is the state (unknown parameter), $y_{k+1}$ is the output (solar radiation), $S_{k+1}$ is the observation error.
covariance, $K_{k+1}$ is the filter gain, and $\gamma$ is the design parameter. In STEP 1, the state $\hat{x}_{k+1|k}$ and observation values $y_{k+1|k}$ one step ahead, that is, the unknown parameters, the solar radiation, and the state error covariance, are predicted. In STEP 2, observation error covariance $S_{k+1}$ is calculated based on the difference between the actual and predicted values. In STEP 3, the filter gain $K_{k+1}$ is calculated based on the observation error covariance $S_{k+1}$ calculated in STEP 2. The state and state error covariance are then updated using the filter gain $K_{k+1}$. The unknown parameters are estimated by repeating the calculations using the $H_{\infty}$ filter algorithm. The prediction is conducted by substituting these estimated values for the unknown parameters in the solar radiation prediction model. The predicted solar radiation value is then used as the predicted external disturbance affecting the comfortable indoor environment, as described in the next section.

3. HVAC OPTIMAL OPERATION

The purpose of the present study is to maintain a comfortable indoor temperatures, while enabling a reduction in electricity costs and a cut in peak electricity demand through the optimal operation of an HVAC system. This section describes how this goal is achieved through the application of MPC, where the operation control is based on the prediction of the changes in both indoor temperature and electricity bill costs from future solar radiation. Only solar radiation is considered as the source of disturbances affecting a comfortable indoor temperature, and the solar radiation prediction value described in the previous section is used as the predicted disturbance value. Moreover, the adopted HVAC system enables the management of conflicting air-conditioning requirements in different rooms such as heating in one room and cooling in another.

3.1 HVAC model

The HVAC model is shown in Fig. 4. As the figure shows, AHU and VAV indicate the air handling unit and variable air volume equipment, respectively. The AHU comprises a cooling coil and a fan to blow the cooled air, whereas the VAV comprises a heating coil and a damper to control the airflow. The model used in this study uses a cooling coil for central air-conditioning, and heating coils in each room for local air-conditioning. Adopting a hybrid system of central and local air-conditioning enables the management of conflicting air-conditioning requirements among the rooms. The proposed HVAC system has an added advantage in that it can be used with existing central air-conditioning systems in buildings.

3.2 Dynamics

The heat transfer in Roomi, is given by the following equation, where $i$ is the room number.

$$C_i \frac{dT_i}{dt} = u_i(t) + \frac{T_{wo}(t) - T_i(t)}{R_i} + P_{d,i}(t)$$

(12)

where $C_i$ [kJ/K] is the heat capacity of each room, $T_i$ [K] is the temperature in each room, $T_{wo}$ [K] is the equivalent outdoor temperature considering the solar radiation, $R_i$ [K/kW] is the total thermal resistance, $u_i$ [kW] is the power used in cooling, $u_i$ [kW] is the power used in heating, and $P_{d,i}$ [kW] is the external disturbance. Although an external disturbance may result from human body temperature, and heat generated from the use of electrical appliances, these are not considered in the proposed model for the sake of simplicity. Therefore, the only external disturbance considered in this study is solar radiation passing through the windows. Using a backward Euler discretization, equation (13) is derived from equation (12), where $T_s$ [s] is the sampling time.

$$T_i(k+1) = A_i T_i(k) + B_{c,i} u_i(k) + B_{d,i} d_i(k)$$

(13)

$$d_i(k) = \begin{bmatrix} P_{d,i} \end{bmatrix}^{T}$$

Considering Room1-Roomn, the state and space can be expressed through the following equations.

$$\dot{x}(t) = Ax(t) + Bu(t) + Bd(t)$$

(14)

$$y(t) = x(t)$$

(15)

The following describes the state, output, input, and external disturbance considered. The state/output is given by $y(k) = x(t) = \begin{bmatrix} T_1(t) \cdots T_n(t) \end{bmatrix}^{T} \in \mathbb{R}^{n \times 1}$; the temperature in each room/input is given by $u(t) = \begin{bmatrix} u_1(t) \cdots u_n(t) \end{bmatrix}^{T} \in \mathbb{R}^{n \times 1}$, where $u_1$ is the power used in cooling, and $u_i$ is the power used in heating. Moreover, the external disturbance from solar radiation/indoor temperature is given by $d(t) = \begin{bmatrix} d_1(t) \cdots d_n(t) \end{bmatrix}^{T} \in \mathbb{R}^{n \times 1}$. In addition, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times 1}$, and $B_d \in \mathbb{R}^{n \times 1}$, are expressed using the following matrices.

$$A = \begin{bmatrix} A_1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & A_n \end{bmatrix}$$

$$B = \begin{bmatrix} B_{c,1} & B_1 & \cdots & 0 \\ 0 & B_2 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & B_{c,n} \end{bmatrix}$$

$$B_d = \begin{bmatrix} B_{d,1} & 0 & \cdots & 0 \\ 0 & B_{d,2} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & B_{d,n} \end{bmatrix}$$

Fig. 4. HVAC model

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3.3 Evaluation function

Evaluation function $J$ is expressed through the following equations.

$$\min_u J = \sum_{j=0}^{H_p-1} ||\mathbf{y}(k+j) - \mathbf{y}_{ref}(k+j)||_Q^2$$

$$+ \sum_{j=0}^{H_p-1} ||\mathbf{P}(k+j)\mathbf{u}(k+j)||_R^2$$

(16)

$$\begin{align*}
(x(k+1) &= A(x(k) + Bu(k) + B_d(d(k)) \\
y(k) &= x(k) \\
y_{min}^{k+1} \leq y(k+1) \leq y_{max}^{k+1} \\
u_{min}^{k} \leq u(k) \leq u_{max}^{k}
\end{align*}$$

(17)

where $H_p$ is the evaluation period, $Q$ and $R$ are the weights, $y_{ref}$ is the temperature setting, $P$ is the electricity costs, and input $u(k)$ is the optimization variable with respect to $J$. The evaluation function $J$ is a trade-off between the two terms on the right-hand side of the equation, where the first term assesses the comfortable temperature, and the second term the electricity bill. Equation (17) expresses the state/space, the range of comfortable temperature, and the constraints in the power consumption.

Moreover, MPC refers to the control method, where for each instant in time, a finite assessment interval considering the time horizon in the future is calculated in real time, and only the initial solution for the optimum input sequence is taken as the actual initial input and treated as the control target. In the proposed method, the evaluation function $J$ is estimated for each assessment interval, and the comfortable indoor temperature in each room is controlled, by considering only the first solution for the optimum input sequence as the initial input for additional indoor control targets for each room.

4. SIMULATION

4.1 Solar radiation prediction condition

The period of prediction was set as September 1 to September 30, 2014, and the ambient temperature and weather forecast data were obtained from the Japan Meteorological Agency. In the solar radiation prediction model, order $L$ was set as $L = 2$, and solar radiation data for the past 90 days with respect to the prediction day were used. The solar power generation data observed using equipment on the roof of building 25 on the Yagami campus of Keio University were applied to generate the solar radiation data using equation (18). The mean relative error (MRE) was used in the assessment of the prediction accuracy.

$$\text{Radiation} = \frac{G}{W_{total} \times K}$$

(18)

$$\text{MRE} = \frac{1}{N} \sum_{k=1}^{N} \frac{|y(k) - \hat{y}(k)|}{W_{total}} \times 100 \%$$

(19)

where $\text{Radiation}$ is the solar radiation, $G$ is the power generated, $W_{total}$ is the rated output, $K$ is the power generation efficiency, $N$ is the number of observations, and the superscript $\hat{}$ indicates the predicted value. For the purpose of this study, power generation efficiency $K$ is taken as 1, assuming no loss owing to a drop in temperature or presence of dirt. The parameters used in the filtering were set as follows (Table I). A Kalman filter (KF), which is not robust for handling outliers, was used in the prediction for comparison purposes.

4.2 HVAC operation conditions

The operation time was set to 9:00 AM to 6:00 PM from September 1 to September 30, 2014. Three rooms with different volumes were chosen, and the size of the rooms were such that Room1 < Room2 < Room3. Consequently, heat transmission in Room1 was the most efficient, whereas in Room3 it was the least efficient. The constraints imposed were 24°C to 28°C for a comfortable temperature, 0-10 [kW] for the electricity consumption range, and higher charges for electricity cost $P$ for higher temperature/solar radiation and during the time of high power demand.

The sampling time $T_s$ is 1800[s], and also set point temperature $T_{ref}$ is 26°C.

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 50 \end{bmatrix} \quad W = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

(20)

4.3 Simulation results

The predicted solar radiation, indoor temperature after HVAC operation in each room (Room1, Room2, Room3), and total power consumed on September 30, 2014 are shown in Figs. 5-7, respectively. Table II shows the MRE of the predicted results using a KF and $H_{\infty}$ filter, the cut in peak power demand, and the reduction in electricity costs for one month (September 1 to September 30, 2014) of operation in buildings 23-25 on Keio University’s Yagami campus. The MRE value shown in Table 2 is the average for one month (September 1 to September 30, 2014).

The use of an $H_{\infty}$ filter resulted in an improvement of 1.15% in the accuracy of solar radiation prediction. With respect to an HVAC operation, Fig. 6 shows that a
comfortable temperature was maintained, and Fig. 7 shows that a cut in peak time (i.e., noon) power demand was realized.

5. CONCLUSION

The present study deals with solar radiation prediction and HVAC operation. In solar radiation prediction, the solar radiation data were first divided into groups of similar data using k-means clustering. Next, the unknown parameters were estimated using a robust outlier prediction method, and the actual solar radiation prediction was conducted by substituting these estimated values for unknown parameters in the model. Using MPC, where the control is based on a prediction of the changes in indoor temperature and electricity costs owing to solar radiation, a comfortable indoor temperature could be maintained, and a reduction in electricity costs and a cut in peak electricity demand were also realized. For future studies, the authors would like to consider even more practical problems such as air-conditioning management, taking into account the CO$_2$ concentration and humidity, and systems that use storage batteries or thermal energy storage for efficient energy use.

REFERENCES


