Scenario-based robust MPC for energy management systems with renewable generators

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Abstract:
This paper addresses photovoltaics (PV) power prediction and energy storage problem which are known to be a key technology in energy management systems (EMS). Extending results of the point prediction of PV power, we first describe a prediction interval (PI) method using a copula, which can express the relation between a multivariable joint distribution and each marginal distribution. Then, resorting to the PI method, the energy storage optimization problem in a building is developed. A scenario robust (SR) optimization theorem, which calculates the robustness of the optimal solution, is applied to the proposed PI method, and hence we obtain an optimal energy storage solution taking the robustness of the solution into account. Additionally, we propose a method which combines a model predictive control (MPC) technique and SR to reduce the total electricity costs. The simulation results finally illustrate the cost reduction and robustness of the proposed method.

Key Words: Prediction Intervals, Robust Optimization, Energy Management

1 Introduction

Recently, environmental problems such as global warming and exhaustion of fossil fuels are one of the most important issues in the world. Hence, an introduction of renewable energy such as wind power and photovoltaics (PV) power is interested and promoted. Also, due to technological innovation, micro and smart grids incorporating with distributed energy systems and power storage systems are attracting attention. That is a reason why it is considered to be certain that construction of electric power system by renewable energy [1]. However, the renewable energy has a characteristic that the power generation is largely influenced by weather. Thus, a prediction method assuming actual operation is required, and further accurate prediction is expected. Nevertheless, in the present, there are not a few prediction errors in any prediction method. Therefore, it is important to develop probabilistic prediction methods.

Research on renewable energy prediction is actively discussed. Regarding PV power, there are methods using SVM [2,3] and neural networks [4,5]. In the method [6], prediction intervals by stochastic prediction are estimated, additionally, there are some other methods [7,8] that estimate intervals about renewable energy. The paper [9] is listed as energy management using such a result of prediction intervals (PI) with copula that is a function that expresses the relationship of multiple probability distributions, and it is main reference in this paper.

In [9], the PI was calculated by using the 2-dimensional copula by the prediction error and the measurement. In this paper, a 3-dimensional copula by prediction error, measurement, and additionally a variation of estimate, is proposed to perform interval prediction. In other words, we find the additional elements in 3-dimensional copula with which the method estimate the prediction intervals more accurately than 2-dimensional copula method. An energy management method using obtained prediction interval is described. In this paper, dealing with energy storage problem in a building as energy management, we aim to operate with robustness due to scenario robust optimization by PI. In addition, a scenario-based robust model predictive control method combining SR and model predictive control (MPC) is proposed. The effectiveness of the proposed method, that is the cost reduction by MPC and the robustness by SR, is confirmed by simulation.

The rest of this paper is organized as follows: Section 2 gives PI algorithm and the result of PI. Section 3 is devoted to an energy management algorithm using MPC and SR. In Section 4, the simulation results of energy management show the benefit of the proposed method.

2 Prediction intervals algorithm

2.1 Prediction target and point forecast

In this paper, we predict 1 hour PV generation in Yagami campus of Keio University. The Generator’s maximum output is 120W. A PI algorithm needs point forecast (PF) and PF result. We use the PF algorithm by Kalman filter, k-means and support vector regression. The prediction situation to be considered is given in Table 1.

Table 1: Prediction situation

<table>
<thead>
<tr>
<th>Location</th>
<th>Yagami Campus of Keio University</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acquisition period for point forecast</td>
<td>42 days from 6/8/2017 to 17/9/2017</td>
</tr>
<tr>
<td>Point forecast operation period</td>
<td>12 days from 18/8/2017 to 28/9/2017</td>
</tr>
<tr>
<td>Interval forecast operation period</td>
<td>12 days from 30/9/2017 to 11/10/2017</td>
</tr>
<tr>
<td>Operation time</td>
<td>from 6:00 to 18:00</td>
</tr>
</tbody>
</table>

2.2 Prediction intervals algorithm using a copula

Fig. 1 is a PI algorithm flowchart. The PI algorithm consists of a PF algorithm and a confidence intervals (CI) algorithm. The characteristic of this PI algorithm is that the PF algorithm is completely independent of the CI algorithm. Hence, this CI algorithm can be combined with any PF algorithm. After describing a copula, we describe the proposed PI algorithm more precisely.
Step 3 Normalization

The definition of a copula is determined by Sklar’s Theorem as variables and calculates their joint distribution. We chose the Gaussian copula and calculate a probability distribution of the prediction error from a conditional probability distribution with fixed parameters. The CI algorithm employed is shown as the following:

\[
F_{PI}(t+1) = H(x_E|x_{TG}) = P(t+1), \Delta P = P(t+1) - P(t).
\]

From the calculated error probability distribution, we can calculate the error CI \( E_{CI} = [-a_E, b_E] \).

By using the error CI \( E_{CI} \) obtained by the CI algorithm and combining \( E_{CI} \) with the estimate \( P \) by PF algorithm, the prediction intervals \( P_{PI} \) is calculated as follows:

\[
P_{PI} = [P - a_E, P + b_E].
\]

2.4 CI algorithm

The CI algorithm employed is shown as the following.

Step 1 Pre-data

Calculate the prediction error \( E \) and the variation of an estimate from 1st before \( \Delta P \) with the past measurement of electricity \( TG \) and the estimate \( P \) obtained from PF result.

Step 2 Classification by positive / negative error

Each data is classified according to when \( E \) is positive and when it is negative. Therefore, it is possible to obtain the error distribution in each positive and negative error and to take into account the different characteristics of the positive and negative error.

Step 3 Normalization

Normalize each data to make it easier to calculate.

Step 4 Probability distribution estimation

Estimate the probability distribution of each normalized data using the kernel density estimation. Hence, we can obtain each distribution \( F_E, F_{TG}, F_{\Delta P} \).

Step 5 Copula parameter setting

Compute the copula parameter \( \Sigma \) in the Gaussian copula. It’s not necessary that the Gaussian copula is chosen. However, the elliptical copula (such as Gaussian copula and t-copula etc.) can be easily extended three or more dimension [10].

Step 6 Generation of joint distribution

The joint distribution \( H \) is generated using the parameter \( \Sigma \) obtained in Step 5. \( H \) consists of \( E, TG \), and \( \Delta P \) in method using three-dimensional (3D) copula. On the other hand, \( H \) consists of only \( E \) and \( TG \) in two-dimensional (2D) copula.

Step 7 Calculation of the error confidence intervals

The prediction error probability distribution \( \hat{F}_E(t+1) \), at the time \( t \), is calculated from the conditional probability of \( H(x_E, x_{TG}, x_{\Delta P}) \) with the estimate \( P(t+1) \) by PF algorithm, as the following:

\[
\hat{F}_E(t+1) = H(x_E|x_{TG}) = P(t+1), \Delta P = P(t+1) - P(t).
\]

A. 2 dimensional-copula (Error, Measurement)

At first, we check the result of 2D-copula.

As shown in Fig. 2, more data are included in each CI than the theoretical value, especially in \( 1 - \alpha_i = 0.1, 0.2, \ldots, 0.9 \).

B. 3 dimensional-copula (Error, Measurement, Variation of estimate)

Then, the result of 3D-copula is depicted as follows.
As shown in Fig. 3, the difference between the theoretical value and the number of measurement is reduced as compared with the case of the 2D-copula. We also show these results quantitatively.

The prediction results are compared using mean absolute error (MAE) between the ratio of the number of measurement within each CI and its theoretical value, and its values are listed in the following Table 2.

<table>
<thead>
<tr>
<th>Table 2: MAE</th>
</tr>
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<tbody>
<tr>
<td>A. 2D-copula</td>
</tr>
<tr>
<td>MAE [%]</td>
</tr>
</tbody>
</table>

One can realize that MAE of 3D-copula decreases by 55.3% compared with 2D-copula. Therefore, the superiority of 3D-copula is shown.

3 Energy Management

We assume a building with a photovoltaic generator and a battery on the rooftop as shown in Fig. 4 and Fig. 5 is its model.

![Fig. 4: Building](image)

![Fig. 5: Modeling](image)

We want to calculate the optimal utility power $w^*$ with the building demand $L$, the PV generation $TG$, a electricity price $\lambda$ and battery charge $H$.

It is assumed that all of the following assumptions hold.

**Assumption 1** The PV generation $TG$ and the demand $TL$ up to the present time are known.

**Assumption 2** Initial battery charge $H_0$ is known.

**Assumption 3** The PV forecast $P$ and the demand forecast $L$ on that day are all done by the initial time.

**Assumption 4** The electricity price $\lambda$ at that day is known.

### 3.1 Optimization using MPC

The evaluation functions and constraints which are employed in the MPC algorithm are given as follows:

$$ P_{MPC} : \min_{w_{t+1}} J_{t+1} $$

$$ = w_{t+1}^T R_{t+1} w_{t+1} + (\hat{H}_{t+1} - H_{t+1}^F)^T A_{t+1} (\hat{H}_{t+1} - H_{t+1}^F) $$

$$ \text{s.t. } TL_{t+1} = w_{t+1} + TG_{t+1} + \Delta H_{t+1} $$

$$ H \in [H_{min}, H_{max}] $$

$$ w_{t+1} \geq 0. $$

The optimal utility power $w^*$ is obtained as a minimization problem of the evaluation function $J$. The matrix $R$ indicates the weighting on a electricity price $\lambda$, $A$ indicates weighting on a battery target value $H^F$, and $\hat{H}$ is a battery charge estimate. The three constraint expressions are matching demand and supply, constraints of battery capacity, and constraints of positive utility power, respectively. In addition, the estimated charge / discharge amount $\Delta \hat{H}$ is defined as follows:

$$ \Delta \hat{H}_{t+1} = H_t - \hat{H}_{t+1}. $$

Since the actual power generation $TG$ and the actual demand $TL$ are unknown at the time of calculation, we calculate using each predicted value $P, L$:

$$ TG_{t+1} = P_{t+1} $$

$$ TL_{t+1} = L_{t+1}. $$

Each matrix is defined as follows.

$$ w_{t+1} = (w_{t+1}, \ldots, w_{t+N_P})^T $$

$$ \hat{H}_{t+1} = (H_{t+1}, \ldots, H_{t+N_P})^T $$

$$ H_{t+1}^F = (H_{t+1}^F, \ldots, H_{t+N_P}^F)^T $$

$$ TL_{t+1} = (TL_{t+1}, \ldots, TL_{t+N_P})^T $$

$$ L_{t+1} = (L_{t+1}, \ldots, L_{t+N_P})^T $$

$$ TG_{t+1} = (TG_{t+1}, \ldots, TG_{t+N_P})^T $$

$$ P_{t+1} = (P_{t+1}, \ldots, P_{t+N_P})^T $$

$$ R_{t+1} = diag(R_{t+1}, \ldots, R_{t+N_P}) $$

$$ A_{t+1} = diag(A_{t+1}, \ldots, A_{t+N_P}). $$

Assuming that the operation period of one day is $T$, the operation time is $t = 0, \ldots, T - 1$. Predictive horizon and control horizon are both $N_P = N_c = T - t$. The evaluation function $J$ is a function of $w$. Regarding the constraints, (6), (7), (10), and (11) lead to following equations.

$$ w_{t+k} \in [w_{t+k}^{min}, w_{t+k}^{max}] $$

$$ w_{t+k}^{min} = L_{t+k} - P_{t+k} - H_{t+k-1} + H^{min} $$

$$ w_{t+k}^{max} = L_{t+k} - P_{t+k} - H_{t+k-1} + H^{max} $$

When (21), (22), (23) are calculated in $k = 2, \ldots, N_p$, an unknown future battery charge amount is necessary. Hence,
we sum the equations as follows:

\[
\begin{align*}
w_{t+k} &= L_{t+k} - P_{t+k} + H_{t+k} - H_{t+k-1} \\
w_{t+k-1} &= L_{t+k-1} - P_{t+k-1} + H_{t+k-1} - H_{t+k-2} \\
&\vdots \\
w_{t+1} &= L_{t+1} - P_{t+1} + H_{t} - H_{t+1}
\end{align*}
\]

\[
\Rightarrow \sum_{i=1}^{k} w_{t+i} = \sum_{i=1}^{k} L_{t+i} - \sum_{i=1}^{k} P_{t+i} + H_{t+k} - H_t.
\]

(24)

Considering \(H_{t+k} \in [H_{\text{min}}, H_{\text{max}}]\), we have

\[
\sum_{i=1}^{k} w_{t+i} \in \left[sw_{t+k}^{\text{min}}, sw_{t+k}^{\text{max}}\right]
\]

(25)

\[
\begin{align*}
sw_{t+k}^{\text{min}} &= \sum_{i=1}^{k} L_{t+i} - \sum_{i=1}^{k} P_{t+i} - H_t + H_{\text{min}} \\
sw_{t+k}^{\text{max}} &= \sum_{i=1}^{k} L_{t+i} - \sum_{i=1}^{k} P_{t+i} - H_t + H_{\text{max}}.
\end{align*}
\]

(26)

(27)

As a matrix express, hence, we get

\[
Ew_{t+1} \leq b
\]

(28)

\[
E = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
1 & 1 & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
1 & 0 & \cdots & 1 \\
-1 & 0 & \cdots & 0 \\
-1 & -1 & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
-1 & -1 & \cdots & -1
\end{bmatrix}, \quad b = \begin{bmatrix}
sw_{t+1}^{\text{max}} \\
sw_{t+1}^{\text{min}} \\
\vdots \\
sw_{t+1}^{\text{max}} \\
sw_{t+1}^{\text{min}} \\
\vdots \\
sw_{t+1}^{\text{min}} \\
sw_{t+1}^{\text{min}} 
\end{bmatrix}
\]

(29)

From the above, the optimal solution \(w_{t+1}\) can be obtained. We operate with the obtained first component of \(w_{t+1}\), that is \(w_{t+1}\), and update the battery capacity with the following:

\[
H_{t+1} = H_t - TL_{t+1} + TG_{t+1} + w_{t+1}.
\]

(30)

As an advantage of the MPC algorithm, it is expected to reduce the total costs by purchasing more electricity at a low price and suppressing purchasing at a high price.

3.2 Scenario robust optimization

The PI obtained in the previous section is combined with this scenario robust optimization theory (SR) and applied to energy management. This theory aims to find the optimal solution that satisfies, at desired probability or more, the constraints that are only known stochastically, [11, 12].

Let \(\delta^{(1)}, \ldots, \delta^{(N)}\) according to the same probability \(P\) are each scenario. A scenario has an input and output \(\delta^{(i)} = (u(i), y(i))\), and there is no the same scenario. Then, we consider the following minimization problem including uncertainty in constraints.

\[
\text{NCSP}_N : \arg \min_{x \in X} f(x)
\]

subject to \(x \in X_{\delta^{(i)}}\) for all \(i = 1, \ldots, N\) (32)

where the convexity of the subsets \(X, X_{\delta}\) and the function \(f\) is not necessary. Constraint \(X_{\delta^{(i)}}\) is determined by scenario \(\delta^{(i)}\), \(x \in X\) is an optimization variable, and \(x^{\ast}_N\) is the solution of NCSP\(_N\). The following definitions are provided.

Definition 1: Violation probability

Let \(x\) be a given point in \(X\). The violation probability of \(x\) is defined as

\[
V(x) \doteq P\{\delta : x \notin X_{\delta}\}.
\]

(33)

Definition 2: Support set

Consider a sample \((\delta^{(1)}, \ldots, \delta^{(N)})\), and let \(x^{\ast}_N\) be the corresponding solution of program NCSP\(_N\). When the solution \(x^{\ast}_N\) does not change even if the constraints except \(x \in X_{\delta^{(i)}}\) \((i = 1, \ldots, N)\) are excluded, support set \(Z\) is defined as follows.

\[
Z = \{\delta^{(1)}, \ldots, \delta^{(i)}\} \subseteq \{\delta^{(1)}, \ldots, \delta^{(N)}\}
\]

(34)

An algorithm to find an irreducible support set is as follows.

Algorithm 1 searching support set algorithm

Require: \(L, \delta^{(i)}, N, \text{NCSP}_N\)
Ensure: index of support set \(\{i_1, \ldots, i_k\}\)

1: Compute index set of \(\{i_1, \ldots, N\}\)
2: for \(i = 1, \ldots, N\) do
3: Compute solution \(x^\ast\) \(\text{NCSP}_N\) by set \(L' \leftarrow L\backslash \delta^{(i)}\).
4: if \(x^\ast = x^{\ast}_N\) then
5: Update set \(L \leftarrow L'\).
6: end if
7: end for
8: Compute index set of \(L\).
9: return \(\{i_1, \ldots, i_k\}\).

When the number of support set is \(s^\ast_N\), the following Theorem 1 holds.

Theorem 1 [12]

When the solution of NCSP\(_N\) uniquely exists, let \(\beta \in (0, 1)\) is confidence parameter. Define the function \(\varepsilon : \{0, \ldots, N\} \rightarrow [0, 1]\) as follows:

\[
\varepsilon(k) \doteq \begin{cases}
1 & \text{if } k = N \\
1 - \sqrt{\frac{\beta}{N^2}} & \text{otherwise}
\end{cases}
\]

(35)

Using the number of support sets \(s^\ast_N\), the following (36) is established.

\[
P^N\{V(x^{\ast}_N) > \varepsilon(s^\ast_N)\} \leq \beta
\]

(36)

3.3 Scenario-based robust MPC

A Scenario-based robust MPC method that combines MPC and scenario robust optimization theory (SR) is explained as follows. The evaluation function \(J\) and relational expression are the same as \(P_{\text{MPC}}\). Based on the SR theory, the constraints are given as follows.

\[
\sum_{i=1}^{k} w_{t+i} \in [sw_{t+k}^{\text{min}}, sw_{t+k}^{\text{max}}].
\]

(37)
with 1250 Wh. Therefore, it is realistic enough because the area of the roof is 200 m².

The electricity price is based on Tokyo Electric Power Company’s plan “Peak shift plan” [14].

The simulation evaluation is performed in the following four patterns.

**Method 1** One step optimization with PF

**Method 2** One step optimization with SR by PI

**Method 3** MPC optimization with PF

**Method 4** MPC optimization with SR by PI

The simulation parameters are shown in Table 3.

### Table 3: Parameters of Energy Management Systems

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight of Electrical Rate</td>
<td>$R$</td>
<td>diag(12.25, 29.08, 54.77, 54.77, 54.77, 29.08, 29.08)</td>
</tr>
<tr>
<td>Weight of Battery</td>
<td>$A$</td>
<td>diag(100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100)</td>
</tr>
<tr>
<td>Initial Battery Charge [kWh]</td>
<td>$H_0$</td>
<td>5</td>
</tr>
<tr>
<td>Battery Lower Limit [kWh]</td>
<td>$H_{\min}$</td>
<td>0</td>
</tr>
<tr>
<td>Battery Upper Limit [kWh]</td>
<td>$H^{\max}$</td>
<td>10</td>
</tr>
<tr>
<td>Battery target value [kWh]</td>
<td>$H^T$</td>
<td>5</td>
</tr>
<tr>
<td>Simulation Period [h]</td>
<td>$T$</td>
<td>12</td>
</tr>
<tr>
<td>Number of Scenarios</td>
<td>$N$</td>
<td>200</td>
</tr>
<tr>
<td>Sampling Time [h]</td>
<td>$\beta$</td>
<td>1</td>
</tr>
<tr>
<td>Confidence Level</td>
<td>$\epsilon$</td>
<td>1 - $\beta$</td>
</tr>
</tbody>
</table>

### 4.2 Simulation result

From Table 4, Methods 3 and 4 using MPC have lower total cost than Methods 1 and 2 not using MPC. Additionally, it is confirmed that Methods 2 and 4 using SR by PI suppress the excess power and shortage power than Methods 1 and 3 not using SR.

### Table 4: Result of Energy Management

<table>
<thead>
<tr>
<th>Method</th>
<th>Total Cost(k yen)</th>
<th>Excess Power(kWh)</th>
<th>Shortage Power(kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method 1</td>
<td>107.92</td>
<td>0</td>
<td>138.25</td>
</tr>
<tr>
<td>Method 2</td>
<td>107.67</td>
<td>0</td>
<td>13.68</td>
</tr>
<tr>
<td>Method 3</td>
<td>104.25</td>
<td>5.07</td>
<td>17.09</td>
</tr>
<tr>
<td>Method 4</td>
<td>108.61</td>
<td>0</td>
<td>4.32</td>
</tr>
</tbody>
</table>

One can see in Table 4 that the results of total cost in methods 3 and 4 using the MPC algorithm are lower than in methods 1 and 2 not using the MPC algorithm. Besides, one can also see that the amount of battery violation (excess and shortage power) in methods 2 and 4 using SR are lower than in methods 1 and 3, not using SR.

Fig. 7. the simulation result of 3-Oct. with Method 3, shows that there are violations of the upper and lower battery constraints at 13:00, 16:00, and 18:00, owing to unexpected power generation and demand. Meanwhile, in Fig. 8, the simulation result with Method 4, there is no violation of constraints at that time because of using SR that can expect the estimated error of generation and demand.

However, as Table 4, Fig. 7, and Fig. 8 showing, the total cost of Method 4 is higher than Method 3, that means the optimality in method 4 is inferior by in method 3. That is because, in the optimization problem, there is generally the relation of a trade-off between the optimality and robustness.

Considering these facts, it is confirmed that suppression of cost which is the effect of MPC and robustness to battery constraints by SR.

In addition, it can be said that the number of support set in SR is one because it almost does not become that the constraints at each scenario are exactly the same in numerical calculation. In consequence, (41), (42) are yielded by substituting $N = 200$, $\beta = 0.2$, $k = 1$ into (35).
Therefore, we can confirm that the violation probability is 5.95% or less with an 80% reliability.

Fig. 7: Energy Management result by method 3 on October 3rd

Fig. 8: Energy Management result by method 4 on October 3rd

5 Conclusion

In this paper, firstly, we proposed the prediction intervals method using the 3D-copula and confirmed the effectiveness of the proposed method by a prediction verification. Then, in the energy storage problem, we discussed the method using the scenario robust theory by the obtained prediction intervals. Moreover, by combining model predictive control and scenario robust theory, we proposed the scenario-based robust MPC method with both robustness by SR and cost reduction by MPC. Finally, the simulation result showed its effectiveness.

References