Merging of Connected and Automated Vehicles at Roundabout using Model Predictive Control

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Abstract: This paper addresses a merging problem of connected and automated vehicles at single lane with two different freeways roundabout. First, we assume that the merging vehicle already knew the environment of the roadway beforehand. After that, we formulate the optimization problem based on model predictive control framework which includes lane keeping and collision avoidance. Finally, we show in the simulation results that the proposed solution is capable of merging the vehicle at the roundabout effectively.

Keywords: Connected and automated vehicles (CAVs), vehicle coordination, merging control, roundabouts.

1. INTRODUCTION

A broad utilization of vehicles every year causes traffic congestion around the globe getting worse especially in urban areas. Sometimes, the problem itself is caused by driver reactions to various disturbances, for instance, a single driver, all of a sudden, hitting the brake too hard. Even so, this is not the primary source of the traffic congestion where [1] reported that the main cause of the problem is merging of vehicles at different transportation segments. Studies have shown that traffic congestion can bring a huge negative impact on health, economic, environment, and many other things. Therefore, many solutions for reducing the congestion have been proposed, and one of the ideas is by properly coordinating the merging vehicles. In achieving this idea, the vehicles must have the ability to communicate with each other and are capable of operating without any intervention of a human. This kind of vehicles is known as connected and automated vehicles (CAVs). Moreover, with this technology, it is believed that the percentage of traffic accidents can be potentially reduced too.

In recent years, there are a lot of research efforts have been conducted on CAVs particularly on how to successfully merge them into different transportation segments. Several merging scenarios at different transportation segments have been investigated by researchers which are highway on-ramp [2], [3], intersections [4], [5], and roundabouts [6]. From analyzing this literature, various issues that correspond to the merging scenarios are identified. In [2] and [4], they had formulated the problem of optimal vehicle coordination at merging roadways and intersections respectively regarding fuel consumption and travel time under the hard constraint of collision avoidance. It is shown in these works that their approaches can reduce both fuel consumption and travel time significantly. Different in [3], where they had focused on coordinating vehicles at merging roadways with multiple main lanes. On the other hand, [5] had derived a merging vehicle model which includes steering model. With this advanced model, a more practical implication on implementing numerous approaches can be provided. Last but not least, [6] had studied how to coordinate vehicles to achieve smooth traffic flow at roundabouts optimally. The study also had been furthered down to see the effect of mixed-traffic environment, i.e., environment with both CAVs and non-CAVs. The results showed that even with the high penetration of CAVs (e.g., 80%), but travel time and fuel savings are much less than a network of CAVs.

In this paper, the goal will be on how to steer vehicles so that they can merge into a roundabout successfully under the hard constraint of collision avoidance. Considering the difficulty in merging problem, the first stage of this research only assume a single-lane roundabout with two different freeways and each freeway has only two CAVs maximum. A decentralized optimization-based control approach based on Model Predictive Control (MPC) is proposed to achieve this goal. Although many researchers had utilized this control scheme for the merging problems [3], [7–9], however, no results had been obtained for merging of vehicles at the roundabout.

2. PROBLEM FORMULATION

In this section, we will discuss how the roundabout and vehicle are modeled as well as defining the control objective.

2.1. Modeling framework

As mentioned previously, this study only assumes a single-lane roundabout with two different freeways. Fig. 1 shows a single-lane with two different ways (i.e., vehicle traveling eastbound and westbound) roundabout used as our roadway model inspired from [6]. As can be seen from the figure, the roundabout is classified into three main zones represented in green, red, and gray colors. The green zone means a control

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zone where a vehicle can reach signal from the coordinator. Note that, the coordinator is not involved in any decision on the vehicle but to share some information between the vehicles and also to determine the ID of the vehicle based on who enters the merging control first. Next, the zone in red denoted the merging zone where in this zone, the potential lateral collision might happen between two or more vehicles. As for the gray zone, it is to denote the arc length from the exit of the control zone (east side) until the entry of the merging zone.

where  is the width of the lane and  is the magnitude of the potential function.

As the vehicle model to be used, each of the vehicles  is considered as a mass point and modeled as a discrete-time second order linear system [3] given as,

\[
x_i(k+1) = Ax_i(k) + Bu_{iz}(k),
\]

\[
y_i(k+1) = Ay_i(k) + Bu_{iy}(k),
\]

\[
x_i(k) = [x_i(k), v_{ix}(k)]^T,
\]

\[
y_i(k) = [y_i(k), v_{iy}(k)]^T,
\]

\[
A = \begin{bmatrix} 1 & t_s \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ t_s \end{bmatrix},
\]

where  is the total number of vehicles inside the control zone,  is a step time,  and  are the position in  and  coordinate,  and  are control inputs in  and  coordinate, and  and  are the speeds in  and  coordinate.

Fig. 1: Merging scene at roundabouts

Referring to Fig. 1, we define:

- \(\alpha\) : Length of control zone
- \(\alpha_r\) : Arc length of vehicle traveling westbound
- \(\beta\) : Arc length of the merging zone

Besides that, each of vehicles  also is given certain constraints as in Eq. (6) to ensure that the control input and vehicle speed are within a given admissible range which is typically set to meet passengers comfort and to satisfy speed limits on roadways set by the authority.

\[
u_{iz\min}(k) \leq u_{iz}(k) \leq u_{iz\max},
\]

\[
u_{iy\min}(k) \leq u_{iy}(k) \leq u_{iy\max},
\]

\[
0 \leq v_{ix}(k) \leq v_{ix\max},
\]

where  and  are the minimum and maximum inputs, and  and  are the minimum and maximum speeds. Note that the minimum speed limit of  is set to be higher and equal to 0 because to prevent the vehicles from moving backward.

\[
0 \leq y \leq y_{0} + \kappa
\]

\[
y \geq y_{0} - \kappa
\]

2.2. Merging control

In this research, we let the vehicle run according to the shape of a roadway. A road potential function is designed to keep the vehicle inside the roadway. In designing the road potential function and getting a better illustration of the roadway model, first, we convert the roadway model from Fig. 1 into a Cartesian plane as shown in Fig. 2.

Fig. 2: Roundabout model in a Cartesian plane

From the above figure, the road potential function is derived as shown in Eq. (7). Please note that, aside from this equation, the values of the potential function are zero.

\[
P(x(k), y(k)) = \begin{cases} \end{cases}
\]

\[
a_0(y - \gamma_0 - \kappa), \quad \text{if} \quad x < x_{c1} \quad \text{or} \quad x > x_{c2} \\
- a_0(y - \gamma_0 + \kappa), \quad \text{if} \quad x \leq x_{c1} \quad \text{or} \quad x > x_{c2} \\
-a_b(y - \gamma_0 - \kappa), \quad \text{if} \quad x < x_{c1} \quad \text{and} \quad y \geq \sqrt{\gamma_1^2 - (x - x_m)^2} + \gamma_0 \\
-a_b(y + \sqrt{\gamma_1^2 - (x - x_m)^2} + \gamma_0), \quad \text{if} \quad x_{c1} \leq x \leq x_{c2} \quad \text{and} \quad y \geq \sqrt{\gamma_1^2 - (x - x_m)^2} + \gamma_0 \\
-a_b(y - \sqrt{\gamma_1^2 - (x - x_m)^2} - \gamma_0), \quad \text{if} \quad x_{c1} \leq x \leq x_{c2} \quad \text{and} \quad y \leq (\sqrt{\gamma_1^2 - (x - x_m)^2} + \gamma_0) \\
-a_b(y - \sqrt{(\gamma_1 - \kappa)^2 - (x - x_m)^2} + \gamma_0), \quad \text{if} \quad x_{c1} + \kappa \leq x \leq x_{c2} + \kappa \\
-a_b(y + \sqrt{(\gamma_1 - \kappa)^2 - (x - x_m)^2} + \gamma_0), \quad \text{if} \quad x_{c1} + \kappa \leq x \leq x_{c2} - \kappa \\
-a_b(y + \sqrt{(\gamma_1 - \kappa)^2 - (x - x_m)^2} + \gamma_0) \quad \text{and} \quad y \leq 0
\]
The road potential function in Eq. (7) is only to keep and maneuver the vehicles according to the shape of the roadway. However, we also need to maintain the position of the vehicles along the center of their corresponding lane. To achieve that, we design a line following function for both vehicles traveling eastbound, \( i \in E(k) \) and westbound, \( i \in W(k) \) as in Eq. (8) and Eq. (9) respectively.

\[
L(x(k), y(k), \omega(k)) = \begin{cases} 
R(\theta(k), \omega(k)), & \forall i \in E(k) \text{ and } x \geq x_{e1} + \frac{\gamma}{2} \\
R(\theta(k), \omega(k)), & \forall i \in W(k) \text{ and } x \leq x_{e2} - \frac{\gamma}{2} 
\end{cases}
\]

\[
R(\theta(k), \omega(k)) = \begin{cases} 
\cos(\theta - \omega) + x_m & r \\
\sin(\theta - \omega) + \gamma_0, & r
\end{cases}
\]

where \( R \) is the roundabout function, \( \theta \) is the roundabout angle (refer Fig. 2), \( \omega \) (rad/s) is the angular velocity obtained from \( \omega = v/r \), and \( r \) is radius required for generating vehicle’s path to travel inside the roundabout (refer Fig. 2).

### 2.3. Collision Avoidance

In merging problem, collision avoidance is one of the most important issues that need to be addressed. In this paper, we divide collision avoidance into two cases which are rear-end and lateral collisions. Typically, rear-end collision might happen when two or more consecutive vehicles are traveling on the same lane. Lateral collision might happen inside the merging area where two or more vehicles from different directions want to merge into the merging area at the same time.

To ensure the absence of rear-end collision for vehicles traveling eastbound and westbound, we impose the following constraint into our optimization problem.

\[
\min(||p_i(k) - p_j(k)||) \geq \Delta,
\]

where \( p_i \) and \( p_j \) are the positions of vehicle \( i \) and neighbor vehicle \( j \) in \( x \) and \( y \) direction, and \( \Delta \) is the minimum distance difference which guarantees the movement of the vehicle on the same lane without collision.

Eq. (10) cannot be used directly to avoid a lateral collision. This is because the vehicles need to decide among themselves who will merge into the merging area first to avoid the lateral collision. As the strategy for this problem, we utilized the following solution where, first, we consider the predicted arrival time that need to be computed by vehicle \( i \) to merge into the merging area \( \beta \) as

\[
k^e_i = \frac{\alpha}{v_i(k)} + \lambda_i \frac{\alpha v_i}{v_i(k)},
\]

\[
\lambda_i = \begin{cases} 
0, & \text{if } \forall i \in E(k) \\
1, & \text{if } \forall i \in W(k).
\end{cases}
\]

Note that, aside from computing the predicted arrival time for themselves, vehicle \( i \) also needs to compute the predicted arrival time for other vehicles traveling in different directions. By using these information, priority index for all vehicles \( N \) can be obtained as follows.

\[
\begin{cases} 
||p_i(k) - p_j(k)|| \geq \delta & \text{if } k^e_i \geq \min(k^e_j) \\
||p_j(k) - p_i(k)|| \geq \delta & \text{if } k^e_i < \min(k^e_j).
\end{cases}
\]

In general, Eq. (13) means that if the vehicle \( i \) reaches the merging area slower compared to vehicle \( j \), then, vehicle \( i \) needs to give way to vehicle \( j \) to pass the merging area first and vice versa.

### 2.4. Objective function

The objective function for each vehicle is set as follows.

\[
J_x(u_x) = \sum_{t=0}^{T_p-1} \left( w_1 \left( v_r - v_x(t) \right)^2 + w_2 \left( u_x(t) \right)^2 \right),
\]

\[
J_y(u_y) = \sum_{t=0}^{T_p-1} \left( w_3 \left( L(x, y, \omega) - p \right)^2 + w_4 \left( u_y(t) \right)^2 + w_5 \left( P(x, y) \right)^2 \right),
\]

where \( w_1 \) to \( w_5 \) are the weight coefficients that normalize the terms in Eq. (14) and Eq. (15), and \( v_r \) is the reference speed.

From the objective function above, the optimization problem is formulated as

\[
\begin{align*}
\text{minimize } & J_x(u_x), J_y(u_y), \\
\text{subject to } & (1) - (6), (10), \text{ and } (13),
\end{align*}
\]

and this problem is solved using \texttt{fmincon} function provided in Matlab.

### 3. SIMULATION RESULTS

In this section, the result will be discussed by considering two case studies (1) Coordination of three vehicles, one is traveling eastbound, and another two are traveling westbound (2) Coordination of four vehicles, two vehicles for each direction. The parameter
values for these two case studies are provided in Table 1. Note that some of the roundabout and vehicle parameter values are considered based on [6] because the values are based on real-time environment.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value [unit]</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>300 [m]</td>
</tr>
<tr>
<td>$\alpha_r$</td>
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</tr>
<tr>
<td>$\beta$</td>
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<td>$\gamma_0$</td>
<td>0 [m]</td>
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<td>$\gamma_1$</td>
<td>32 [m]</td>
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<tr>
<td>$\gamma_2$</td>
<td>$-32$ [m]</td>
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<td>$x_{c1}$</td>
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<td>$x_m$</td>
<td>332 [m]</td>
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<td>$x_{c2}$</td>
<td>364 [m]</td>
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<td>$\kappa$</td>
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<td>$\theta(i \in E)$</td>
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<tr>
<td>$u_{x_{\max}}$</td>
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<td>$v_{x_{\max}}$</td>
<td>15.6 [m/s] $\approx 56$ [km/h]</td>
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<td>$\Delta$</td>
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</tbody>
</table>

Case study 1: Merging of 3 vehicles

In this study, we will coordinate three vehicles where $i = 1$ (represents in red) is traveling eastbound and another two, $i = 2$ (represents in blue) and $i = 3$ (represents in green) are traveling westbound. The purpose of this scenario is to validate either the controller embedded into each of the vehicles is capable of coordinating the vehicles without having the lateral collision issue.

![Fig. 3: Position of vehicles N](image)

Fig. 3 shows the positions of vehicles $N$ at iteration $k = 300, 320, 340, 360$. As can be observed from the figures, vehicle one starts to slow down at $k = 320$ and $k = 340$. After that, at $k = 360$, the vehicle one slowly to accelerate back to merge into the roundabout. From these figures also, it can be confirmed that the vehicle $N$ are not having a lateral
collision with each other.

Fig. 4 shows the inputs and speeds of vehicle N. From these figures, it is shown that all vehicle N are satisfying all the inputs and output constraints i.e., the minimum and maximum accelerations as well as the minimum and maximum speeds.

Case study 2: Merging of 4 vehicles

In this study, we will coordinate four vehicles where \( i = 4 \) (represents in black) is added into the simulation. Note that this newly added vehicle is traveling eastbound. This means that now we have two vehicles traveling eastbound and two vehicles traveling westbound where both front-end and lateral collisions might happen.

Fig. 5 shows the positions of vehicles N at iteration \( k = 300, 320, 340, 360 \). As can be seen from the figures, vehicle one starts to slow down at \( k = 320 \). After that, at \( k = 340 \), vehicle four also starts to decelerate to avoid the front-end collision with vehicle one. Next, at \( k = 360 \) both of vehicles one and vehicle four accelerate back to merge into the roundabout slowly.

(a) Input and speed at x coordinate
(b) Input and speed at y coordinate

Fig. 4: Input and speed of vehicle N

(c) Position of vehicles at \( k = 340 \)
(d) Position of vehicles at \( k = 360 \)

Fig. 5: Position of vehicles N
Fig. 6: Input and speed of vehicle N

Fig. 6 shows the inputs and speeds of vehicle N. Same as in Fig. 5, it is shown that all vehicle N are satisfying all the inputs and output constraints.

4. CONCLUSION

This paper had addressed the merging control of connected and automated vehicles (CAVs) at a single lane with two different ways roundabout under the hard constraint of collision avoidance. To address the problem, we had proposed an optimization framework based on decentralized Model Predictive Control (MPC). From the simulation result, it was shown that our proposed method could successfully merge four CAVs into the roundabout without having any collision issue including front-end and lateral collisions. For the future work, we will extend the model of the CAVs which include the turning of the vehicle as well as analyzing the travel time and fuel consumption in the scenarios of more than thirty vehicles.

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