Passivity-Short-based Stability Analysis on Electricity Market Trading System Considering Negative Price

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Abstract—This paper deals with a distributed decision method for determining the electricity price considering the negative price in the market trading. In addition, we analyze the stability and convergence properties for the market trading system using the concept of passivity-short. At first, mathematical models of market participants' behavior which maximize their own profits are discussed. Then, we construct a trading system based on dynamic pricing considering plug and play operation that each market participant intermittently participates in market trading. As for the optimal electricity price, even if the optimal price might be negative, we indicate the stability of the system can be guaranteed by using passivity-short which is a relaxed concept of passivity. Finally, numerical simulation results illustrate that each value decided by the market trading converges to the optimal value.

I. INTRODUCTION

In recent years, energy demand has been increasing worldwide and energy sustainability are becoming a major concern. In order to solve the problems, demand response has been extensively studied. In this method, demand and supply are controlled from both the consumer and generator sides. Among many kinds of demand response, there are different studies on dynamic pricing which changes electricity price depending on the time or season. Using this method, economic benefits for consumers increase when compared to a fixed price. However, in order to properly determine the electricity price in the market, it is necessary to consider the benefits of both the consumers and generators. In particular, in order to ensure stable operation of the power system, we have to keep the power balance at all time. Therefore, in the power market where market participants are to pursue their own objectives, it is necessary to decide an appropriate electricity price so as to satisfy the above-mentioned principle of simultaneous equal quantity.

For the problems, many papers on dynamic pricing method and its corresponding distributed power supply and demand management method have been published. In particular, the power supply and demand management method which achieves the maximization of social welfare in a distributed manner has been proposed in [1]–[3], where the economic behavior model of consumers is modeled by using the utility function expressing the degree of financial satisfaction that consumers are obtained by consuming electricity. Furthermore, distributed market algorithms using which participants in each area repeat transactions to decide the power supply and demand within the electric power system have been proposed in [4], [5]. In addition, a plug-and-play operation that allows market participants to intermittently participate in transactions is presented in [6], where stability analysis in electricity market trading system is based on passivity. Here, passivity refers to the energy balance relationship between input and output of the system, and is used in many kinds of control problems, including stability analysis of the teleoperation robot, design of optimal energy management algorithm for physically fused building air conditioning, the formation control for multi agent system [7]–[11].

However, in the above papers, the occurrences of negative prices in market transactions are not considered. Negative price is a phenomenon in which the market price enters a negative area when a generation surplus exceeding demand occurs. Specifically, for power generation that requires a large cost for shut-down and start-up such as nuclear power and thermal power, as long as it does not exceed the cost spent on it, the generators pay money and supply power to consumers. Thereby avoiding the shut-down of the generator and reducing the cost. In this paper, we analyze the stability when this negative price occurs in market transactions based on the concept of passivity-short which is a relaxed concept of passivity.

In Section II, we first describe the economic behavior model of each market participant. Next, we describe the social welfare maximization problem in Section III, and we show that the transaction amount determination system of each market participant is passivity-short in Section IV. In addition, it shows that each value decided in electricity market trading system interconnecting the transaction amount decision system of each market participant converges to the optimal value by repeating market transactions. Finally, in Section V, we verify the effectiveness of the proposed method using numerical simulation results.

II. PROBLEM FORMULATION

A. Market model

A schematic diagram of the electricity market trading model is shown in Fig. 1. In this paper, there are multiple consumers and generators in each area, and the market manager, ISO, decides the electricity price and adjusts the supply and demand amount. Also, we consider that there are multiple areas in power grid and the consumers intermittently participate in the market. Note that, $\mathcal{A}$ denotes a set of areas.
in the power market model in our paper, and $|\mathcal{A}|$ represents the number of these areas.

B. Consumer

At first, we formulate the consumer behavior model considering the occurrence of negative price. With respect to consumers, $\mathcal{L}_l$ represents the set of consumers in the area $l$ participating in the market trading, $P_{L,i}^l, i \in \mathcal{L}_l$ is the electricity demand for consumer $i$ in area $l$, and $\lambda^l$ is electricity price. We introduce Assumption 1 for utility function $v^l_i(\cdot)$ of the consumer $i$. Here, the utility function represents financial satisfaction of each consumer for the power consumption.

**Assumption 1:** Utility function $v^l_i(\cdot)$ is in $C^2$ $[0, P_{L,i}^{l,\text{max}}]$, non-negative and a strictly concave function.

Then, the consumer welfare function $W_{L,i}(P_{L,i}^l, \lambda^l)$ is defined as follows:

\[
W_{L,i}(P_{L,i}^l, \lambda^l) := v^l_i(P_{L,i}^l) - \lambda^l P_{L,i}^l, \forall i \in \mathcal{L}_l, \forall l \in \mathcal{A}.
\]

Since the consumers act to maximize own profits, the economic behavior model that determines the electricity demand is expressed as follows:

\[
\max_{P_{L,i}^l} W_{L,i}(P_{L,i}^l, \lambda^l), \quad \text{s.t. } P_{L,i}^{l,\text{min}} \leq P_{L,i}^l \leq P_{L,i}^{l,\text{max}};
\]

where $P_{L,i}^{l,\text{min}}, P_{L,i}^{l,\text{max}}$ are the upper and lower bounds on the electricity demand amount of each consumer.

C. Generator

Next, we formulate the generator behavior model. With respect to generators, $\mathcal{G}_i$ represents the set of generators in the area $l$ participating in the market trading, $P_{G,i}^l, i \in \mathcal{G}_i$ is the electricity supply for generator $i$ in area $l$. Also, supply amount by base load power sources such as thermal and nuclear, and supply amount by renewable energy such as wind and sunlight are given as $P_{G,bi}^l, P_{G,ri}^l, i \in \mathcal{G}_i$ respectively. Here, with respect to the cost function representing the power generation cost, the base load power cost function is $c^l_{bi}(P_{G,bi}^l)$ and the renewable energy cost function is $c^l_{ri}(P_{G,ri}^l)$, then we introduce Assumptions 2 and 3 respectively.

**Assumption 2:** The base load power cost function $c^l_{bi}(\cdot)$ is in $C^2$ $[0, \infty)$, non-negative and a strictly convex function.

**Assumption 3:** The renewable energy cost function $c^l_{ri}(\cdot)$ is in $C^1 [0, P_{G,ri}^{l,\text{max}}]$ and a convex function.

Then, the generator welfare function $W_{G,i}^l(F_{G,bi}^l, F_{G,ri}^l, \lambda^l)$ is defined as follows:

\[
W_{G,i}^l(\cdot) := \lambda^l P_{G,i}^l - c^l_{bi}(P_{G,bi}^l) - c^l_{ri}(P_{G,ri}^l), \forall i \in \mathcal{G}_i, \forall l \in \mathcal{A}.
\]

Since the generators act to maximize own profits, the economic behavior model that determines the electricity supply is expressed as follows:

\[
\max_{P_{G,b}, P_{G,ri}^l} W_{G,i}^l(P_{G,b}, P_{G,ri}^l, \lambda^l),
\]

\[
\text{s.t. } P_{G,bi}^l + P_{G,ri}^l = P_{G,i}^l, \quad P_{G,bi}^{l,\text{min}} \leq P_{G,bi}^l \leq P_{G,bi}^{l,\text{max}}, \quad P_{G,ri}^{l,\text{min}} \leq P_{G,ri}^l \leq P_{G,ri}^{l,\text{max}}.
\]

where $P_{G,bi}^{l,\text{min}}, P_{G,bi}^{l,\text{max}}$ are the upper and lower bounds on the electricity supply amount by base load powers, and $P_{G,ri}^{l,\text{min}}, P_{G,ri}^{l,\text{max}}$ are the upper and lower bounds on the electricity supply amount by renewable energy.

III. SOCIAL WELFARE MAXIMIZATION PROBLEM

In this section, we consider the social welfare maximization problem. Then, using the consumer and generator model, we formulate the social welfare function $W(P_L, P_G)$ as follows.

\[
W(\cdot) := \sum_{i \in \mathcal{A}} \left[ \sum_{l \in \mathcal{L}_l} v^l_i(\cdot) - \sum_{i \in \mathcal{G}_i} \{c^l_{bi}(\cdot) - c^l_{ri}(\cdot)\} \right].
\]

Here, the social welfare maximization problem is to derive an optimal solution that maximizes the profit of each market participant while achieving agreement of supply and demand, and it is expressed as follows.

\[
\max_{P_L, P_G} W(P_L, P_G),
\]

\[
\text{s.t. } P_G = P_L,
\]

\[
P_{G,bi}^l + P_{G,ri}^l = P_{G,i}^l, \forall i \in \mathcal{G}_i, \forall l \in \mathcal{A},
\]

\[
P_{G,bi}^{l,\text{min}} \leq P_{G,bi}^l \leq P_{G,bi}^{l,\text{max}}, \forall i \in \mathcal{G}_i, \forall l \in \mathcal{A},
\]

\[
P_{G,ri}^{l,\text{min}} \leq P_{G,ri}^l \leq P_{G,ri}^{l,\text{max}}, \forall i \in \mathcal{G}_i, \forall l \in \mathcal{A},
\]

where,

\[
P_G = [P_L^1 \cdots P_L^{|\mathcal{A}|}]^T \in \mathbb{R}^{|\mathcal{A}|}, \quad P_G := \sum_{i \in \mathcal{G}_i} P_{G,i}^l,
\]

\[
P_L = [P_L^1 \cdots P_L^{|\mathcal{A}|}]^T \in \mathbb{R}^{|\mathcal{A}|}, \quad P_L := \sum_{i \in \mathcal{L}_l} P_{L,i}^l.
\]

When the Lagrange multiplier is $\lambda_0 = [\lambda_0^1 \cdots \lambda_0^{|\mathcal{A}|}]^T \in \mathbb{R}^{|\mathcal{A}|}$, the dual problem for the optimization problem is as follows:

\[
\min_{\lambda_0} \max_{P_L, P_G} W(P_L, P_G) + \lambda_0^T (P_G - P_L),
\]

\[
\text{s.t. } (10) - (13).
\]
Then, Lagrange multiplier $\lambda^*_i$ is equivalent to the price $\lambda^i$. Note that, in the rest of this paper, the notations for the variables are unified, so both variables are represented using $\lambda^i, i \in A$. Additionally, the optimal solution for each variable is represented using a superscript $(\cdot)^*$ respectively.

IV. PASSIVITY-SHORT BASED STABILITY ANALYSIS

In this section, we describe the concept of passivity-short. Here, consider the following nonlinear system [12].

$$
\begin{align*}
\dot{x}(t) &= f(x(t)) + g(x(t))u(t), \\
y(t) &= h(x(t)),
\end{align*}
(18)
$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input and $y \in \mathbb{R}^m$ is the control output. Then, the definition of dissipativity is shown below.

**Definition 1:** System (18) is said to be dissipative with respect to a storage function $V(x)$ and the supply rate $\Phi(x,u)$ if the following inequality holds,

$$
V(x(t)) - V(x(0)) \leq \int_0^t \Phi(x(\tau), u(\tau))d\tau.
(19)
$$

For the inequality (19), both the storage function $V(x(t))$ and the supply rate $\Phi(x(\tau), u(\tau))$ are selectable. Here, concept of passivity is defined as follows:

**Definition 2:** With respect to inequality (19), the supply rate $\Phi$ is expressed in a quadratic form as follows. Here, $\eta(x)$ is a positive semi-definite function.

$$
\Phi(x,u,y) = -\eta(x) + u^Ty + \frac{\rho}{2}\|y\|^2.
(20)
$$

When $\rho > 0$, system (18) is said to be passivity-short. Also, when $\rho \leq 0$, the system is passive and if $\rho < 0$, it is strictly passive. Here, $\rho$ is called impact coefficient.

Note that, for arbitrary functions $f(x)$, with $x(\geq 0)$, it is defined as follows [7].

$$
(f(x))_x := \begin{cases} 
  f(x), & \text{if } x > 0, \text{ or } x = 0 \text{ and } f(x) \geq 0, \\
  0, & \text{if } x = 0 \text{ and } f(x) < 0.
\end{cases}
(21)
$$

A. Consumer demand decision system

We describe the dynamic model for determining the demand amount by consumers in each area. By applying dual decomposition to (16), the demand amount determination problem by consumers is expressed as follows:

$$
\begin{align*}
\min_{\lambda^i, \mu^i_L \geq 0, \mu^i_L \geq 0} \max_{P^i_L} & \hat{W}_L(P^i_L, \lambda^i, \mu^i_L, \mu^i_L), \\
\hat{W}(P^i_L, \lambda^i, \mu^i_L, \mu^i_L) := v^i_l(P^i_L) - \lambda^i(P^i_L) \\
& - \mu^i_L(P^i_L) - P^i_L - P^i_L^{\max},
\end{align*}
(22)
$$

where, $\mu^i_L, \mu^i_L$ are the Lagrange multipliers for the upper and lower bounds on the electricity demand. By partially differentiating (22) with respect to each variable, the dynamic model for updating the demand is given by (24)-(26), and the dynamic models for updating the Lagrange multipliers are given by (27), (28). Here, it is shown in [13] that the controller includes the property of proportional and integral action, therefore, we employ PI controller. In [14], PI control system is also utilized.

$$
\begin{align*}
\frac{d}{dt}x^*_1 &= x^*_2, \\
\frac{d}{dt}x^*_2 &= v^i_l(P^i_L) - \lambda^i + \mu^i_L - \mu^i_L, \\
\frac{d}{dt}P^i_L &= k_{L_1}x^*_1 + k_{P_L}x^*_2,
\end{align*}
(24)
$$

$$
\begin{align*}
\frac{d}{dt}\mu^i_L &= k_{\mu_L}x^*_1 + k_{P_L}x^*_2, \\
\frac{d}{dt}P^i_L &= k_{P_L}x^*_1 - k_{\mu_L}x^*_2 + k_{P_L}x^*_2,
\end{align*}
(25)
$$

where $k_{L_1}, k_{P_L}, k_{\mu_L}, k_{\mu_L}$ are positive gains. Also, we connect each model as shown in Fig. 2.

![Fig. 2. Block diagram of consumer demand decision system](image)

Here, $\bar{x} := x - x^*$ denotes the deviation of an arbitrary variable $x$ from its optimal solution $x^*$. Then, the following Lemma 1 provides the property between input (price) and output (demand).

**Lemma 1:** Suppose that Assumptions 1-3 hold. Then, the dynamic model for updating the demand (24)-(26) is passivity-short from input $\ddot{x}^*_i, \dddot{x}^*_i$ to output $\dddot{x}^*_i = P^i_L$.

**Proof:** Consider the following positive definite function.

$$
V^{I}_{P_L,i} := \frac{1}{2k_{P_L,i}} \left[ k_{L_1,i}(\ddot{x}^*_i)^2 + (k_{L_1,i} \dot{x}^*_i + k_{P_L,i} x^*_2)^2 \right].
(29)
$$

Then, the time derivative of the above positive function along trajectory is given by:

$$
\frac{d}{dt}V^{I}_{P_L,i} = \frac{k_{L_1,i}^2}{k_{P_L,i}}(\ddot{x}^*_i)^2 + \frac{k_{L_1,i}}{k_{P_L,i}}(\ddot{x}^*_i)^2 + \frac{k_{L_1,i}}{k_{P_L,i}}|\dddot{x}^*_i|^2 + \frac{k_{L_1,i}}{k_{P_L,i}}|\dddot{x}^*_i|^2 + \frac{k_{L_1,i}}{k_{P_L,i}}|\dddot{x}^*_i|^2.
(30)
$$

Also, it follows from (22) and (23) that the following equation holds regarding their optimal solutions and optimal Lagrange multipliers:

$$
\dddot{x}^*_i(P^i_L) - \lambda^i + \mu^i_L - \mu^i_L = 0.
(31)
$$
Additionally, from \( \frac{d}{dt} x_{1,L_i} = v_i^l(P_{L_i}^l) - \lambda^l + \mu_i^L - \mu_i^{L^+} \), \( y_{P_{L_i}^l}^l = P_{L_i}^l \), (30) is as follows:

\[
\frac{d}{dt} V_{P_{L_i}^l} = - \frac{k_i^{L_i}}{k_{PL_i}^l} |\bar{x}_{1,L_i}|^2 + \bar{P}_{L_i}^l(v_i^l(P_{L_i}^l) - v_i^l(P_{L_i}^*)^2) + \bar{P}_{L_i}^l(-\lambda^l + \mu_i^L - \mu_i^{L^+}) + \frac{k_i^{L_i}}{k_{PL_i}^l} |\bar{P}_{L_i}^l|^2. \tag{32}
\]

Here, \(-\frac{k_i^{L_i}}{k_{PL_i}^l} |\bar{x}_{1,L_i}|^2\) is negative, and \( \bar{P}_{L_i}^l(v_i^l(P_{L_i}^l) - v_i^l(P_{L_i}^*)^2)\) is negative because the utility function \( v_i^l(P_{L_i}^l) \) is a strictly concave. Therefore, the dynamic model for updating the demand is passivity-short from input \( \bar{u}_{P_{L_i}^l} = -\lambda^l + \mu_i^L - \mu_i^{L^+} \) to output \( \bar{y}_{P_{L_i}^l} = \bar{P}_{L_i}^l \).

Subsequently, the following lemma holds with respect to the dynamic models (27) and (28) for updating the Lagrange multipliers.

**Lemma 2:** Suppose that Assumptions 1-3 hold. Then, the dynamic model for updating the Lagrange multiplier (27) is passive from input \( \bar{u}_{\mu_i^{L^+}} = \bar{P}_{L_i}^l \) to output \( \bar{y}_{\mu_i^{L^+}} = -\bar{\mu}_i^{L^+} \).

**Proof:** Consider the following positive definite function.

\[
V_{\mu_i^{L^+}} := \frac{1}{2} (\frac{\mu_i^{L^+}}{k_{\mu_i^{L^+}}})^2. \tag{33}
\]

Then, the time derivative of the above positive function along trajectory is given by:

\[
\frac{d}{dt} V_{\mu_i^{L^+}} = \frac{1}{k_{\mu_i^{L^+}}} \frac{\mu_i^{L^+}}{k_{\mu_i^{L^+}}} \frac{d}{dt} \mu_i^{L^+} = \bar{V}_{\mu_i^{L^+}} - \bar{\mu}_i^{L^+}. \tag{34}
\]

Therefore, the dynamic model for updating the Lagrange multiplier (27) is passive from input \( \bar{u}_{\mu_i^{L^+}} = \bar{P}_{L_i}^l \) to output \( \bar{y}_{\mu_i^{L^+}} = -\bar{\mu}_i^{L^+} \).

**Lemma 3:** Suppose that Assumptions 1-3 hold. Then, the dynamic model for updating the Lagrange multiplier (28) is passive from input \( \bar{u}_{\mu_i^{L^-}} = \bar{P}_{L_i}^l \) to output \( \bar{y}_{\mu_i^{L^-}} = \bar{\mu}_i^{L^-} \).

**Proof:** This proof is similar to Lemma 2 and is thus, omitted.

From the Lemma 1, the dynamic model for updating the demand is passivity-short, and from the Lemmas 2-3, the dynamic models for updating the Lagrange multipliers are passive, so the following Lemma 4 for the consumer demand decision system holds.

**Lemma 4:** Suppose that Assumptions 1-3 hold. Then, the consumer demand decision system (24)-(28) is passivity-short from input \( \bar{u}_{P_{L_i}} = -\lambda^l \) to output \( \bar{y}_{P_{L_i}} = \bar{P}_{L_i}^l \).

**Proof:** Consider the following positive definite function.

\[
V_{P_{L_i}} := V_{P_{L_i}}^l + V_{\mu_i^{L^+}}^l + V_{\mu_i^{L^-}}^l. \tag{35}
\]

Then, the time derivative of the above positive function along trajectory is given by:

\[
\frac{d}{dt} V_{P_{L_i}} = - \frac{k_i^{L_i}}{k_{PL_i}^l} |\bar{x}_{1,L_i}|^2 + \bar{P}_{L_i}^l(v_i^l(P_{L_i}^l) - v_i^l(P_{L_i}^*)^2) + \bar{P}_{L_i}^l(-\lambda^l + \mu_i^L - \mu_i^{L^+}) + \frac{k_i^{L_i}}{k_{PL_i}^l} |\bar{P}_{L_i}^l|^2. \tag{36}
\]

Therefore, the consumer demand decision system (24)-(28) is passivity-short from input \( \bar{u}_{P_{L_i}} = -\lambda^l \) to output \( \bar{y}_{P_{L_i}} = \bar{P}_{L_i}^l \).

**B. Generator supply decision system**

We describe the dynamic model for determining the supply amount generated by each area. Similar to consumers, we obtain the following equation.

\[
\begin{align*}
\min_{\lambda^l, \mu_i^{G^-}, \mu_i^{G^+}} & \quad \max_{\mu_i^{G^-}, \mu_i^{G^+}} \bar{W}_{G_i}(\cdot), \\
\bar{W}_{G_i}(\cdot) := & \bar{\lambda}^l(P_{G_i}^l - P_{G_i}) + \bar{\mu}_i^{G^-}(P_{G_i}^l - P_{G_i}^{\min}) \\
& - \bar{\mu}_i^{G^+}(P_{G_i}^l - P_{G_i}^{\max}) - \bar{\mu}_i^{G^-}(P_{G_i}^l - P_{G_i}^{\max}) - \bar{\mu}_i^{G^+}(P_{G_i}^l - P_{G_i}^{\min}) \geq 0
\end{align*}
\]

where \( \mu_i^{G^-}, \mu_i^{G^+}, \mu_i^{G^-}, \mu_i^{G^+} \geq 0 \) are Lagrange multipliers for upper and lower bounds of each power generation. Then, the dynamic model for updating supply is given by (39)-(41) and the dynamic models for updating Lagrange multipliers are given by (42), (43). Also, supply decision model base load power and renewable energy can be expressed in the same way, so we define \( G_{bi} \) or \( G_{ri} \) as \( G_{ni} \cdot i = b, r \).

\[
\begin{align*}
\frac{d}{dt} x_{1,G_{ni}} &= x_{2,G_{ni}}, \\
\frac{d}{dt} x_{2,G_{ni}} &= \lambda^l - \mu_i^{G^-} + \mu_i^{G^+}, \\
y_{P_{G_i}^l} &= P_{G_i}^l = k_{G_{ni}} x_{1,G_{ni}} + k_{P_{G_i}} x_{2,G_{ni}}, \\
\frac{d}{dt} y_{P_{G_i}^l} &= k_{P_{G_i}} x_{1,G_{ni}} + k_{P_{G_i}} x_{2,G_{ni}}, \\
\frac{d}{dt} y_{P_{G_i}^l} + P_{G_i}^{\max} &+ k_{P_{G_i}} x_{2,G_{ni}} \quad \text{are positive gains, then, the following lemma holds.}
\end{align*}
\]

**Lemma 5:** Suppose that Assumptions 1-3 hold. Then, the generator supply decision system (39)-(43) is passivity-short from input \( \bar{u}_{P_{G_i}} = -\lambda^l \) to output \( \bar{y}_{P_{G_i}} = -P_{G_i} \).

**Proof:** This proofs are similar to Lemmas 1-4 and are thus, omitted.

**C. ISO price updating system**

Finally, we consider the electricity prices updating by the ISO, which acts as market managers. Then, ISO price updating system is defined as follows:

\[
\begin{align*}
u_{\lambda} &= -(P_G - P_L), \\
\frac{d}{dt} x_{\lambda} &= -(P_G - P_L), \\
y_{\lambda} &= \lambda = k_{I_{\lambda}} x_{\lambda} + k_{P_{\lambda}} u_{\lambda}.
\end{align*}
\]
where $\lambda = [\lambda^1, \ldots, \lambda^{|A|}]^T \in \mathbb{R}^{|A|}$, and $k_{I\lambda}, k_{P\lambda}$ are positive gains. Then, the input-output property of this pricing mechanism is given by Lemma 6 below.

**Lemma 6:** Suppose that Assumptions 1-3 hold. Then, ISO price updating system (44)-(46) is strictly passive from input $\tilde{u}_\lambda = -(\tilde{P}_G - \tilde{P}_L)$ to output $\tilde{y}_\lambda = \lambda$.

**Proof:** Consider the following positive definite function.

$$V_\lambda := \sum_{l \in A} \frac{k_{I\lambda}}{2} (x_{l\lambda})^2.$$  \hfill (47)

Then, the time derivative of the above positive function along trajectory is given by:

$$\frac{d}{dt} V_\lambda = \sum_{l \in A} \left(K_{I\lambda} \frac{d}{dt} x_{l\lambda} \right)$$

$$= -k_{P\lambda} \|-(\tilde{P}_G - \tilde{P}_L)\|^2 + \lambda^T (-\tilde{P}_G + \tilde{P}_L).$$  \hfill (48)

Therefore, ISO price updating system (44)-(46) is strictly passive from input $\tilde{u}_\lambda = -(\tilde{P}_G - \tilde{P}_L)$ to output $\tilde{y}_\lambda = \lambda$.

**D. Electricity market trading system**

Finally, we analyze overall stability for the electricity market trading system considering the negative price on passivity-short as shown in Fig. 3. Note that, $D_l \in \mathbb{R}^{||A|\times|A|}$ is $|A|$ column vector, where the $l$th element is 1 and other elements are 0.

![Electricity market trading system in area l](image)

Fig. 3. Electricity market trading system in area $l$

We proved that the consumer demand decision system and the generator supply decision system are passivity-short from Lemma 4,5 and ISO price updating system is strictly passive from Lemma 6. Therefore, for the convergence of the solution derived by the electricity market trading system, the following theorem holds.

**Theorem 1:** Suppose that Assumptions 1-3 hold. Then, in the electricity market trading system with the consumer demand decision system, the generator supply decision system, and ISO price updating system as shown in Fig. 3, the solutions derived in those systems converge to the optimal solutions $P^*_L, P^*_G$ in social welfare maximization problem (8)-(13) and the optimal Lagrange multiplier $\lambda^*$ in dual problem (16)-(17).

**Proof:** From Lemma 4, the consumer demand decision system is passivity-short from input $-\lambda^*$ to output $\tilde{P}_L$. Also, from Lemma 5, the generator supply decision system is passivity-short from input $-\lambda^*$ to output $-\tilde{P}_G$. Therefore, as shown in Fig. 3, the system in which is connected in parallel is also passivity-short, and additionally, the system with multiplicity by $D_l^T, D_l$ is also passivity-short. Also, from Lemma 6, ISO price updating system is strictly passive from input $-(\tilde{P}_G - \tilde{P}_L)$ to output $\lambda$.

Therefore, in the electricity market trading system, the solutions derived in those systems converge to the optimal solutions $P^*_L, P^*_G$ in social welfare maximization problem (8)-(13) and the optimal Lagrange multiplier $\lambda^*$ in dual problem (16)-(17).

V. SIMULATION VERIFICATION

In this section, we verify the effectiveness of proposed method presented by numerical simulation.

A. Simulation conditions

In this simulation, we use the power grid model with two areas, and suppose that the respective number of consumers and generators participating in market trading are $|C_l| = 3, |G_l| = 1, \forall l \in A$. Additionally, the utility function and the cost function are set using the quadratic function: $v^l(P^L_l) = a^l_1 P^L_l + b^l_1 P^L_l^2$ and $c^l(P^G_l) = a^l_2 P^G_l + b^l_2 P^G_l^2$, and values of gains are set as follows: $k_{I1} = 1.0 \times 10^4, k_{P1} = 1.0 \times 10^{-1}, k_{I2} = 1.0 \times 10^1, k_{P2} = 1.0 \times 10^{-2}, k_{I3} = 1.0 \times 10^{-1}, k_{I4} = 1.0 \times 10^{-1}, k_{P3} = 1.0 \times 10^{-2}$, also, upper and lower bounds and coefficient parameters are set as follows: $[P^L_{G_{min}}, P^L_{G_{max}}, P^L_{L_{min}}, P^L_{L_{max}}] = [120, 200, 170], [P^G_{G_{min}}, P^G_{G_{max}}, P^G_{L_{min}}, P^G_{L_{max}}] = [250, 150, 100], [P^L_{min}, P^L_{min}, P^L_{min}, P^L_{min}] = [0, 0, 0, 0], [P^G_{max}, P^G_{max}, P^G_{max}, P^G_{max}] = [\infty, \infty, \infty, \infty], [a^l_1, a^l_2, a^l_3, a^l_4] = [-7.98 - 9.58 - 6.39] \times 10^{-2}, [b^l_1, b^l_2, b^l_3, b^l_4] = [2.80, 3.37, 2.25] \times 10^{-2}, a^l_{G_{1}, G_{1}} = 7.16 \times 10^{-2}, b^l_{G_{1}, G_{1}} = 3.02 \times 10^1, a^l_{G_{1}, G_{1}} = 7.48 \times 10^{-2}$. In addition, even when each market participant joins or withdraws trading system on the way, in order to show that it converges to the optimal demand, supply, and price according to the change in the number of participants, we set that each consumer has a different time schedule to participate in trading from the beginning. Specifically, the consumer 1, 2 and 3, respectively, participate in its trading from 0s to 60s, at all time, and from 0s to 30s and from 90s to 120s.

B. Simulation results

The results of this simulation are shown in Fig. 4-7. Fig. 4, 5 show electricity demand in each area, Fig. 6 shows electricity price $\lambda^*, l \in A$, and Fig. 7 shows power $P^L_l - P^G_l, l \in A$. In each figure, the broken line represents the optimal value obtained by solving the social welfare maximization problem.
From the simulation results, it can be confirmed that a negative price is generated when supply surplus occurs. Also, even when a negative price occurs in market transactions, the consumers determine the positive demand amount. From the results, in proposed method, the positive demand amount is can be determined not only if a positive price occurs, but also if a negative price occurs. Therefore, it can be confirmed that it is an electricity market trading system considering the occurrence of negative price. Also, even if the number of market participants participating in the market transactions changes, it can be confirmed that the solid line and the broken line coincide in each figure. Consequently, it turns out that each value converges to the optimal value obtained by the social welfare maximization problem and its dual problem.

Therefore, it is confirmed that electricity market trading system considering negative price is the plug and play operation.

In this paper, we analyzed the stability and convergence properties for the market trading system using the concept of passivity-short. Specifically, we represented the consumer demand decision system, the generator supply decision system, and the ISO price updating system as dynamic models, and analyze a stability based on the concept of passivity-short for electricity market trading system with those systems. As a result, the demand, supply and price converged to the optimal solution to maximize social welfare, and by market transactions in the proposed system, we showed that social welfare maximization was achieved in a distributed manner. Here, by analyzing the stability for electricity market trading system based on the concept of passivity-short, even in the case of a negative price, the electricity market trading system had the characteristics of plug and play operation. Finally, the effectiveness of proposed method was confirmed by numerical simulation.

REFERENCES


