Fuel Consumption Optimization for a Power-Split HEV via Gain-Scheduled Model Predictive Control

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Abstract: In this paper, we propose a fuel consumption optimization technique for a power-split hybrid electric vehicle (HEV) via gain-scheduled model predictive control. The control algorithms for the hybrid powertrain choose the power split between the internal combustion engine and electric motor of the HEV in order to minimize the fuel consumption. The operating conditions of the HEV change during driving in real time by road conditions and driver demand. Therefore, it is necessary to have optimal control in real time and to consider the time-varying motor condition. We model an HEV and propose a gain-scheduled model predictive control for a power-split HEV. Using the proposed technique, the optimal fuel consumption for a power-split HEV is achieved. Finally, the proposed approach is validated by numerical simulations.

Keywords: Gain-Scheduled Model Predictive Control, Hybrid Electric Vehicle, Optimal Control.

1. INTRODUCTION

In recent years, the number of cars has been increasing in the world. An environmental technology for cars has been developed in response to environmental concerns and problems of rising fuel cost. The hybrid electric vehicle (HEV) is provided by electric motors and an internal combustion engine and is reduced emissions and fuel consumption by combining their output[1]. The improvement of fuel economy is achieved by using surplus energy that is recovered from engine power and the regenerative energy reused, as shown in Fig. 1, and allow running a smaller combustion engine in a higher efficiency region. However, the control of the HEV with a battery and two driving forces is more complex than a conventional car.

Therefore, the control of the HEV to optimize fuel consumption is being actively studied [2][3][4][5][6]. Therein, the model predictive control [2] is believed to be an effective means. However, in previous studies, the state of the battery, which depends on the varying motor speed, cannot be simulated. The controlling the battery within its physical constraints is necessary to express the varying state of motor speed.

In this paper, we propose a fuel consumption optimization technique for a power-split HEV as shown in Fig. 2 via gain-scheduled model predictive control [7]. We make possible an optimal control for a variable state system and represent the varying state by combining gain-scheduled model with conventional model predictive control. Thus, it is possible to optimize fuel consumption for an HEV considering not only the state of the engine, EM1, and EM2 but also the state of the battery. The scheduled parameter used in the gain-scheduled control is obtained by linear interpolation between the minimum and maximum of the EM1 and EM2 rotational speed.

At first, we model a power-split HEV and describe about the general model predictive control. Then the gain-scheduled model predictive control proposes. Finally, the proposed control law is validated by a numerical simulation.

2. PROBLEM SETTING

2.1. Fuel Economy Optimization Problem

In this paper, the fuel economy optimization problem is considered for the optimal distribution of power in the HEV powertrain to maximize fuel economy and combustion efficiency. This problem needs to consider not only the optimization of fuel consumption but also the constraint of the battery state, drivability.

2.2. HEV Modeling

In this section, we consider the model of a power-split HEV as shown in Fig. 2.
2.2.1. Vehicle Dynamics
The motion of the HEV is formulated as
\[ M \ddot{v}(t) = \tau_d(t) - \tau_a(t) - \tau_f(t) - \tau_g(t) \] (1)
where
\[ \tau_d(t) = \frac{\tau_d(t)}{R_w} \] (driving force),
\[ \tau_a(t) = \frac{1}{2} \rho A_f C_d v^2(t) \] (air resistance),
\[ \tau_f(t) = \mu M g \cos(\theta(t)) \] (rolling resistance),
\[ \tau_g(t) = M g \sin(\theta(t)) \] (gradient resistance),
here \( M \) [kg] is the vehicle weight and \( R_w \) [m] is the tire radius, \( \rho \) [kg/m\(^3\)] is the air density, \( C_d \) is the air resistance coefficient, \( A_f \) [m\(^2\)] is the vehicle frontal projected area, \( \mu \) is the rolling resistance coefficient, \( g \) [m/s\(^2\)] is the gravitational acceleration, \( \theta \) [rad] is the road gradient, \( v \) [m/s] is the vehicle speed, \( \tau_d \) [Nm] is the driver-demanded torque, \( \tau_s \) [s] is time. The driver-demanded torque, which is a measurable disturbance to the car, becomes Eq. (6) based on Eq. (1).
\[ \tau_d(t) = R_w(M \ddot{v}(t) + \frac{1}{2} \rho A_f C_d v^2(t) + M g \cos(\theta(t)) + M g \sin(\theta(t))) \] (6)

2.2.2. Powertrain Dynamics
We formulate the equation of state of EM1, EM2, and the engine to configure the powertrain of the HEV. EM1, EM2, and the engine are connected as shown in Fig. 2 by planetary gears. The relationships among the speed of EM1, EM2, and the engine are connected as shown in Fig. 2.
\[ \dot{\omega}_{em1}(t) = \tau_{em1}(t) + f_p(t) R_s, \] (8)
\[ \dot{\omega}_{em2}(t) = \tau_{em2}(t) - \frac{\tau_d(t)}{G_f} + f_p(t) R_r, \] (9)
\[ \dot{\omega}_{eng}(t) = \tau_{eng}(t) - f_p(t)(R_s + R_r), \] (10)
where \( I_{em1} \) [kg \cdot m\(^2\)], \( I_{em2} \) [kg \cdot m\(^2\)], and \( I_{eng} \) [kg \cdot m\(^2\)] are inertia moments of EM1, EM2, and the engine, \( G_f \) is the differential gear ratio, \( f_p \) [N] is the interaction force caused by contact with the planetary gear, \( \tau_{em1} \) [Nm], \( \tau_{em2} \) [Nm], and \( \tau_{eng} \) [Nm] are EM1, EM2, and engine torque, respectively. \( f_p(t) \) [N] is expressed in Eq. (11) as follows from Eqs. (7) and (8).
\[ f_p(t) = \frac{I_{em1}}{R_s^2} [(R_r + R_s) \dot{\omega}_{eng}(t) - R_r \dot{\omega}_{em2}(t)] - \frac{\tau_{em1}(t)}{R_s} \] (11)

The motion equations for EN2 and engine are formulated in Eq. (12) as follows from Eqs. (9), (10), and (11).
\[ \begin{pmatrix} I_{em2} & R_s^2 \frac{R_r + R_s}{R_r} \frac{R_s}{R_r} I_{em1} - R_s^2 \frac{R_r + R_s}{R_r} I_{em1} \\
\frac{R_r}{R_r^2} \frac{R_s}{R_r} I_{em2} & I_{em2} + \left( \frac{R_r}{R_r^2} \right)^2 I_{em1} \end{pmatrix} \begin{pmatrix} \omega_{em2}(t) \\
\omega_{em1}(t) \end{pmatrix} = \begin{pmatrix} \tau_{em2}(t) \\
\tau_{em1}(t) \end{pmatrix} + \tau_d(t) \] (12)

2.2.3. Battery Dynamics
The battery needs to be controlled within its constraints to prevent over-charging or over-discharging. The state of charge is \( \text{soc} \ [%] \).
\[ \text{soc}^{\text{min}} \leq \text{soc}(t) \leq \text{soc}^{\text{max}} \] (13)
The state equation for the battery can be given as (8)[9]
\[ \dot{\text{soc}}(t) = - \frac{V_{soc} \sqrt{V_{soc}^2 - 4 p_{batt}(t) R_{batt}}}{2 C_{batt} R_{batt}} \] (14)
where \( C_{batt} \) [Ah] is the battery capacity, \( p_{batt} \) [W] is the battery power, \( V_{soc} \) [V] is the open circuit voltage, and \( R_{batt} \) [\( \Omega \)] is the battery resistance. The battery power in Eq. (15) is a function of the rotational speed and torque. Here the efficient of battery charge and discharge are not considered.
\[ p_{batt}(t) = \tau_{em1}(t) \omega_{em1}(t) + \tau_{em2}(t) \omega_{em2}(t) \] (15)
Eqs. (14) and (15) represent the relationship between \( \dot{\text{soc}} \) and \( p_{batt} \) in Fig. 3. Therefore, the equation for the battery state can be formulated as Eq. (17), which can be considered from the approximate straight line in Fig. 3.
\[ \dot{\text{soc}}(t) = S_o p_{batt}(t) \] (16)
\[ \text{soc}(t) = S_o \text{soc}_{em1}(t) + \omega_{em1}(t) + \omega_{em2}(t) \] (17)
where \( S_o \) is the battery coefficient. This equation for the battery state has time-varying states, which are \( \omega_{em1}(t) \) and \( \omega_{em2}(t) \).

2.2.4. Fuel Consumption Model
We next consider the fuel consumption model. The instantaneous fuel consumption \( \dot{m}_f \) [g/s] is expressed in Fig. 4 from the relationship between torque and speed. Then, we consider the approximate straight line passing through the origin in Fig. 4. This line can express as
\[ \dot{m}_f(t) = E_0 \tau_d(t) \] (18)
where \( E_0 \) is the fuel consumption coefficient, which is average of slope on the each engine rotational speed.
2.2.5. State Space Representation

We show a state-space representation of the HEV by using the equations derived in the previous section. The $B$ matrix is time-varying because the battery has a time-varying state.

$$\dot{x}(t) = Ax(t) + Bu(t) + B_d(t) \quad \text{(19)}$$
$$y(t) = Cx(t) + Du(t) \quad \text{(20)}$$

subject to

$$u_{\text{min}} \leq u(t) \leq u_{\text{max}} \quad \text{(21)}$$
$$y_{\text{min}} \leq y(t) \leq y_{\text{max}} \quad \text{(22)}$$

where

$$x(t) = \begin{bmatrix} \omega_{\text{eng}}(t) \\ \omega_{\text{em}2}(t) \\ m_f(t) \\ \text{soc}(t) \end{bmatrix}, \quad y(t) = \begin{bmatrix} \omega_{\text{eng}}(t) \\ \omega_{\text{em}2}(t) \\ m_f(t) \\ \text{soc}(t) \end{bmatrix}, \quad d(t) = \tau_d(t),$$

$$u(t) = \begin{bmatrix} \tau_{\text{eng}}(t) \\ \tau_{\text{em}1}(t) \end{bmatrix}, \quad A = \alpha \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B(t) = \alpha \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{R_e}{R_{\text{em}2}} & -\frac{R_e}{R_{\text{em}2}} & 0 & 0 \\ 0 & 0 & \omega_{\text{em}1}(t) & \omega_{\text{em}2}(t) \end{bmatrix},$$

$$\dot{\mathbf{B}} = \alpha \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\alpha = \begin{bmatrix} I_{\text{eng}} + \left(\frac{R_e}{R_{\text{em}1}} + \frac{R_e}{R_{\text{em}2}}\right)^2 I_{\text{em}1} - \frac{R_e}{R_{\text{em}1}} I_{\text{em}1} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1}.$$

subject to

$$\dot{x}(t) = Ax(t) + Bu(t) + B_d(t) \quad \text{(24)}$$
$$y(t) = Cx(t) + Du(t) \quad \text{(25)}$$

$$u_{\text{min}} \leq u(t) \leq u_{\text{max}} \quad \text{(26)}$$
$$y_{\text{min}} \leq y(t) \leq y_{\text{max}} \quad \text{(27)}$$

$$t \geq 0, \quad \Delta t > 0 \quad \text{(28)}$$

2.3. Control Purpose

The cost function can be formulated as the quadratic equation that minimizes the deviation of the target value and state.

$$\min_u \int_t^{t+\Delta t} (y(\tau) - r(\tau))^T Q(y(\tau) - r(\tau)) + u(\tau)^T R u(\tau) d\tau \quad \text{(23)}$$

subject to

$$J = \sum_{i=0}^{H_p} \|y(k+i) - \tilde{r}(k+i)\|^2_Q$$
$$+ \sum_{i=0}^{H_s} \|\Delta \tilde{u}(k+i)\|^2_R \quad \text{(29)}$$
\[ \hat{x}(k + 1 + i) = A\hat{x}(k + i) + B\hat{u}(k + i) + \tilde{B}\hat{d}(k + i) \quad (30) \]
\[ \hat{y}(k + i) = C\hat{x}(k + i) + D\hat{u}(k + i) \quad (31) \]
\[ y_{\text{min}} \leq \hat{y}(k + i) \leq y_{\text{max}} \quad (32) \]
\[ u_{\text{min}} \leq \hat{u}(k + i) \leq u_{\text{max}} \quad (33) \]
\[ \Delta \hat{u}(k + i) = 0 \quad \text{for} \quad i = H_u + 1, \ldots, H_p \quad (34) \]

where \( H_u \) is the control horizon, \( H_p \) is the predictive horizon, \( \hat{r} \) is the reference trajectory, \( \hat{y} \) is the predictive output, \( k \) is the current time, \( i \) is the step, \( Q, R \) are weights, \( \hat{u} \) is the predictive control input, and \( \Delta \hat{u} \) is expressed by \( \Delta \hat{u}(k + i) = \hat{u}(k + i) - \hat{u}(k + i - 1) \).

### 3.2. Predictive Output

The predictive state and predictive input is given as
\[ \dot{x}(k + 1) = Ax(k) + Bu(k) + \tilde{B}\hat{d}(k) \]
\[ \vdots \]
\[ \dot{x}(k + H_p) = AH_p x(k) + A^{H_p - 1} Bu(k) + \cdots + B u(k + H_p - 1) + A^{H_p - 1} \tilde{B}\hat{d}(k) + \cdots + B\hat{d}(k + H_p - 1) \]
\[ \hat{u}(k) = \Delta \hat{u}(k) + u(k - 1) \]
\[ \vdots \]
\[ \hat{u}(k + H_u - 1) = \Delta \hat{u}(k + H_u) + \cdots + u(k - 1) \quad (38) \]

The predicted output is determined by substituting the estimated input and the predicted state from Eq. (39). The predictive output from \( \Delta U(k) = [\Delta \hat{u}(k) \cdots \Delta \hat{u}(k + H_u - 1)]^T \) is formulated as
\[ Y(k) = \Psi x(k) + \Upsilon u(k - 1) + \Theta \Delta U(k) + \Xi D_m(k) \quad (39) \]

subject to
\[ Y(k) = \begin{bmatrix} \hat{y}(k + 1) \\ \vdots \\ \hat{y}(k + H_p) \end{bmatrix}, D_m(k) = \begin{bmatrix} \hat{d}(k) \\ \vdots \\ \hat{d}(H_p - 1) \\ \hat{d}(H_p) \end{bmatrix} \]
\[ \Psi = C \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^{H_p} \end{bmatrix}, \Upsilon = C \begin{bmatrix} \sum_{i=0}^{H_p - 1} A^i B \\ \vdots \\ \sum_{i=0}^{H_p - 1} A^{H_p - i} B \end{bmatrix} \]
\[ \Theta = C \begin{bmatrix} B \\ AB + B \\ \vdots \\ \sum_{i=0}^{H_p - 1} A^i B \\ \sum_{i=0}^{H_p - 1} A^{H_p - i} B \end{bmatrix} \]
\[ \Xi = C \begin{bmatrix} \tilde{B} \\ A\tilde{B} \\ \tilde{B} \\ \vdots \\ A^{H_p - 1}\tilde{B} \\ A^{H_p - 2}\tilde{B} \end{bmatrix} \]

### 3.3. Quadratic Programming Problem

First \( G \) and \( \Phi \) are given by Eqs. (40) and (41) as
\[ G = 2\Theta^T Q e(k) \]
\[ \Phi = \Theta^T Q \Theta + R \]

where \( e(k) \) is the error in the predicted output and the reference trajectory. The quadratic programming problem is formulated as in Eqs. (42) and (43) with \( G \) and \( \Phi \).
\[ \min_{\Delta U(k)} \frac{1}{2} \Delta U^T(k) G \Delta U(k) - \frac{1}{2} \Delta U^T(k) \Phi \Delta U(k) + \Omega \Delta U(k) \leq \omega \]

The optimal \( \Delta U^*(k) \) is obtained by calculating Eqs. (42) and (43). The prediction control at the current time becomes \( \Delta \hat{u}^*(k) \). Then, the optimal solution at the current time is determined by Eq. (44) in accordance with the previous inputs.
\[ u(k) = u(k - 1) + \Delta \hat{u}^*(k) \quad (44) \]

The current optimal control value is inputted to the control target.

### 4. GAIN-SCHEDULED MODEL PREDICTIVE CONTROL

The general model predictive control is formulated in the previous section, but it is not possible to handle the time-varying state of the battery. Therefore, we propose a novel controller using gain-scheduled model predictive control [7]. We represent the time-varying state by scheduling the gain and (43). The prediction control at the current time becomes \( \Delta \hat{u}^*(k) \). Then, the optimal solution at the current time is determined by Eq. (44) in accordance with the previous inputs.

At first, we decompose \( B \) into 4 matrices, as in Eqs. (45)~(48), including the maximum and minimum values of EM1 and EM2 rotation speed. The \( B \) matrix is expressed as \( B^0_{\text{em}}(a=\text{low} \text{ or } \text{high}, b=\text{em}1 \text{ or } \text{em}2) \).
The four state space representations are utilized for the model predictive control. Then, the optimization problem is calculated by adding up the each quadratic programming problem that multiplied the scheduled parameters. The optimal solution is calculated by Eq. (44).

\[
\min_{\Delta U(k)} \frac{1}{2} \left[ \lambda_{low}^{em1} \frac{1}{2} U^T(k) \Phi_{low}^{em1} \Delta U(k) - G_{low}^{em1} \Delta U(k) \right] + \lambda_{high}^{em1} \frac{1}{2} U^T(k) \Phi_{high}^{em1} \Delta U(k) - G_{high}^{em1} \Delta U(k) \right] + \lambda_{low}^{em2} \frac{1}{2} U^T(k) \Phi_{low}^{em2} \Delta U(k) - G_{low}^{em2} \Delta U(k) \right] + \lambda_{high}^{em2} \frac{1}{2} U^T(k) \Phi_{high}^{em2} \Delta U(k) - G_{high}^{em2} \Delta U(k) \right] \] (49)

subject to \( \Omega \Delta U(k) \leq \omega \) (50)

4.1. Linear Interpolation of the Scheduled Parameters

The scheduled parameters \( \lambda_{low}^{em1}, \lambda_{high}^{em1}, \lambda_{low}^{em2}, \lambda_{high}^{em2} \) are calculated by the actually measured \( \omega_{em1}(t) \), \( \omega_{em2}(t) \). The scheduled parameters are defined near of the maximum rotational speed and the minimum rotational speed as

\[
\omega_{low}^{em1} - \delta \leq \omega_{em1}(t) \leq \omega_{low}^{em1} + \delta, \lambda_{low}^{em1} = 1, \lambda_{high}^{em1} = 0 \] (51)

\[
\omega_{high}^{em1} - \delta \leq \omega_{em1}(t) \leq \omega_{high}^{em1} + \delta, \lambda_{low}^{em1} = 0, \lambda_{high}^{em1} = 1 \] (52)

\[
\omega_{low}^{em2} - \delta \leq \omega_{em2}(t) \leq \omega_{low}^{em2} + \delta, \lambda_{low}^{em2} = 1, \lambda_{high}^{em2} = 0 \] (53)

\[
\omega_{high}^{em2} - \delta \leq \omega_{em2}(t) \leq \omega_{high}^{em2} + \delta, \lambda_{low}^{em2} = 0, \lambda_{high}^{em2} = 1 \] (54)

where \( \delta \) is an acceptable error. The scheduled parameters in between the maximum rotational speed and the minimum rotational speed are considered by using a linear interpolation as shown in Fig. 5.

\[
\lambda_{high}^{em1}(t) = \frac{\omega_{em1}(t) - \omega_{low}^{em1}}{\omega_{high}^{em1} - \omega_{low}^{em1} - 2\delta} \] (55)

\[
\lambda_{low}^{em1}(t) = 1 - \lambda_{high}^{em1}(t), \] (56)

\[
\lambda_{high}^{em2}(t) = \frac{\omega_{em2}(t) - \omega_{low}^{em2}}{\omega_{high}^{em2} - \omega_{low}^{em2} - 2\delta} \] (57)

\[
\lambda_{low}^{em2}(t) = 1 - \lambda_{high}^{em2}(t). \] (58)

5. SIMULATION

5.1. Simulation Condition

The proposed approach is validated by numerical simulations. A software application called GT-suite in Matlab Simulink is used to simulate. The target vehicle speed as shown in Fig. 6. The driving environment does not consider traffic congestion, weather, or the grade of the roadway. Case 1 is the result by the proposed method, and Case 2 is the result by Rule Base.

\[
R = diag(1.0 \times 10^{-5}, 1.0 \times 10^{-5}, 1.0 \times 10^{-5}) \] (59)

\[
Q = diag(1, 1, 1) \] (60)

5.2. Simulation Result

The simulation results of Case 1 and Case 2 are indicated in Figs. 8~15. The actual vehicle speed is intended to follow the target vehicle speed in both cases. The SOC is controlled within the constraints. By the proposed method, the engine is operated at the optimal point.
The fuel economy and soc of Case 1 and Case 2 are indicated in Table 2. Case 1 improved 5.11% over Case 2. The soc is almost the same in both cases.

6. CONCLUSIONS

In this paper, we have proposed the fuel consumption optimization for a power-split HEV via gain-scheduled model predictive control. The proposed controller can improve fuel economy over the Rule Base and express the time-varying battery state. Also, the vehicle speed and the soc is considered by the controller. But in the case, we don’t consider about the engine characteristic. These factors and the start time of the engine should be considered in the future.

REFERENCES