Abstract—This paper deals with the Simultaneous Localization and Mapping (SLAM) problem via the $H_{\infty}$ filter with compensations for intermittent observations. In order to estimate positions of a robot and landmarks accurately under intermittent observations, we propose the method with a novel filter which detects and compensates intermittent observations by comparing the obtained data with their estimations. This paper also shows the convergence of the estimated error covariance matrices. With simulation and experimental results, we confirm that the state of the robot and the environmental conditions are estimated accurately via the proposed filter under intermittent observations and that the derived theorems of the convergence are correct.

I. INTRODUCTION

Recently, as the use of and the expectations for autonomous mobile robots have increased, the simultaneous localization and mapping (SLAM) problem has been studied more actively, and many solutions have been proposed [1]. In the SLAM problem, a robot estimates its own position and environmental conditions with observation data of feature points such as remarkable landmarks. However, because of obstacles between the robot and landmarks or the limit of the sensor range, the robot sometimes cannot obtain proper observations and makes the estimation accuracy become worse.

Such an intermittent observation problem has been studied not only in the SLAM problem but also in various fields. The paper [2] analyzed the state estimation with the Switching Kalman Filter (SKF) under the condition in which the observation data becomes intermittently. Also, the paper [3] dealt with a detection and compensation for intermittent observations in a sensor network problem. The SLAM problem considering this problem is described in [4]. According to this paper, with the SKF, the error covariance matrix is bounded even if there are intermittent observations. However, this thesis does not describe the way to detect intermittent observations or the observations are lost partially. In addition, the noise is assumed to be White Gaussian noises to use the Kalman filter. Meanwhile, a method for the SLAM problem via the $H_{\infty}$ filter is proposed in [5], [6]. The $H_{\infty}$ filter only needs to assume that the energies of noises are bounded. However, these methods do not consider the intermittent observation problem.

Hence, in this paper, we propose the solution via the $H_{\infty}$ filter with compensations for intermittent observations. The proposed method detects intermittent observations by comparing the obtained observation data and their estimations in each step and compensates with a diagonal matrix whose components are switched between 0 and 1. Deriving the filter gain with this matrix, the proposed method can update their estimations without the effects of intermittent observations. As a result, the estimation accuracy is prevented to become worse.

In the following sections, at first, the SLAM problem model considering intermittent observations is presented. Then, we propose the novel algorithm in the SLAM problem with the detection and compensation for intermittent observations. Next, the convergence of the estimated error covariance matrix is proven. Finally, we show the simulation and experimental results, and discuss the effectiveness of the proposed method.

II. PROBLEM FORMULATION

Figures 1 and 2 show the system configuration and the general model of the SLAM problem, respectively. We consider that a mobile robot moves in the $X$-$Y$ plane in which there are $M$ unknown landmarks. Then, the robot estimates its own position and positions of these landmarks simultaneously with the intermittent observation data which also contain uncertainties. The SLAM problem is represented by two models; the process model and the observation model.

The process model represents the state transition of the robot and the landmarks. We define $\mathbf{x}_R := [\theta_R \ x_R \ y_R]^{T} \in \mathbb{R}^{3}$ as the robot state vector at a certain time $k$ which consists of

- $\theta_R$: robot orientation
- $x_R$: robot $x$-coordinate
- $y_R$: robot $y$-coordinate

Hence, in this paper, we propose the solution via the $H_{\infty}$ filter with compensations for intermittent observations. The proposed method detects intermittent observations by comparing the obtained observation data and their estimations
its attitude angle and its $X$-$Y$ position, $p_i := [x_i, y_i]^T \in \mathbb{R}^2$ as the $X$-$Y$ position of the $i$th landmark and $p_{all} := [p_1^T, p_2^T, \ldots, p_M^T]^T \in \mathbb{R}^{2M}$ as the summarized state of all $M$ landmarks. Here, assuming that landmarks are stationary, with a straight-line approximation by Euler’s method, the update equation of the whole state of the system, $x_k := [x_{R_k}^T, p_{all}^T]^T \in \mathbb{R}^{3+2M}$, is represented as follows.

$$
x_{k+1} = f(x_k, v_k, o_k) + e_1_k 
$$

$$
f(x_k, v_k, o_k) := \begin{bmatrix} 
\theta_{R_k} + T o_k \\
\frac{x_{R_k} + T v_k \cos\theta_{R_k}}{y_{R_k} + T v_k \sin\theta_{R_k}} \\
\end{bmatrix} \tag{1}
$$

$$
\text{where } f \in \mathbb{R}^{3+2M} \text{ is a non-linear function that defines the state transition, and } v_k, o_k \text{ and } T \text{ are the robot velocity input, turning rate input and the sampling time, respectively. Meanwhile, since landmarks are stationary, there is no state transition of the landmarks positions. In (1), } e_1_k \in \mathbb{R}^{3+2M} \text{ is the process noise whose mean and covariance are } 0 \text{ and } Q_k \in \mathbb{R}^{(2M+3) \times (2M+3)}.
$$

Next, the observation model calculates and measures the observation. In the same equation, $\eta_k \in \mathbb{R}^n$ is the improper observed value caused by an intermittent observation. In order to distinguish the symbols used in the general models from the other symbols, we use a different typeface such as $x_k$ in the general model.

$$
x_{k+1} = F_k x_k + G_k w_k
$$

$$
y_k = E_k h_k x_k + v_k + (I - E_k) y_k' \tag{9}
$$

where $x_k \in \mathbb{R}^n$ is the state vector, $y_k \in \mathbb{R}^p$ is the observation vector, and $F_k \in \mathbb{R}^{n \times n}$, $G_k \in \mathbb{R}^{n \times r}$ and $H_k \in \mathbb{R}^{p \times n}$, respectively. Here, $E_k \in \mathbb{R}^{n \times n}$ is a diagonal matrix and its components follow Bernoulli process. In addition, $w_k \in \mathbb{R}^r$ and $v_k \in \mathbb{R}^p$ are the noise vectors which satisfy $\sum_{k=0}^N \|w_k\|^2 < \infty$, $\sum_{k=0}^N \|v_k\|^2 < \infty$ for a given $N$. Consider to estimate linear combinations of the state vector $x_k = E_k x_k$ based on the observation data. The $H_\infty$ filtering problem is the problem of finding the estimate $\hat{z}_k = \hat{z}_k$ which satisfies the conditional equation (11) for $\gamma > 0$.

$$
\sup_{x_0, e_1, e_2} \sum_{k=0}^N \|z_k - \hat{z}_k\|^2 < \gamma^2 
$$

where $\|x\|_p^2$ represents $x^T P x$, and $P_0 > 0$, $Q_k > 0$ and $R_k > 0$ are the weighting matrices for the initial state $x_0$, bounded energy noises $w_k$ and $v_k$. Equation (11) means that the ratio of the energy of the estimated error and those of the noises is smaller than a certain value $\gamma^2$ for any bounded energy noises.

Here, we set the following assumption on the weighting matrix of the observation noise $R_k$.

**Assumption 2:** the weighting matrix of observation noise $R_k$ is a diagonal matrix.

Then, the following lemma holds on the existence condition of the $H_\infty$ filter considering intermittent observations.

**Lemma 1:** Suppose that Assumption 2 holds. Considering the $H_\infty$ filtering problem for a system with intermittent observations represented in (9) and (10), the sufficient and necessary condition for the existence of a unique solution to the problem is to satisfy the following two conditions.

1. The Riccati equation, (12), has a positive definite solution.

$$
P_{k+1} = F_k P_k F_k^T + G_k^T Q_k G_k - 0 \tag{12}
$$

$$
\Psi_k := I + (H_k^T R_k^{-1} E_k H_k - \gamma^2 L_k^T L_k) P_k \tag{13}
$$

2. Equation (14) has a positive definite solution.

$$
P_k^{-1} + H_k^T R_k^{-1} E_k H_k - \gamma^2 L_k^T L_k > 0, \ k = 0, 1, \ldots, N \tag{14}
$$

**Proof:** Since this can be proven with almost the same way as the the conventional $H_\infty$ filter described in [7], the detailed proof is omitted in this paper.

Next, consider to apply this proposed filter to the SLAM problem.
Assumption 3: Process noise $\epsilon_{1k}$ and observation noise $\epsilon_{2k}$ are independent of each other. In addition, they are bounded deterministic noises which satisfy the following equation.

$$\sum_{k=0}^{N} \| \epsilon_{1k} \|^2 < \infty, \quad \sum_{k=0}^{N} \| \epsilon_{2k} \|^2 < \infty$$  \hspace{1cm} (15)

Since the models in the SLAM problem are represented by non-linear functions as described in the previous section, we linearize the models with Taylor expansion in such a way to the solution in [8] that uses the extended Kalman filter. Here, $F_k$ and $H_k$ denote the Jacobian matrices of the state transfer function $f$ and observation function $h$, respectively. Their components are calculated as follows with the values estimated by the filter described after.

$$F_k := \frac{\partial f(x,u)}{\partial x} = \begin{bmatrix} F_v & 0_{2M \times 3} & 0_{3 \times 2M} \end{bmatrix}$$  \hspace{1cm} (16)

$$H_k := \frac{\partial h(x)}{\partial x} = [H_v \mid H_p]$$  \hspace{1cm} (17)

where

$$F_v := \begin{bmatrix} 1 & 0 & 0 \\ -v_k \cos \theta_k & 1 & 0 \\ -v_k \sin \theta_k & 0 & 1 \end{bmatrix}$$  \hspace{1cm} (18)

$$H_v := \begin{bmatrix} -H_{1v}^T & -H_{2v}^T & \cdots & -H_{Mv}^T \end{bmatrix}, \quad H_p = [e \ A_i]$$  \hspace{1cm} (19)

$$\epsilon := \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad A_i := \begin{bmatrix} \frac{d\theta_i}{dt} & \frac{d\gamma_i}{dt} & \frac{dr_i}{dt} \end{bmatrix} (i = 1, 2, \cdots, M)$$  \hspace{1cm} (20)

From (16) and (18), the rank of $F_k$ is $2M + 3$ in every step which is the dimension of the state. Then, from Lemma 1, the following theorem is obtained for the condition to exist a unique solution to the $H_{\alpha}$ filter considering intermittent observations applied to the SLAM problem.

**Theorem 1:** If Assumption 1 and 2 hold, the sufficiency and necessary condition for the existence of a unique solution to the $H_{\alpha}$ filter considering intermittent observations applied to the SLAM problem is to satisfy (22).

$$P_k^{-1} + H_k^T R_k^{-1} E_k H_k - \gamma^2 I > 0, \quad k = 0, 1, \cdots, N$$  \hspace{1cm} (22)

**Proof:** First, both $G_k$ and $L_k$ are identity matrices in the SLAM problem. Hence, if a unique solution exists, (12) and (14) are satisfied. Therefore, the condition (22) is also satisfied. Then, the necessity of this condition is proved. Next, consider its sufficiency. Substituting $\Psi_k$ as (13) into (12), the Riccati equation, we obtain the following.

$$P_{k+1} = F_k P_k F_k^T + Q_k$$  \hspace{1cm} (23)

$$K_k = P_k H_k^T (E_k H_k P_k H_k^T + R_k)^{-1} E_k$$  \hspace{1cm} (30)

$$P_{k+1} = F_k P_k F_k^T + Q_k$$  \hspace{1cm} (24)

$$\Psi_k := I + (H_k^T R_k^{-1} E_k H_k - \gamma^2 I) P_k$$  \hspace{1cm} (32)

In addition to the conventional $H_{\alpha}$ filter algorithm, this proposed algorithm detects intermittent observations and determines the value of $\eta_k$ by using the difference $|r_k|$ between the relative distance $r_k$ and its estimation $\hat{r}_k$ with the threshold $\hat{r}_{lim}$ in step 3. Then, updating the estimations of the states using the gain derived with $E_k$ in step 4, the proposed method prevents from becoming the estimation accuracy bad.

**B. Proposed filtering algorithm in SLAM**

In this subsection, the proposed $H_{\alpha}$ filter algorithm applied to the SLAM problem is described. With the Jacobian matrices $F_k$ and $H_k$, the $H_{\alpha}$ filter considering intermittent observations which estimates the states of the robot and landmarks is given in the following 4 recursive steps. Here, $\hat{r}_{k+1|k}$ and $\hat{r}_{k+1|k+1}$ represent the priori and posteriori estimated value, respectively.

**Step1:** Prediction

$$\hat{x}_{k+1|k} = f(\hat{x}_{k|k}, u_k), \quad \hat{y}_{k+1|k} = h(\hat{x}_{k+1|k})$$  \hspace{1cm} (25)

**Step2:** Observation

$$\hat{y}_{k+1|k} = y_{k+1} - \hat{y}_{k+1|k}$$  \hspace{1cm} (26)

**Step3:** Detection and Compensation

$$E_{k+1} = \text{block diag}\{\eta_{1k+1}, \eta_{2k+1}, \cdots, \eta_{Mk+1}\}$$  \hspace{1cm} (27)

$$\eta_{k+1} = \begin{bmatrix} I_2 \{ |r_{k+1} | \leq \hat{r}_{lim} \} \\ 0_2 \{ |r_{k+1} | > \hat{r}_{lim} \} \end{bmatrix}, \quad r_{k+1} := r_{k+1} - \hat{r}_{k+1}$$  \hspace{1cm} (28)

**Step4:** Update

$$\hat{x}_{k+1|k} = \hat{x}_{k+1|k} + K_{k+1} \hat{y}_{k+1|k}$$  \hspace{1cm} (29)

where $K_{k+1}$ is the filter gain and the estimated state covariance matrix $P_k$ are given as follows.

$$K_k = P_k F_k^T (E_k F_k P_k F_k^T + R_k)^{-1} E_k$$  \hspace{1cm} (30)

$$P_{k+1} = F_k P_k F_k^T + Q_k$$  \hspace{1cm} (31)

$$\Psi_k := I + (H_k F_k^T E_k H_k - \gamma^2 F_k^T) P_k$$  \hspace{1cm} (32)

In the conventional $H_{\alpha}$ filter algorithm, this proposed algorithm detects intermittent observations and determines the value of $\eta_k$ by using the difference $|r_k|$ between the relative distance $r_k$ and its estimation $\hat{r}_k$ with the threshold $\hat{r}_{lim}$ in step 3. Then, updating the estimations of the states using the gain derived with $E_k$ in step 4, the proposed method prevents from becoming the estimation accuracy bad.

**IV. CONVERGENCE PROPERTY**

The convergence of the estimated state error covariance matrix $P_k$ represents the confidence of the estimation [9]. Here, while a robot moves, the Jacobian matrices $F_k$ and $H_k$ are time-varying. Furthermore, since these Jacobian matrices are calculated with estimated values, they change minutely even if the robot is stationary. However, in order to confirm the convergence of our proposed system, we assume that the Jacobian matrix of the state transition function $F_k = I$ and that the covariance of the process noise $Q_k = 0$ of a stationary robot. With this assumptions, we prove the convergence of the error covariance matrix. We define $P_k^0$ as the value of the estimated error covariance matrix at time $k$ when a mobile robot stops and $P_k$ as the value of that at the time when observation has been made $i$ times after a robot stops. Moreover, $W_k$ is defined as follows to simplify equations.

$$W_k := H_k^T R_k^{-1} E_k H_k - \gamma^2 I$$  \hspace{1cm} (33)

Then, when the robot is stationary, the inverse matrix of the error covariance matrix is update with $W_k$ in (33) as follows with the assumptions described before.

$$P_{k+1} = \{F_{k+1}(P_{k+1} + W_{k+1})^{-1} F_{k+1} + Q_{k+1}\}^{-1}$$

388
\[ P_{k+1}^{-1} + W_{k+1} \]  

From the above equation, the inverse of the error covariance matrix when the stationary robot made observations for \( n \) times is represented as follows.

\[ P_k^{-1} = P_k^{0-1} + \sum_{j=0}^{n-1} W_{k+j} \]  

(35)

Meanwhile, we also assume that, if the stationary robot obtains proper observation data, the Jacobian matrices of the observation function, \( H_{k+j} \), are constant in each observation. Then, the following equation holds with the probability \( p_1 (0 < p_1 \leq 1, \ i = 1, 2, \cdots , M) \) represented in (6) and the number of observations, \( n \).

\[ \sum_{j=0}^{n-1} W_{k+j} = \sum_{j=0}^{n-1} (H_{k+j}^T R_{k+j}^{-1} E_{k+j} H_{k+j} - \gamma^{-2} I) \]

\[ = n (H_k^T R_k^{-1} E_k H_k - \gamma^{-2} I) \]  

(36)

\[ E^* := \text{block diag}\{ p_1 I_2, p_2 I_2, \cdots , p_M I_2 \} \]  

(37)

With these equations, the following lemma holds on the determinant of the error covariance matrix when the robot is stationary.

**Lemma 2:** Suppose that Assumptions 1–3 hold and we also assume that the stationary robot continues to observe landmarks. Then, the determinant of the error covariance matrix decreases with its update if the following equation (38) is satisfied.

\[ W^* := H_k^T R_k^{-1} E_k^* H_k - \gamma^{-2} I > 0 \]  

(38)

**Proof:** With (35) and (36), the error covariance matrix \( P_k^e \) is represented with \( W^* \) as follows.

\[ P_k^e = (P_k^{0-1} + \sum_{j=0}^{n-1} W_{k+j})^{-1} \]

\[ = \left\{ P_k^{0-1} + n (H_k^T R_k^{-1} E_k^* H_k - \gamma^{-2} I) \right\}^{-1} \]

\[ = (P_k^{0-1} + nW^*)^{-1} \]  

(39)

Here, if (38) is satisfied, the following equation holds on its determinant \( |P_k^e| \).

\[ |P_k^e| = |P_k^{0-1} + nW^*|^{-1} < |P_k^{0-1}|^{-1} = |P_k^0| \]  

(40)

Therefore, if the stationary robot continues its observation and (38) is satisfied, the determinant of the error covariance matrix decreases with its updating. \( \blacksquare \)

Here, define the initial error covariance matrix when the robot is stationary, \( P_k^{0-1} \), and the sub matrices of \( W^* \) as follows.

\[ P_k^{0-1} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \]  

(41)

\[ W^* = \begin{bmatrix} H_k^T R_k^{-1} E_k^* H_k - \gamma^{-2} I_3 \\ H_k^T R_k^{-1} E_k^* H_k \\ \vdots \\ H_k^T R_k^{-1} E_k^* H_k - \gamma^{-2} I_{5M} \end{bmatrix} \]

\[ = \begin{bmatrix} W_{11}^* & W_{12}^* \\ W_{21}^* & W_{22}^* \end{bmatrix} \]  

(42)

Then, we obtain the following theorem for the convergence of the error covariance matrix of the stationary robot.

**Theorem 2:** We assume that Assumptions 1–3 hold and that the robot is stationary and continues to observe the landmarks. After the stationary robot has observed the landmarks \( n (> 0) \) times satisfying the condition (38) described in Lemma 2, the estimated state error covariance matrix is shown in (43). Moreover, as \( n \to \infty \), the error covariance matrix converges to \( \lim_{n \to \infty} P_k^e = \Theta_3 + 2M \).

\[ P_k^e = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \end{bmatrix} \]  

(43)

where \( P_{11} \) is the error covariance matrix of the robot, \( P_{22} \) is that of the landmarks, and \( P_{12} \) and \( P_{21} \) are the cross-error covariance matrices. These are defined as follows.

\[ P_{11} = n^{-1} (\Xi_{11} - \Xi_{12} ^* \Xi_{22} ^* \Xi_{21} )^{-1} \]

(44)

\[ P_{12} = - P_{11} \Xi_{12} ^* \Xi_{22} ^* \]

(45)

\[ P_{21} = - \Xi_{22} ^* \Xi_{12} \]

(46)

\[ P_{22} = - \Xi_{22} ^* \Xi_{21} + n^{-1} \Xi_{22} \]

(47)

\[ \Xi_{ij} := n^{-1} p_{kj} + W_{ij} \]  

(48)

**Proof:** Assuming that a robot is stationary, \( F_k = I \) and \( Q_k = 0 \). Then, from (39), (41) and (42), the inverse matrix of the covariance matrix of the robot has stopped and made observations \( n \) times is represented with \( \Xi_{ij} \) in (48) as follows.

\[ P_k^{e-1} = P_k^{0-1} + nW^* \]

\[ = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} + n \begin{bmatrix} W_{11}^* & W_{12}^* \\ W_{21}^* & W_{22}^* \end{bmatrix} \]

\[ = n (\Xi_{11} - \Xi_{12} ^* \Xi_{22} ^* \Xi_{21} )^{-1} + n W^* \]

(49)

Then, applying the inverse matrix lemma for (49), the error covariance matrix \( P_k^e \) is represented as (43) and its components are represented as (44)–(47).

Here, if \( n \to \infty \), \( \Xi_{ij} \) in (48) becomes as follows.

\[ \lim_{n \to \infty} \Xi_{ij} = W_{ij}^* \]  

(50)

In addition, \( P_{11} \) in (44) also converges to \( \lim_{n \to \infty} P_{11} = 0 \). Therefore, since all the components of (43) converge to 0, the estimated error covariance matrix \( P_k^e \) converges to \( \lim_{n \to \infty} P_k^e = 0 \). The convergence of the system has thus been proved theoretically. \( \blacksquare \)

V. SIMULATION

In this section, we validate the proposed method and compare its estimation accuracy with the conventional method in simulation.

A. Simulation Conditions

The parameters used in this simulation are shown in Table I. The velocity and turning rate of the robot are shown in Figs. 3 and 4, respectively. In addition, the process noise and observation noise were constructed from uniformly distributed random numbers within the ranges shown in Table I. In this simulation, whole or the part of proper observation data of landmarks can be obtained intermittently as shown in Fig. 5. In this figure, the improper observation data caused from obstacles are represented by higher or lower values than proper ones and those caused from a sensor fault are represented by 0.
TABLE I. SIMULATION PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation time (s)</td>
<td>( t_{	ext{sim}} )</td>
<td>1000</td>
</tr>
<tr>
<td>Initial state of the robot</td>
<td>( \mathbf{x}_0 )</td>
<td>( \begin{bmatrix} 0 \ 0 \ \cdots \ 0 \end{bmatrix} )</td>
</tr>
<tr>
<td>Initial state error covariance matrix</td>
<td>( P_0 )</td>
<td>( \begin{bmatrix} 0.1 &amp; 0.01 \ 0.01 &amp; 0.01 \end{bmatrix} )</td>
</tr>
<tr>
<td>Covariance matrix of process noise</td>
<td>( Q )</td>
<td>( \begin{bmatrix} 0.01 &amp; 0.01 \ 0.01 &amp; 0.01 \end{bmatrix} )</td>
</tr>
<tr>
<td>Covariance matrix of observation noise</td>
<td>( R )</td>
<td>0.1</td>
</tr>
<tr>
<td>Process Noise</td>
<td>( \mathbf{W}_\text{true} )</td>
<td>0.01</td>
</tr>
<tr>
<td>Observation Noise</td>
<td>( \mathbf{V}_\text{true} )</td>
<td>0.01</td>
</tr>
<tr>
<td>Design parameter</td>
<td>( f_{\text{th}} )</td>
<td>20</td>
</tr>
</tbody>
</table>

Fig. 3. Velocity Fig. 4. Angular velocity

B. Simulation Results

Figure 6 shows the components of the intermittent compensation matrix. Comparing these results and the obtained observation data in Fig. 5, each \( \mathbf{n}_i \) becomes 0 corresponding to the intermittent observation of each landmark. Therefore, we can confirm detections and compensations for intermittent observations with our proposed method.

First, the results of estimations are shown in Fig. 7. In this figure, each line and mark represent the true values and estimations of the robot path and landmark position via each filter, respectively. From this figure, because of intermittent observations, the estimations via the conventional \( H_i \) filter (HF) are far from the true value. Meanwhile, the results with our proposed method and the switching Kalman filter (SKF) are not affected by intermittent observations. Next, consider the error covariance matrix shown in Fig. 8. In this figure, while the value with the SKF increases sharply when there is at least one intermittent observation, that of with the proposed method increases only when all observation data can not be obtained such as in 250s-280s. This is because, even if some observation data lost, our proposed method can calculate its filter gain with the other proper observation data. In addition, both covariance matrices go back to the original values when the proper observation data is obtained again. Furthermore, we can confirm the value via the proposed method converges after the robot stops at 1200s. Last, Figs. 9 and 10 show the root mean squared error (RMSE) of the estimated robot and landmark positions, respectively. From both figures, the errors via the proposed method become smaller than that of via the SKF. Therefore, it is shown that our proposed method prevents deterioration of estimation accuracy from intermittent observations compared with the SKF.

VI. EXPERIMENT

Next, we validated the proposed method with an experiment using the simulation program and the data obtained from an actual robot.

A. Experimental setup and conditions

Figure 11 shows the setup and the overview of the experiment. Experimental verification was carried out under the parameters shown in Table II. We gave the input signals shown in Fig. 12 to the Amigobot thorough a wireless LAN and allowed the robot to move around the landmark in a circle with a radius of one meter. The Amigobot obtained the relative distances to the landmarks with sonar sensors. We used Matlab to calculate the estimated robot trajectory and the positions of the landmarks with each of the filters and to compare them with their true values obtained by using a camera set above.

B. Experimental results

Figure 13 shows the obtained observation data in this experiment. Since we made the robot move as the right side
of Fig. 11, the observed data is to become close to 1000 [mm]. However, because of the obstacle or the miss of the sonar sensor, the Amigobot obtained the proper observation data intermittently. Comparing this result, the component of the intermittent compensation matrix become 0 corresponding to the intermittent observation in Fig. 14.

Figures 15–17 show the results of the estimated positions of the robot and the landmark, the estimated error covariance matrices, and the root mean squared errors (RMSE) of the position of the landmark, respectively. Since we could not synchronize the sampling time of the Amigobot and the above camera, the RMSE of the robot position in each step was omitted. First, Fig. 15 shows that the conventional $H_{\infty}$ filter (HF) can not estimate correctly because of intermittent observations. Meanwhile, our proposed method and the SKF can continue to estimate the robot and landmark position until the end of the experiment. Next, from Fig. 16, though the error covariance matrices via the proposed method and the SKF increase because of intermittent observations, they go back to the original values when the proper observation data can be obtained again. In addition, the value via the proposed method converges after the robot stops and we can confirm the correctness of the derived theorem. Last, Fig. 17 shows the error via the proposed method is small compared with the SKF, and then we show that our proposed method is useful to prevent the deterioration of estimation accuracy from intermittent observation.

VII. CONCLUSION

In this paper, we dealt with the SLAM problem via the $H_{\infty}$ filter considering intermittent observations. Our proposed method detects intermittent observations by comparing the obtained observation data and their estimations. Then, by using the filter gain with the compensation matrix $E_k$, the proposed method prevents the deterioration of estimation accuracy from intermittent observation. In this paper, we proved the convergence of the estimated error covariance matrix for the proposed algorithm. Using both simulation and experimental results, we confirmed the convergence of the estimated error covariance matrix and that the proposed method is useful to prevent the deterioration of estimation accuracy from intermittent observation compared with conventional methods.

REFERENCES