Consensus-based Cooperative Formation Control with Collision Avoidance for a Multi-UAV System

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Abstract—In this paper, we study cooperative control problems with a multi-UAV system. Specifically, we propose a consensus-based cooperative formation control algorithm with collision-avoidance capability for a group of multiple unmanned aerial vehicles (UAVs). To cooperatively fly in formation, a consensus-based algorithm and a leader-follower structure are applied to the UAVs. Collisions among the UAVs can occur while they are flying with the cooperative control algorithm. Therefore, we describe how the cooperatively controlled UAVs avoid collisions while they are flying to desired position for formation. The collision-avoidance strategy is based on an artificial potential approach. The convergence is guaranteed even when the cooperative formation control algorithm and the collision-avoidance control algorithm are simultaneously applied to the quadrotors. Finally, the proposed approach is validated by some simulations.

I. INTRODUCTION

In recent years, cooperative control problems with a multi-vehicle system have attracted a lot of attention from many researchers [1]. These cooperative control technologies are expected to be applied to actual vehicles such as UAVs, artificial satellites, and autonomous mobile observation robots as well as distributed sensor networks.

Formation control problems for a multi-vehicle system have been studied very well in recent years, and many control algorithms have been developed for the problems. A major strategy for formation control is to apply a consensus algorithm. Consensus-based cooperative control, which is a distributed approach, has advantage of having network flexibility. Most works of the cooperative control problems using a consensus-based algorithm assume that vehicles are expressed as a first-order system [2]–[8]. The results in a first-order system can be directly extended to a second-order system [4], [8]. Works of a linear system, however, are not directly extended from the results in a first-order system, because it is difficult to extend the algorithms for a first-order system directly to the problems with a linear system. Therefore, most authors who consider a linear system mainly use complicated ways such as an optimization approach [9] and a linear matrix inequality (LMI) [10]. Generally, a model of UAVs is expressed as a more complicated system than a first-order system.

We have been studying the cooperative control problems with a multi-vehicle system [11]–[14], especially focusing on formation control problems using a consensus algorithm. In [13], we expressed the dynamics of UAVs in the horizontal plane as a fourth-order system, then proposed a consensus-based control algorithm for a group of the UAVs to fly in formation cooperatively. However, collisions are not considered, and collisions can occur while the UAVs are flying with the consensus-based control algorithm.

Control problems with collision or obstacle avoidance have attracted a lot of attention from many researchers in recent years, and many control algorithms have been developed for the problems. The major strategies are rule-based approaches and optimization-based approaches. One of the rule-based approaches is an artificial potential field based approach [15], [16], and one of the optimization-approaches is a model predictive control (MPC) based approach [17], [18]. An artificial potential field has been extensively applied to autonomous robot navigation. Attractive potential fields are assigned to target points and repulsive potential fields are assigned to obstacles, and then vehicles move along the negative gradient of the composite of the potential fields.

The consensus-based control algorithm for formation we proposed guarantees the convergence to the position for desired formation [13]. However, the control algorithm in [13] was designed for formation flying in the horizontal plane, not in three-dimensional space. Hence, a consensus-based cooperative control algorithm for formation flying in the vertical direction is proposed in this paper. Then, the convergence is guaranteed when the consensus-based control algorithm for formation in the vertical direction and the artificial potential-based collision-avoidance algorithm are simultaneously applied to the UAVs.

An outline of this paper is as follows. In Section II, how to model a quadrotor and a multi-UAV system is stated, and then a control objective is defined. The main control objective of this study is to fly in formation cooperatively while the UAVs avoid collisions. In Section III, we propose a control algorithm for the problem defined in Section II. Then, we provide convergence analysis for control algorithms proposed in this paper. Section IV presents simulation results to validate the proposed approach, and finally, concluding remarks are stated in Section V.

II. PROBLEM STATEMENT

This section presents how to linearized quadrotor in the vertical direction and a multi-UAV system is modeled mathematically, and then a control objective is defined.

A. Modeling a Quadrotor

Suppose that there are $N$ quadrotors that have the same motion characteristics. A multi-UAV system consists of $N$
quadrotors and a leader. Each of the quadrotors has four propellers and a controller. The controller gives each of the propellers control commands individually. To simply model the quadrotor [19], let us make a few assumptions as follows. First, the quadrotor flies slowly enough to ignore external aerodynamic forces such as aerodynamic drag and blade vortex interface acting on the quadrotor. Second, the propellers respond to thrust commands fast enough to ignore time delay from when the controller gives the propellers the thrust commands till the propellers actually produce the commanded thrust forces. Last, the yawing moment is never produced. Under the condition that hovering at a constant altitude is an equilibrium point of a nonlinear quadrotor system, a linearized model of the quadrotor in the vertical direction is given by

$$\frac{dh_i}{dt} = \frac{h_i^{(0)}}{\hat{T}_{\text{total}, i}}, \quad i \in \{1, 2, \cdots, N\},$$

where $h_i = h$, $h^{(1)} = -w$, and $\hat{T}_{\text{total}} = T_{\text{total}}/m$. Note that $T_{\text{total}}/4$ is equally assigned to each of the propellers. The meaning of the symbols above is shown in Table I. Note that a linearized model of the quadrotor in the horizontal plane is expressed as a fourth-order system in [13].

B. Modeling a Multi-UAV System

A multi-UAV system is modeled as a group of dynamical system in which multiple UAVs and a leader exchange information with one another. To mathematically describe this network, we use graph theory [20].

We use a graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ to model information interaction among $N$ UAVs, where $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$ is a set of nodes, and $\mathcal{A} \in \mathcal{V} \times \mathcal{V}$ is a set of edges. The edge $(v_i, v_j)$ in the edge set of the graph denotes that there is a network path from the UAV $i$ to the UAV $j$. This means that the UAV $j$ can obtain and use information from the UAV $i$.

A directed tree is a digraph where every node has exactly one parent node except for one node, called a root. The root has no parent and has a directed path to every other node. Also, a directed spanning tree of $\mathcal{G}$ is a tree that contains all nodes of $\mathcal{G}$.

Let $\mathcal{A} \in \mathbb{R}^{N \times N}$, $\mathcal{D} \in \mathbb{R}^{N \times N}$, and $\mathcal{L} = \mathcal{D} \mathcal{A}$ be an adjacency matrix, a degree matrix, and a graph Laplacian matrix related to the graph $\mathcal{G}$, respectively. The component of the adjacency matrix $A_{i,j}$ is given by

$$a_{ij} = \begin{cases} 1, & \text{for } (v_j, v_i) \in \mathcal{A} \\ 0, & \text{otherwise} \end{cases}$$

This means that $a_{ij}$ is set to one if the UAV $i$ is obtaining information from the UAV $j$ through a network, otherwise $a_{ij}$ is set to zero.

The degree matrix $\mathcal{D}$ is an in-degree matrix given by

$$\mathcal{D} = \text{diag}(\text{deg}(v_1), \text{deg}(v_2), \cdots, \text{deg}(v_N)),$$

where $\text{deg}(v_i)$ is a number of communication links arriving at the node $v_i$.

The graph Laplacian matrix $\mathcal{L}$ is defined as

$$\mathcal{L} = \mathcal{D} - \mathcal{A}.$$  

The graph Laplacian has the following properties: if a graph has or contains a directed spanning tree, the graph Laplacian $\mathcal{L}$ has a single eigenvalue at zero, and all nonzero eigenvalues of the graph Laplacian have positive real part.

C. A Control Objective

In this paper, let us consider a mission as follows: $N$ quadrotors fly in formation following their leader while the quadrotors are avoiding collisions among them. Specifically, each of the quadrotors will converge to a time-variant desired position while avoiding collisions among them. The time-variant desired positions are determined by a desired geometric configuration of formation.

To achieve this control objective, we make a set of assumptions as follows:

Assumption 1: Every quadrotor must have a connection from a leader on the network, but all of the quadrotors do not necessarily have a direct connection from the leader. In addition, the network between the quadrotors must be a bidirectional connection.

Assumption 2: Movement of the leader must be independent from any quadrotor, that is, movement of the leader is not affected by any quadrotors.

III. PROPOSED APPROACH

In this section, we propose a control algorithm to achieve the control objective as mentioned in Section II. C.

As described in Section II. A, the quadrotor model in the horizontal plane is expressed as a fourth-orders system and the model in the vertical direction is expressed as a second-order system. Hence, to fly in formation in three-dimensional space, separate control algorithms are applied for formation flying in the horizontal plane and in the vertical direction. The control algorithm for formation flying in the horizontal plane is described in [13].

To avoid collisions among the quadrotors, we apply a control strategy that the quadrotors take evasive action only in the vertical direction, not in the horizontal plane.

A. Formation in the Vertical Direction

For formation flying in the horizontal plane, we separately consider an objective of a group of quadrotors and an objective of each of the quadrotors. The first objective is that a group of quadrotors cooperatively flies in formation. The other objective is that each of the quadrotors generates geometric configuration of the formation. We apply a
consensus-based cooperative control algorithm to achieve the former cooperative objective and also apply a leader-follower structure to achieve the latter individual objective. The leader individually provides the only directly connected quadrotors with its own position and desired positions for formation. The advantage of the control algorithm proposed by the authors is that all of the quadrotors are not necessarily directly connected from the leader.

A control law for the quadrotor \( i \) is given by

\[
\hat{T}_{\text{total}}(t) = -\sum_{j=1}^{N+1} a_{ij} \left[ \sum_{k=0}^{1} \gamma_k (\hat{h}_j^{(k)} - \hat{h}_j^{(k)}) \right],
\]

\( i \in \{1, 2, \ldots, N\} \),

(5)

\[
\hat{h}_j^{(k)} = h_j^{(k)} - d_{h_j}, \quad j \in \{1, 2, \ldots, N+1\}, \quad k \in \{0, 1\},
\]

(6)

where the subscript of \( N + 1 \) denotes a leader, and \( \gamma_k \in \mathbb{R} \), \( k \in \{0, 1\} \) are positive control gains. Also, \( h_j^{(k)} \in \mathbb{R} \), \( k \in \{0, 1\} \) is the state of the quadrotor \( j \) in the vertical direction, and \( d_{h_j} \in \mathbb{R} \), \( k \in \{0, 1\} \) is the desired relative state between the quadrotor \( j \) and the leader in the vertical direction. As defined in (2), \( a_{ij} \) is the value that indicates whether the quadrotor \( i \) uses information from the quadrotor \( j \) or not, that is, \( a_{ij} = 1 \) if using some information, otherwise \( a_{ij} = 0 \).

B. Collision Avoidance

In this section, we propose a collision-avoidance strategy for the multiple UAVs controlled with the cooperative control protocols (5). The basic direction of collision-avoidance is that the quadrotors take evasive action only in the vertical direction, not in the horizontal plane, if the risk of collisions among the quadrotors increases. As shown in Fig. 1, a safety region is set around the center of the gravity of the quadrotor. The region is formed of cylindrical shape whose hight and radius are \( 2\Delta H \) and \( \Delta R \), respectively.

If the safety region of a quadrotor hits against that of the other quadrotors, all of the quadrotors whose safety regions are overlapping with one another begin to take evasive action. The evasive action is continuously taken until the overlaps of the safety region are avoided.

Here, we define \( r_i \) and \( h_i \) as the position in the horizontal plane and the altitude of the quadrotor \( i \), respectively. Also, \( |r_{ij}| \) is the relative distance in the horizontal plane between the quadrotor \( i \) and the quadrotor \( j \), and \( |h_{ij}| \) is the altitude gap between them.

\[
r_{ij} = r_i - r_j, \quad h_{ij} = h_i - h_j,
\]

\[
|r_{ij}| = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}, \quad |h_{ij}| = \sqrt{(h_i - h_j)^2},
\]

\( i, j \in \{1, 2, \ldots, N\} \).

(7)

An artificial potential field produced by the quadrotor \( i \) and the quadrotor \( j \) is defined as

\[
U_{ij} = \begin{cases} 
K_h \left( \frac{1}{|h_{ij}| + 1} - \frac{1}{2\Delta H + 1} \right)^2, & \text{for } |h_{ij}| \leq 2\Delta H \land |r_{ij}| \leq 2\Delta R, \\
0, & \text{for otherwise}
\end{cases}
\]

\( i, j \in \{1, 2, \ldots, N\} \),

(8)

where \( K_h \in \mathbb{R} \) is a positive control parameter. The artificial potential decreases as the altitude gap between the quadrotor \( i \) and the quadrotor \( j \) closes. In addition, the artificial potential is zero when there is no overlap between the safety regions of the quadrotor \( i \) and that of the quadrotor \( j \).

From (8), the average artificial potential field produced by the quadrotor \( i \) and the others is given by

\[
U_i = \frac{1}{N-1} \sum_{j=1, j\neq i}^{N} U_{ij}, \quad i \in \{1, 2, \ldots, N\}.
\]

(9)

From (9), the total artificial potential field produced by every quadrotor is given by

\[
U_c = \sum_{i=1}^{N} U_i.
\]

(10)

We propose artificial force for the quadrotor \( i \) to avoid collisions among the quadrotors:

\[
f_{ca_i} = -\nabla h_i U_c, \quad i \in \{1, 2, \ldots, N\}.
\]

(11)

Note that there is no local minima because the potential field \( U_c \) is composed of only the repulsive potential fields. (11) is expressed in a vector form as

\[
f_{ca} = -\nabla U_c = -\left[ \frac{\partial U_c}{\partial h_1}, \frac{\partial U_c}{\partial h_2}, \ldots, \frac{\partial U_c}{\partial h_N} \right]^T.
\]

(12)

The artificial force acts in the opposite direction of the potential gradient, that is, the force acts so as to decrease the potential. Therefore, the artificial force acts so as to get the sum of the altitude gap wider.

C. Formation with Collision Avoidance

In this section, we propose a formation control algorithm with collision-avoidance capability as follows:

\[
\hat{T}_{\text{total}} = f_{form} + f_{ca_i}, \quad i \in \{1, 2, \ldots, N\},
\]

(13)

where \( f_{form} \) is the control protocol (5) for formation flying in the vertical direction, and \( f_{ca_i} \) is the control protocol (11) for collision-avoidance.

**Theorem 1:** Suppose that a multi-UAV system composed of a leader and \( N(\geq 2) \) quadrotors expressed as (1).
Also, the assumptions 1 and 2 are satisfied. If the control protocol (5) with positive control gains $\gamma_k, k \in \{0, 1\}$ and the control protocol (11) with a positive control parameter $K_h$ are simultaneously applied to each of the quadrotors, then each of the quadrotors with collision-avoidance capability will asymptotically converge to the desired position in the vertical direction.

Proof: Applying the control protocol (13) to the quadrotor $i$ expressed as (2), we can get

$$
\dot{h}_{i}^{(1)} = - \sum_{j=1}^{N+1} a_{ij} \left[ \sum_{k=0}^{1} \gamma_k (\dot{h}_{i}^{(k)} - \dot{h}_{j}^{(k)}) \right] + f_{ca},
$$

$$
i \in \{1, 2, \cdots, N\}. \quad (14)
$$

We define the new state vector $\dot{h}^{(k)} = \left[ \begin{array}{c} h_{1}^{(k)} \\ h_{2}^{(k)} \\ \vdots \\ h_{N}^{(k)} \\ 0 \end{array} \right] \in \mathbb{R}^{(N+1)}$ and $\dot{h}_{i}^{(k)} = \left[ \begin{array}{c} \dot{h}_{i}^{(k)} \\ \dot{h}_{i}^{(k)} \\ \vdots \\ \dot{h}_{i}^{(k)} \end{array} \right] \in \mathbb{R}^{N+1}$, $k \in \{0, 1\}$. Also, the new force vector $f_{ca} = [f_{ca}^{(0)}, 0]^T \in \mathbb{R}^{N+1}$ is defined. Using $\dot{h}_{i}^{(k)}, \dot{h}_{i}^{(k)}$, and $f_{ca}$ to express (14) in a matrix form, we can rewrite (14) as

$$
\dot{h}^{(1)} = -\gamma_0 L \dot{h}^{(0)} - \gamma_1 L \dot{h}^{(1)} + f_{ca},
$$

where the matrix $L \in \mathbb{R}^{(N+1) \times (N+1)}$ is the graph Laplacian of the multi-UAV system. Note that the leader is regarded as the UAV $(N+1)$. Here, the vector $\dot{h}^{(k)}$ consists of the states of the quadrotors and commands from the leader. Hence, we separate the states of the quadrotors from the others. The following identities concerning the rows of the graph Laplacian $L$ always hold.

$$
a_{i(N+1)} = \sum_{j=1}^{N+1} a_{ij} - a_{i1} - a_{i2} - \cdots - a_{iN},
$$

$$
a_{ii} = 0, \quad i \in \{1, 2, \cdots, N\}. \quad (16)
$$

From $d_{k}^{(k)} = 0, k \in \{0, 1\}$ and (16), (15) is separately expressed as

$$
\dot{h}^{(1)} = -\gamma_0 M h^{(0)} - \gamma_1 M h^{(1)} + \gamma_0 M \dot{h}_{N+1}^{(0)} + \gamma_1 M \dot{h}_{N+1}^{(1)} + \gamma_0 M d_{h}^{(0)} + \gamma_1 M d_{h}^{(1)} + f_{ca},
$$

where the matrix $M \in \mathbb{R}^{N \times N}$ is defined as (18). And also,

$$
h^{(k)} = [h_{1}^{(k)}, h_{2}^{(k)}, \cdots, h_{N}^{(k)}]^{T} \in \mathbb{R}^{N}, \quad \dot{h}^{(k)} = [\dot{h}_{1}^{(k)}, \dot{h}_{2}^{(k)}, \cdots, \dot{h}_{N}^{(k)}]^{T} \in \mathbb{R}^{N}, \quad \square
$$

is the kronecker product, and $1_N = [1 \ 1 \ \cdots \ \cdots \ \cdots \ 1]^{T} \in \mathbb{R}^{N}$. Note that the matrix $M$ is similar to the graph Laplacian, but not equal to the matrix.

$$
M = \begin{bmatrix}
\sum_{j=1}^{N} a_{ij} & -a_{i1} & \cdots & -a_{iN} \\
-a_{i1} & \sum_{j=1}^{N} a_{j1} & \cdots & -a_{j2} \\
& \ddots & \ddots & \ddots \\
& & -a_{N1} & -a_{N2} & \cdots & -a_{NJ}
\end{bmatrix}
$$

$$
(18)
$$

Equation (17) can be rewritten in a matrix-vector form as

$$
\begin{aligned}
\frac{d}{dt} \left[ \begin{array}{c} \dot{h}^{(0)} \\ \dot{h}^{(1)} \end{array} \right] = & \left[ \begin{array}{c}
0_N \\
-\gamma_0 M \\
\gamma_1 M
\end{array} \right] \left[ \begin{array}{c} h^{(0)} \\ h^{(1)} \\ f_{ca} \end{array} \right] \\
+ & \left[ \begin{array}{c}
0_N \\
-\gamma_0 M \\
\gamma_1 M
\end{array} \right] \left[ \begin{array}{c} \dot{h}^{(0)} \\ \dot{h}^{(1)} \end{array} \right] \\
+ & \left[ \begin{array}{c} 0_N \\
\gamma_0 M \\
\gamma_1 M
\end{array} \right] \left[ \begin{array}{c} \dot{h}^{(0)} \\ \dot{h}^{(1)} \end{array} \right] + \left[ \begin{array}{c} d_{h}^{(0)} \\ d_{h}^{(1)} \end{array} \right],
\end{aligned}
$$

where $I_N \in \mathbb{R}^{N \times N}$ is a $N$-dimensional unit matrix, $0_N \in \mathbb{R}^{N \times N}$ is a $N$-dimensional zero matrix, and $0_N \in \mathbb{R}^N$ is a $N$-dimensional zero vector.

Now, to examine the stability of the differential equation (19), let us consider the homogeneous equation of (19) expressed as

$$
\frac{d}{dt} \left[ \begin{array}{c} h^{(0)} \\ h^{(1)} \end{array} \right] = \left[ \begin{array}{c}
0_N \\
-\gamma_0 M \\
-\gamma_1 M
\end{array} \right] \left[ \begin{array}{c} h^{(0)} \\ h^{(1)} \end{array} \right] + \left[ \begin{array}{c} 0_N \\ f_{ca} \end{array} \right]. \quad (20)
$$

Here, we construct the following Lyapunov candidate function $V$. This function is composed of the total energy of the multi-UAV system.

$$
V = \frac{1}{2} h^T \dot{h} + \frac{1}{2} \gamma_0 h^T M h + U_c.
$$

The time derivative of the function $V$ is given by

$$
\dot{V} = \dot{h}^T (\dot{h} + \gamma_0 M h) + \dot{U}_c
$$

$$
= -\gamma_1 h^T M \dot{h} + \dot{h}^T f_{ca} + \dot{U}_c. \quad (22)
$$

From (11), we can get

$$
\dot{U}_c = \dot{h}^T \nabla U_c = -\dot{h}^T f_{ca}. \quad (23)
$$

Thus, from (22) and (23), we can get

$$
\dot{V} = -\gamma_1 h^T M \dot{h}. \quad (24)
$$

First, let us consider the Lyapunov candidate function $V$ (21). The graph composed of the multi-UAV system has a directed spanning tree if the assumptions 1 and 2 are satisfied. Therefore, from the property of the graph Laplacian, $L$ has a single eigenvalue at zero, and all nonzero eigenvalues of the graph Laplacian have positive real part. From this property and the relationship between the graph Laplacian $L$ and the matrix $M$, the matrix $M$ is positive definite, that is $M > 0$. In addition, we have $U_c \geq 0$ if the parameter of the artificial potential $K_h$ in (8) is positive. Thus, $V$ is the sum of three terms that are always positive or zero if the control gain $\gamma_0$ is positive and the control parameter of the artificial potential $K_h$ is positive. Furthermore, we need all of them to be zero for $V$ to be zero. Specifically, we have $V = 0$ if and only if $h = 0, \dot{h} = 0,$ and $U_c = 0$ simultaneously. Note that $U_c = 0$ if there is no overlap of the safety regions between every pair of the distinct quadrotors.

Next, let us consider the derivative of the Lyapunov candidate function $\dot{V}$ (24). From $M > 0$, we can always get $\dot{V} \leq 0$ if the control gain $\gamma_1$ is positive. Also, we get $\dot{V} = 0$ if and only if $\dot{h} = 0$, and also $\dot{V} \neq 0$ along any solution of the differential equation (20) except $h = 0$, $\dot{h} = 0$, and $U_c = 0$ simultaneously hold.

The asymptotic stability can be solved by Lyapunov theorem involving LaSalle’s principle [21].

Here, the particular solution of (19) in the case of no overlap of the safety region can be given by

$$
\begin{aligned}
\left[ \begin{array}{c} h^{(0)} \\ h^{(1)} \end{array} \right] = & \left[ \begin{array}{c}
\hat{h}^{(0)}_{N+1} \\ \hat{h}^{(1)}_{N+1}
\end{array} \right] + \left[ \begin{array}{c}
d_{h}^{(0)} \\ d_{h}^{(1)}
\end{array} \right].
\end{aligned}
$$

This validity can be confirmed by substituting (25) for (19). To confirm this validity, we use $\hat{h}^{(2)}_{N+1} = 0$ and $d_{h}^{(2)} = 0$
because the leader provides each of the quadrotors with the desired state, not the desired input. Note that $\delta_{1}^{(2)}_h$ and $\delta_{h}^{(2)}$ denote the input of the leader and the desired relative input between the quadrotor and the leader, respectively.

The general solution of the non-homogeneous differential equation (19) is the sum of the particular solution and the general solution of the homogeneous equation. Hence, if the control gains $\gamma_k$, $k \in \{0, 1\}$ and the control parameter of the artificial potential $K_h$ are selected to be positive, the general solution of (19) asymptotically converges to:

$$
\begin{bmatrix}
\dot{h}^{(0)}_1 \\
\dot{h}^{(1)}_1
\end{bmatrix} \rightarrow \begin{bmatrix}
\ddot{h}^{(0)}_{N+1} \\
\ddot{h}^{(1)}_{N+1}
\end{bmatrix} + \begin{bmatrix}
\delta_h^{(0)}_1 \\
\delta_h^{(1)}_1
\end{bmatrix}, \quad \text{as } t \to \infty. \quad (26)
$$

This results in the convergence to only the command from the leader.

From the element of the first row block in (26), it is proved that each of the quadrotors with collision-avoidance capability will asymptotically converge to the desired position in the vertical direction.

IV. SIMULATION RESULTS

In this section, we present some simulation results to validate the performance of the proposed control algorithm.

A. Simulation Setup

We consider a group of three quadrotors and a leader, and also consider a single network that has a directed spanning tree and has interactive communication links between every pair of the distinct quadrotors as shown in Fig. 2.

The leader flies at a constant altitude of 2.0(m). Each of the quadrotors keeps a certain relative altitude from the leader. The relative altitudes are steadily 0(m), and the information is provided by the leader as the desired relative altitude $d_{h_i} = 0, \forall i \in \{1, 2\}$. In other other words, the desired altitudes for three quadrotors are steadily 2.0(m).

The control gains for formation in the vertical direction and the control parameter for collision-avoidance are selected so as to satisfy all of the conditions: $\gamma_0 = 1.0$, $\gamma_1 = 1.8$, and $K_h = 5.0$. Each of the quadrotors has the safety region whose parameters are $\Delta H = 0.7$(m) and $\Delta R = 3.0$(m).

The quadrotors are judged to be in collision with each other if the altitude gap is less than 0.4(m) and the relative distance in the horizontal plane is less than 2.0(m). The mass of the quadrotor is 0.42(kg).

Two simulations are carried to validate the proposed control algorithms. In CASE I, the simulation is carried out without the collision-avoidance algorithm. On the other hand, in CASE II, the simulation is carried out with the collision-avoidance algorithm. To cooperatively fly in formation in the horizontal plane, the formation control algorithm in [13] is applied to the quadrotors.

B. Simulation Results

We show the simulation results. Fig. 3 show the results of CASE I, and Fig. 4 show the results of CASE II.

In Figs. 3 and 4, the third graph from the top shows the time history of the thrust command for formation flying, the fourth graph from the top shows the time history of the thrust command for collision-avoidance, and the fifth graph from the top shows the time history of the sum of them. Also, the last two graphs from the top show the relative distance in the horizontal plane and the altitude gap between the quadrotors, respectively. The boundary of the safety region and that of the collision are illustrated in these figure. If some quadrotors are in collision with each other, collision-flags associated with the quadrotors in collision are marked.

Fig. 3 shows that the first and the third quadrotors are in collision with each other in about 5(sec). On the other hand, Fig. 4 shows that this collision is avoided by collision-avoidance algorithm. This figure shows that the controller provides the thrust command for collision-avoidance while there are overlaps of the safety region and also shows that each of the quadrotors converges to the desired altitude after the overlaps are avoided. These results indicate the validity of the formation control strategy in the vertical direction with collision-avoidance capability.

V. CONCLUSIONS AND FUTURE WORK

We briefly showed how a linearized model of quadrotors is expressed and how a multi-UAV system is mathematically modeled using graph theory, then provided a formation control algorithm to cooperatively fly in formation in three-dimensional space. For formation in the horizontal plane and in the vertical direction, the separate formation control algorithms were applied to the quadrotors. To avoid collisions among the quadrotors, a collision-avoidance strategy for the multiple UAVs controlled with the cooperative formation control algorithms was proposed. The basic direction of collision-avoidance was that the quadrotors take evasive action only in the vertical direction. For the collision-avoidance algorithm, an artificial potential field based approach was applied. In addition, repulsive potential fields were assigned to the altitude gap between the quadrotors. This also resulted in no local minima. We proposed a control strategy to simultaneously apply the cooperative formation control algorithm and the collision-avoidance control algorithm, then proved the convergence if the control gains satisfy the conditions.

Our extensive simulation results showed that the proposed control algorithm was validated and effective for formation flying with collision-avoidance capability.

Conducting experiments to validate the proposed formation control algorithms will be a challenging issue.

We did not consider avoiding the excess of operational limitations imposed on the UAVs in this paper. Our approach can be easily extended to a control strategy for this...
avoidance. Also, the UAVs were expressed as a fourth-order system and a second-order system. Hence, studying and designing a control algorithm considering this avoidance for a more general system will be a challenging issue for future work in this area.

REFERENCES