Cooperative Capturing for Multi-Agent System with Reaction Force from Target Object

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Abstract: This paper proposes a cooperative control of multi-agent system to capture target object. When the agents capture the target object, they may be subjected to reaction force from it. Therefore, we have to consider the physical contact between the agent and target object. First, we introduce the second order model to consider reaction force from target object. The network topology among agents is time-varying and connected, and information of target object is allowed to be available to at least one agent. Second, we show the proposed control law guarantees the agents capture the target object. The control law consists of graph theory and consensus algorithm, and we analyze the convergence property. Finally, simulation results show the effectiveness of the proposed control law.

Keywords: Multi-Agent System, Cooperative Capturing, Distributed Cooperative Control

1. INTRODUCTION

Multi-agent system that consists of multiple agents has been studied actively in recent years. Multi-agent system is composed of multiple autonomous agents and they exchange information for each other to work cooperatively. Vehicles and satellites are used for agents and they can achieve the desired formation or attitude [1, 2]. Agents can perform difficult or complex task by cooperative behavior. Furthermore, information exchange gives the system robustness, superior fault-tolerance and improvement of work efficiency [3].

Many research about consensus problem for multi-agent system have been reported and the control law that drive the agents to desired state under communication constraints is proposed in [4]. Higher order model for agents has been also studied extensively, in [5], the authors present distributed consensus protocols for higher order dynamical systems.

As a control problem of using consensus problem, there is a target capturing by multiple agents. Cooperative capturing have received recent attention in [6-9]. [6] discusses a moving target enclosing without knowing the target’s velocity. In [7], capturing a target by a team of kinematically controlled non-holonomic Dubins-like vehicles based on range-only measurements is proposed. A control law is proposed for a group of agents whose purpose is to search for, detect, and track a spill in an unknown environment in [8] and Target-capturing strategy for multiple vehicles with dynamic network topology is presented in [9]. There are many researches for cooperative capturing in this way. However, most researches do not considered reaction force from a target object.

In this paper, we propose cooperative control law to capture the target object with reaction force motivated by [9]. We represent the model of agent as second order model to consider reaction force from the target object. The network among the agents is time-varying and connected. Moreover, target’s information is available to at least one of the agents. Our control law consists of graph theory and consensus algorithm, and we analyze the convergence. The control law guarantees cooperative capturing in spite of the existence of reaction force. Finally, we show the effectiveness of our control law by numerical simulation results.

2. PROBLEM FORMULATION

When agents perform cooperative capturing, we give the desired position of capturing to agents. However, depending on the desired position, agents may contact target object. Therefore, we need to assume the model considering reaction force from target object. First, we set agents and target object as follows:

- Agent and target object are rigid body.
- Agents and target object are circular object, and their radius are \( l_i, l_t \in \mathbb{R} \).
- Agents and target object can move all direction.
- Agents don’t have robot arms, and they contact target object directly.
- The input of target object is only the force from agents, and target object can’t move by itself.

We show model, network and control objective for agents and target object mentioned above.

2.1 Model

We set the model for \( N \) agents and one target object in Cartesian coordinates. Second order model considering reaction force is given by

\[
m_i \ddot{r}_i = u_i + F_i
\]

\[
m_t \ddot{r}_t = - \sum_{j=1}^{N} F_j
\]

where \( i = 1, \ldots, N, \ r_i, r_t \in \mathbb{R}^2 \) denote, respectively, the position of agent and target object, \( u_i \in \mathbb{R}^2 \) is the control input of agent \( i \) and \( F_i \in \mathbb{R}^2 \) is reaction force subjected to agent \( i \) from target object. Moreover, \( m_i, m_t \in \mathbb{R} \) denote the respective mass. Fig. 1 shows the model of agents and target object.
Now, we define reaction force when an agent contacts a target object as follows:
\[ F_i \cdot \Delta t_i = \begin{cases} -\frac{m_1 m_2}{m_1 + m_2} (1 + e) r_{it} \cdot \parallel r_{it} \parallel < l_i + l_t \\ 0 \end{cases} \quad (\text{otherwise}) \tag{3} \]
where \( \Delta t_i \in \mathbb{R}^2 \) is contact time, \( e \in \mathbb{R} \) is the coefficient of restitution, \( r_{it} = r_i - r_t \), \( \parallel \cdot \parallel \) is Euclidean norm. Agent and target object are rigid body as mentioned above, and we use mass model. Therefore, we express reaction force as impulse. As it can be noticed for Eq. (3), reaction force is valid when agent contacts target object.

### 2.2 Network

We express the network topology of the system as a graph. A graph consists of a node and an edge. With \( v_i \) denotes the \( i^{th} \) node of a graph consists of \( N \) nodes, a node set is represented \( V = \{ v_1, v_2, \ldots, v_N \} \) and an edge set is represented \( E \subset V \times V \). An edge \( (v_i, v_j) \) is expressed by an arrow going from a node \( v_j \) to a node \( v_i \). Furthermore, a neighborhood set of \( i^{th} \) node is \( N_i = \{ v_j \in V : (v_j, v_i) \in E \} \). A graph consists of undirected edge is called a connected graph and the other graph is called a directed graph. Nodes \( v_i \) and \( v_j \) are connected defined that a graph has a path from \( v_i \) to \( v_j \). If arbitrary node \( v_i \) to arbitrary node \( v_j \) is connected, the graph is called a connected graph. In a directed graph, that is called a strongly connected graph.

When a graph is expressed algebraic, an adjacency matrix \( A \), a degree matrix \( D \) and graph Laplacian \( L \) are often used. A adjacency matrix \( A = [a_{ij}] \) represents the nodes of a graph are adjacent to other nodes as
\[
a_{ij} = \begin{cases} 1 \quad (v_j \in N_i) \\ 0 \quad (v_j \not\in N_i) \end{cases} \tag{4}\
\]
A degree matrix \( D \) is a diagonal matrix which consists of a degree of each node where a degree is defined as \( d_i = |N_i| \).
\[
D = \text{diag}(d_1, d_2, \ldots, d_N) \tag{5}\
\]
In considering the multi-agent system, the most important matrix is graph Laplacian. Graph Laplacian \( L \) is defined as
\[
L = D - A \tag{6}\
\]
Graph Laplacian has the following properties.

**Property 1:** Graph Laplacian has a zero eigenvalue and the corresponding eigenvector is \( L \mathbf{1}_N = 0 \) where \( \mathbf{1}_N = [1 \ 1 \ \cdots \ 1]^T \in \mathbb{R}^N \).

**Property 2:** If the graph is a connected graph, \( L \) is positive-semidefinite.

Now, we set the network of the system. As shown in Fig. 2, agent can get information of neighboring agent or target object if the distance between each other as follows:
\[
\parallel r_i - r_j \parallel \leq \rho. \tag{7}\
\]
The network topology will change depending on the position of agents and target object. \( \rho \in \mathbb{R} \) is assumed the range of sensor or communication. Thus, the adjacency matrix \( A = [a_{ij}] \in \mathbb{R}^{(N+1) \times (N+1)} \) associated with the network topology is denoted as
\[
a_{ii} = 0 \\
a_{ij} = \begin{cases} 1 & (\parallel r_i - r_j \parallel \leq \rho) \\ 0 & (\text{otherwise}) \end{cases} \tag{9}\
\]
for \( i, j = 1, \ldots, N, N + 1 \). Note that target object is labeled as agent \( N + 1 \) and \( d_{i(N+1)} = 0 \) (\( j = 1, \ldots, N \)). Because agents can obtain information of each other, the network topology among agents is connected graph.

Now, we set some assumptions for the network topology of the system as follows:

**Assumption 1:** The network topology among agents is time-varying but connected.

**Assumption 2:** There exists at least one agent which can get information of target object.

Assumption 1 concerns the network topology among agents. Assumption 1 and 2 mean the system has a directed spanning tree and target object is the root of the directed spanning tree.

\[
\parallel r_i - r_j \parallel \leq \rho \\
\parallel r_i - r_j \parallel > \rho
\]

### 2.3 Control Objective

We consider capturing by \( N \) agents. Capturing is the placing of agents around target object, and we define as follows:

**Control object 1:** Each agent converges to the position \( R_i \) around target object.
\[
\lim_{t \to \infty} \parallel r_i(t) - (r_i(t) + R_i) \parallel = 0 \quad i = 1, \ldots, N \tag{10}\
\]
where \( R_i \in \mathbb{R}^2 \) is a desired position for agent \( i \), and we set it in advance. Fig. 3 shows the situation achieving capturing.
3. PROPOSED CONTROL LAW

We propose a control input for each agent to achieve control objective (10) as

\[
\begin{align*}
    u_i &= \frac{m_i}{\kappa_i} \sum_{j=1}^{N+1} a_{ij} \left\{ -k_1 (\hat{r}_i - \hat{r}_j) \right. \\
    & \quad \left. -k_2 (\hat{r}_i - \hat{r}_j) + \hat{r}_j \right\} - F_i
\end{align*}
\]

(11)

where \( k_1 \leq k_2 \in \mathbb{R} > 0 \) are positive scalar, \( a_{ij} \) is the entry of the adjacency matrix \( A \). \( \hat{r}_i = r_i - R_i, \) \( R_{N+1} = 0 \), \( \kappa_i = \sum_{j=1}^{N+1} a_{ij} \), and \( \kappa_i > 0 \) by assumption 1.

We use consensus algorithm for this control law to converge the error. The first term of Eq. (11) modifies position error, and the second term of Eq. (11) modifies velocity error. Note that, target object is labeled as \( i = N + 1 \)th agent. Agents perform the formation enclosing target object. From the above, the control law guarantees cooperative capturing.

Here, we obtain the following theorem about model (1), (2) and control law (11).

**Theorem 1:** We apply the cooperative capturing control law (11) to the system consists of \( N \geq 2 \) agents (1) and target object (2) satisfying the assumption 1 and 2. Then agents asymptotically achieve control object (10).

**Proof:** Substituting the control law (11) to model of agent (1) can be written as

\[
\begin{align*}
    m_i \ddot{\hat{r}}_i &= \frac{m_i}{\kappa_i} \sum_{j=1}^{N+1} a_{ij} \left\{ -k_1 (\hat{r}_i - \hat{r}_j) \right. \\
    & \quad \left. -k_2 (\hat{r}_i - \hat{r}_j) + \hat{r}_j \right\}.
\end{align*}
\]

(12)

Expanding Eq. (12) gives

\[
\sum_{j=1}^{N+1} a_{ij} (\hat{r}_i - \hat{r}_j) = -k_2 \sum_{j=1}^{N+1} a_{ij} (\hat{r}_i - \hat{r}_j) - k_1 \sum_{j=1}^{N+1} a_{ij} (\hat{r}_i - \hat{r}_j).
\]

(13)

Eq. (13) can be written in matrix form as

\[
\begin{align*}
    \begin{bmatrix}
        \sum_{j=1}^{N+1} a_{1j} & -a_{12} & \cdots & -a_{1(N+1)} \\
        -a_{21} & \sum_{j=1}^{N+1} a_{2j} & \cdots & -a_{2(N+1)} \\
        \vdots & \vdots & \ddots & \vdots \\
        -a_{N1} & -a_{N2} & \cdots & \sum_{j=1}^{N+1} a_{Nj}
    \end{bmatrix}
    \begin{bmatrix}
        \ddot{x}_1 \\
        \ddot{x}_2 \\
        \vdots \\
        \ddot{x}_{N+1}
    \end{bmatrix}
    &= -k_2
    \begin{bmatrix}
        \hat{r}_{i1} \\
        \hat{r}_{i2} \\
        \vdots \\
        \hat{r}_{i(N+1)}
    \end{bmatrix}
    - k_1
    \begin{bmatrix}
        \hat{r}_{11} \\
        \hat{r}_{12} \\
        \vdots \\
        \hat{r}_{1(N+1)}
    \end{bmatrix}
    - F_i
\end{align*}
\]

(14)

With graph Laplacian \( \mathcal{L} = [l_{ij}] \in \mathbb{R}^{(N+1) 	imes (N+1)} \), \( l_{ij} = -a_{ij}, \) \( l_{ii} = \sum_{j \neq i} a_{ij} \) and \( \hat{r} = [\hat{r}_1^T \hat{r}_2^T \cdots \hat{r}_{N+1}^T]^T \), then

\[
(\mathcal{L} \otimes I_2) \hat{r} = -k_2 (\mathcal{L} \otimes I_2) \hat{r} - k_1 (\mathcal{L} \otimes I_2) \hat{r}.
\]

(15)

Letting \( \hat{r}_{ci} = r_i - R_{N+1} \), Eq. (15) can be written as

\[
\begin{align*}
    \begin{bmatrix}
        \sum_{j=1}^{N+1} a_{1j} & -a_{12} & \cdots & -a_{1(N+1)} \\
        -a_{21} & \sum_{j=1}^{N+1} a_{2j} & \cdots & -a_{2(N+1)} \\
        \vdots & \vdots & \ddots & \vdots \\
        -a_{N1} & -a_{N2} & \cdots & \sum_{j=1}^{N+1} a_{Nj}
    \end{bmatrix}
    \begin{bmatrix}
        \ddot{x}_1 \\
        \ddot{x}_2 \\
        \vdots \\
        \ddot{x}_{N+1}
    \end{bmatrix}
    &= -k_2
    \begin{bmatrix}
        \ddot{r}_{c1} \\
        \ddot{r}_{c2} \\
        \vdots \\
        \ddot{r}_{c(N+1)}
    \end{bmatrix}
    - k_1
    \begin{bmatrix}
        \ddot{r}_{c1} \\
        \ddot{r}_{c2} \\
        \vdots \\
        \ddot{r}_{c(N+1)}
    \end{bmatrix}
    - F_i
\end{align*}
\]

(16)

Introducing \( \ddot{\mathcal{L}} = [l_{ij}] \in \mathbb{R}^{N \times N} \), \( \ddot{r}_{ci} = -a_{ij}, \) \( \ddot{r}_{ci} = \sum_{j=1}^{N+1} a_{ij} \) and \( \ddot{r}_{c} = [\ddot{r}_{c1}^T \ddot{r}_{c2}^T \cdots \ddot{r}_{c(N+1)}]^T \), gives

\[
(\ddot{\mathcal{L}} \otimes I_2) \ddot{r}_{c} = -k_2 (\ddot{\mathcal{L}} \otimes I_2) \ddot{r}_{c} - k_1 (\ddot{\mathcal{L}} \otimes I_2) \ddot{r}_{c}
\]

(17)
Now we show that $\tilde{L}$ is positive definite. The network topology among agents is connected by assumption 1. In other words, $x^T (L_N \otimes I_2) x = 0$ where $x = [x_1^T, \ldots, x_N^T]^T$ with $x_i \in \mathbb{R}^2$ and graph Laplacian $L_N$ which is the network topology among $N$ agents, if and only if $x_i = x_j$. Moreover, at least one agent can get information of target object by assumption 2. At least one $a_{i(N+1)}$ equals to 1. Thus, $x^T (\tilde{L} \otimes I_2) x = x^T (L_N \otimes I_2) x + \sum_{i=1}^{N} a_{i(N+1)} x_i^T x_i \geq 0$ and $x^T (\tilde{L} \otimes I_2) x = 0$ if and only if $x = 0$. Therefore, $\tilde{L}$ is positive definite. According to $\tilde{L} > 0$, Eq. (17) can be written as

$$\tilde{r}_e = -k_2 \tilde{r}_e - k_1 \dot{r}_e. \quad (18)$$

Because $k_1, k_2 > 0$, it follows that $\tilde{r}_e \to 0$ as $t \to \infty$ regardless of the network topology. Thus, it follows $r_i \to r_t + R_i$ and each agent converges to the desired position.

4. NUMERICAL SIMULATION

In this section, we apply the control law (11) in simulation to achieve capturing. We consider cooperative capturing for $N = 4$ agents with moving target object. We compare two simulation results where gains $k_1, k_2$ are different. Case 1 has larger $k_1$ than $k_2$ to converge position error faster. Case 2 has larger $k_2$ than $k_1$ to converge velocity error faster. Step size is 0.1[s] in simulation. Simulation parameters are tabulated in Table 1. The desired position of agent $R_i$ is chosen that agents are placed centered on target object.

$$R_i = \xi \left[ \cos \left( \frac{2\pi(i-1)}{n} \right) \sin \left( \frac{2\pi(i-1)}{n} \right) \right]^T \quad (19)$$

Agents are arranged at equal intervals on the circumference of capturing circle. The radius of capturing is chosen $\xi = 0.585[\text{m}]$ because agents contact target object. In addition, the velocity of agents are 0[m/s] and the velocity of target object is 0.25[m/s] at $t = 0$. There exists a Gaussian noise with variance 0.0002 to the reaction force.

<table>
<thead>
<tr>
<th>Table 1 Simulation parameters</th>
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<tbody>
<tr>
<td>mass of agent $m_i$ [kg]</td>
</tr>
<tr>
<td>mass of target $m_t$ [kg]</td>
</tr>
<tr>
<td>radius of agent $l_i$ [m]</td>
</tr>
<tr>
<td>radius of target $l_t$ [m]</td>
</tr>
<tr>
<td>Coefficient of restitution $\rho$</td>
</tr>
<tr>
<td>Sensor range $\rho$ [m]</td>
</tr>
</tbody>
</table>

Case 1. $k_1 = 5$, $k_2 = 3$

The simulation results are shown in Figs. 4 to 7. Fig. 4 shows the trajectory of each agent and target object. Black circle and red circle denote, respectively, agents and target object. Fig. 5 shows final positions of agents and target object, and Fig. 6 shows the position error of each agent for capturing. Fig. 7 shows reaction force of each agent from target object. From Fig. 6, the position errors converge to 0 with bounded errors, and agents are achieving capturing. From Figs. 4, 5, 7, agents are subjected to reaction force, but they capture target object.

Therefore, it can be seen that agents can achieve the capturing target object despite there exists influence of reaction force.
**Case 2.** $k_1 = 3$, $k_2 = 5$

The simulation results are shown in Figs. 8 to 11. Fig. 8 shows the trajectory of each agent and target object. Fig. 9 shows final positions of agents and target object, Fig. 10 shows the position error of each agent, and Fig. 11 shows reaction force from target object. From Figs. 8 to 11, it can be seen that agents are achieving control object (10).

Compared case 1 with case 2, in case 1, the convergence of position error is faster than case 2. This is because $k_1$ in case 1 is larger than $k_1$ in case 2. Thus, agents hit target object and agents are subjected to large reaction force. In contrasts to this, $k_2$ is larger than $k_1$ to converge velocity error faster in case 2, convergence of position error is slow. However, agents are not subjected to reaction force because there is no error between velocity of agents and target object.
5. CONCLUSIONS

In this paper, we studied cooperative capturing by multi-agent system with reaction force from target object. We dealt second order model to consider the influence of reaction force. The network topology among agents was time-varying and connected, and at least one agent could get information of target object. Cooperative control law consisted of graph theory and consensus algorithm has been proposed and its convergence properties have also been given. We showed capturing was achieved asymptotically. Finally, the effectiveness of proposed control law was verified by the numerical simulations.

In future work, we will extend the results to address the issue of conveyance to the desired position. We will also address the force control when agent contact target object.

REFERENCES


