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Predictive Control and Estimation for Systems with Information Structured Constraints

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Abstract: This paper deals with a predictive control and estimation problem with information structured constraints in a constrained finite-time optimal control framework. A large-scale system with information structures is defined as a system in which each subsystem collects spatio-temporally different information, and in general it is necessary to collect and process information in a distributed fashion. We propose a novel predictive control scheme and an estimation law with local information that each distributed subsystem obtains. The effectiveness of the proposed control and estimation law is evaluated through numerical simulations of a simplified micro grid.

Keywords: Predictive control, Estimation, Information structure, Constrained system, Smart/Micro grids

1. INTRODUCTION

Information structured systems consist of multiple sub-systems collecting distributed spatio-temporally different information. Such systems had been extensively studied in the 1970s, and a variety of control methodologies were also proposed and discussed (see [1]).

These systems with information structures receive a lot of attention again (see [2], [3], [4], [5]) in recent years due to the growing interests in new environmental and energy technologies such as smart grid, micro grid, sensor networks, and so on. For example, it is required in the smart grid (Fig. 1) that different power generators (photovoltaic generator, wind farm, fuel cell, micro gas turbine and so on) and power storages cooperate in energetically and environmentally optimal fashion.

Note that this research subject is also closely related to cooperative control theory which has attracted attention over the years in systems and control society. Among numerous research works on systems with information structures, our focus is on the distributed optimal control and estimation approach ([6], [7], [8], [9], [10], [11]). Especially, we focus on the work due to [4], where the information structures are modelled by covariance constraints and a stationary LQG control law is presented based on the model. A finite and infinite time LQG control with covariance constraints is also proposed in [5].

Though the present control method might be useful for another objective, this work is basically motivated by the aforementioned network connected micro grid / smart grid, where it is required to satisfy input and state constraints the system inherently possesses and to use information available on-line such as weather forecast[12].

The main objective of this paper is thus to propose a novel predictive control scheme, which is known to be a useful control methodology in order to meet the above requirements, for systems with information structured constraints. We first present a predictive control scheme for a finite-time optimal control problem. Then, we prove that the constrained finite-time optimal control problem can be reduced to a deterministic convex programming problem. We moreover present an estimation scheme for systems with information structure based on the works due to Rantzer [13].

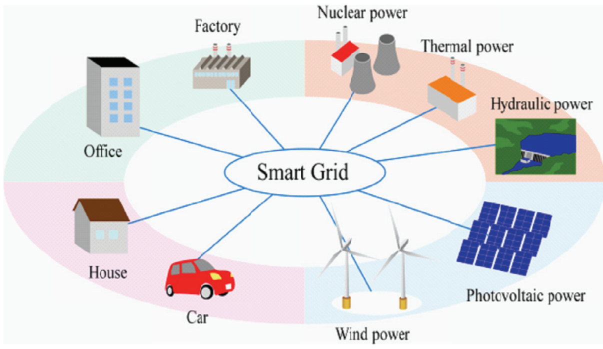


Fig. 1 Smart Grid

Finally the effectiveness of the proposed control and estimation law is shown via a numerical simulation of a simplified micro grid model.

The following notations are used in this paper. $\mathbb{Z}_+$ is the non-negative integer set, $\mathbb{R}^n$ is the n-dimensional real space, $\mathbb{R}^n_+$ is the n-dimensional non-negative real space, $\mathbb{R}^{m\times n}$ is the $m \times n$-dimensional real space, $\mathbf{E}$ is an expectation operator, $Q > 0 (Q \geq 0)$ means that Q is a positive(non-negative) matrix, and $\mathbf{Tr}$ is a trace operator.

2. SYSTEM DESCRIPTION

Consider the following discrete-time linear time invariant system,

$$x(t+1) = Ax(t) + Bu(t) + Fw(t), \quad (1a)$$
$$y(t) = Cx(t) + v(t), \quad (1b)$$

where $t \in \mathbb{Z}_+$, $x(t) \in \mathbb{R}^{n_x}$ is the state, $u(t) \in \mathbb{R}^{n_u}$ is the control input, $y(t) \in \mathbb{R}^{n_y}$ is the measurement, $w(t) \in \mathbb{R}^{n_w}$ and $v(t) \in \mathbb{R}^{n_v}$ are respectively zero mean white process and sensor noises. The system (1) is assumed to be controllable and observable, and satisfies

$$\mathbf{E}\begin{bmatrix} w(t) \\ v(t) \end{bmatrix}\begin{bmatrix} w^T(s) & v^T(s) \end{bmatrix} = \begin{bmatrix} R_{ww} & 0 \\ 0 & R_{vv} \end{bmatrix}\delta_{ts}, \tag{2}$$

$$\delta_{ts} = 1 \quad \text{if} \quad t = s, \quad \delta_{ts} = 0 \quad \text{if} \quad t \neq s,$$

$$\mathbf{E}\ w(t)x^T(s) = 0, \ \ \mathbf{E}\ v(t)x^T(s) = 0 \quad \text{if} \quad t \geq s. \tag{3}$$

A system with information structures where each subsystem collects spatio-temporally different information is considered. Thus, the information available for control and estimation differs from subsystems to other subsystems, which should be included into the system model (1). Here we assume that information propagates through the communication channels at least as fast as it propagates through the plant itself (funnel causality[3] or partially nested information structure[1]). Then, it is known that the difference can be represented by communication delays between subsystems as exemplified below.

We consider a micro grid system with Grid 1-3 in Fig. 2 which is represented by

$$\begin{bmatrix} z_1(t+1) - z_1^{ref}(t+1) \\ z_2(t+1) - z_2^{ref}(t+1) \\ z_3(t+1) - z_3^{ref}(t+1) \end{bmatrix} = \begin{bmatrix} \Xi_{11} & \Xi_{12} & 0 \\ \Xi_{21} & \Xi_{22} & \Xi_{23} \\ 0 & \Xi_{32} & \Xi_{33} \end{bmatrix}\begin{bmatrix} z_1(t) - z_1^{ref}(t) \\ z_2(t) - z_2^{ref}(t) \\ z_3(t) - z_3^{ref}(t) \end{bmatrix}$$

$$+\begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} + \begin{bmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \end{bmatrix} + \begin{bmatrix} \Delta z_1(t) - z_1^{ref}(t+1) \\ \Delta z_2(t) - z_2^{ref}(t+1) \\ \Delta z_3(t) - z_3^{ref}(t+1) \end{bmatrix}, \tag{4a}$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = \begin{bmatrix} z_1(t) - z_1^{ref}(t) \\ z_2(t) - z_2^{ref}(t) \\ z_3(t) - z_3^{ref}(t) \end{bmatrix} + \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix}, \tag{4b}$$

where $z_i(t)$ is the total power of the i-th subsystem, $u_i(t)$ is the power generated by the i-th micro gas turbine, $\Delta z_i(t)$ is by the i-th photo-voltaic generator, $z_i^{ref}(t)$ is the desirable power of i-th subsystem, and $w_i(t)$, $v_i(t)$ are zero mean white noises. In this example, we assume that it takes 1 time step for information to be passed from a subsystem to neighbors.

Note that the power $\Delta z_i(t)$ by the photo-voltaic generator is uncontrollable, whereas $u_i(t)$ by the micro gas turbine is controllable. However, it is possible to gain the predictive information on $z_i^{ref}(t)$ and $\Delta z_i(t)$ over a certain finite future time interval at each time instant by energy demands forecasting[14].

Throughout this paper, we employ this system for ease of explanation under the assumption of $z_i^{ref}(t) = \Delta z_i(t) = 0 \ \forall t \in \mathbb{Z}_+$. Under the above simplifications, we get a simple model represented as

$$\begin{bmatrix} z_1(t+1) \\ z_2(t+1) \\ z_3(t+1) \end{bmatrix} = \begin{bmatrix} \Xi_{11} & \Xi_{12} & 0 \\ \Xi_{21} & \Xi_{22} & \Xi_{23} \\ 0 & \Xi_{32} & \Xi_{33} \end{bmatrix}\begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \end{bmatrix} + \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} + \begin{bmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \end{bmatrix}. \tag{5}$$

Note that all of the following statements hold true without this assumption if the models of z_i^{ref} and Δz_i are available. However, in practical situations, such models

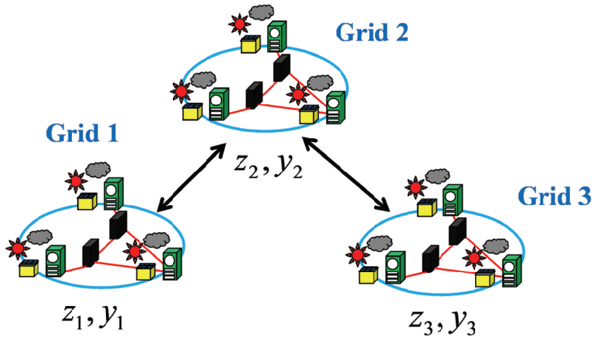


Fig. 2 Connections of Micro Grid Systems

are not obtained and only numerical data is available online. Predictive control we handle in this paper allows us to include such future information available on-line into the control problem easily. This will be demonstrated in Section 4 through numerical simulation results.

It is well-known that one can model information structures by using the covariance constraints for general systems. The constraints can be generally described by

$$\mathbf{E}\ u_i(t)w_j(t-\tau) = 0 \quad \text{if} \ \ \tau \leq \text{dist}(i, j), \tag{6}$$

where $\tau \in \mathbb{Z}_+$, dist(i, j) is the distance between subsystem i and subsystem j. Thus, the model (1a) with covariance constraints (5) can be employed in control problems of systems with information structures.

We next derive a model used for state estimation. The information structures of the other subsystems can be described by covariance constraints. In a general case, the covariance constraints can be represented as

$$\mathbf{E}\ y_i(t)v_j^T(t-\tau) = 0 \quad \text{if} \ \ \tau \leq \text{dist}(i, j) - 1. \tag{7}$$

The micro grid in general have a variety of input and state constraints. We thus include two kinds of constraints into the system description: The first one is the power constraints represented by covariance constraints.

$$\mathbf{E}\ x^T(t)Q_x x(t) + u^T(t)Q_u u(t) \leq \gamma, \ Q_z, Q_u > 0 \tag{8}$$

The second one is the mean polytopic constraints represented by

$$\mathbf{E}\begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \in \mathcal{D} \subset \mathbb{R}^{n_x+n_u}, \tag{9}$$

where $\mathcal{D}$ is a convex polytope including the origin as an interior. In summary, the system model under consideration is given by (1) with constraints (5), (6), (7) and (8).

3. PREDICTIVE CONTROL FOR SYSTEMS WITH INFORMATION STRUCTURED CONSTRAINTS

3.1 Constrained Finite-Time Optimal Control problem

In this section, we propose a state feedback predictive control law i.e. $C = I$ and $v \equiv 0$ for systems with information structures. For this purpose, we first consider the following constrained finite-time optimal control (CFTOC) problem.

Problem 1:

$$\min_{u(0),\cdots,u(N_c-1)} \mathbf{E}\left\{x^T(N_c)P_{N_c}x(N_c) + \sum_{j=0}^{N_c-1}\begin{bmatrix}x(j)\\u(j)\end{bmatrix}^T P \begin{bmatrix}x(j)\\u(j)\end{bmatrix}\right\} \quad (10a)$$

subject to

$$x(k+1) = Ax(k) + Bu(k) + Fw(k),\ x(0) = y_0$$

$$\mathbf{E}\begin{bmatrix}x(k)\\u(k)\end{bmatrix}^T Q_j \begin{bmatrix}x(k)\\u(k)\end{bmatrix} \le \epsilon_j,\ j = 1,2,\ldots,m \quad (10b)$$

$$\mathbf{E}\begin{bmatrix}x(k)\\u(k)\end{bmatrix}^T Q \begin{bmatrix}x(k)\\u(k)\end{bmatrix} < \gamma,\ Q = \begin{bmatrix}Q_x & 0\\ 0 & Q_u\end{bmatrix} \quad (10c)$$

$$\mathbf{E}\begin{bmatrix}x(k)\\u(k)\end{bmatrix} \in \mathcal{D} \subset \mathbb{R}^{n_x+n_u} \quad (10d)$$

$$\mathbf{E}\ x(N_c) \in \mathcal{F}. \quad (10e)$$

$Q = H^TH > 0$ and Q_r are symmetric matrices. Note that $x(0)$ is a probability variable with mean y_0 and variance R_{vv}. (9b) represents the covariance constraint introduced by the information structures, where we relax the constraint by inserting a sufficiently small parameters ϵ_j. The constraint (9c) describes the power constraints (7) and (9d) describes mean constraints (8) on states and inputs, $\mathcal{F}$ is a terminal constraint set, where

$$\mathcal{F} := \left\{\begin{bmatrix}x\\u\end{bmatrix} \middle| M_F\begin{bmatrix}x\\u\end{bmatrix} \le \mathbf{1}\right\}. \quad (11)$$

The present control law determines the control input according to the receding horizon control policy, i.e. the above problem with the initial state $y_0 = x(t)$ is solved, the first one of the computed control moves is implemented and then the optimal control problem is newly solved at the next step with the horizon shifted forward by one time instant. In the following, the numbers of linear constraints in (9d) and (9e) are denoted by N_D and N_F respectively.

3.2 Solution to CFTOC

In this subsection, we present a solution to Problem 1. In terms of this issue, we have the following theorem.

Theorem 1: Problem 1 is reduced to the following deterministic optimization problem.

Problem 2:

$$\min_{s,\lambda} s \text{ subject to} \begin{bmatrix}\bar{\Phi}(\lambda) & \bar{\Psi}(\lambda)\\ * & \bar{\eta}(\lambda)+s\end{bmatrix} > 0. \quad (12)$$

The definitions of $\bar{\Phi}$, $\bar{\Psi}$ and $\bar{\eta}$ are given in the proof.

Proof: First, we define

$$U(0) := \begin{bmatrix}u^T(0) & u^T(1) & \cdots & u^T(N_c-1)\end{bmatrix}^T$$

$$W(0) := \begin{bmatrix}w^T(0) & w^T(1) & \cdots & w^T(N_c-1)\end{bmatrix}^T$$

Then, as well known, $x(k),\ k \in [0, N_c]$ is described by the form of

$$x(k) = \Phi_k U(0) + \Psi_k\begin{bmatrix}x(0)\\W(0)\end{bmatrix} \quad (13)$$

where

$$\Phi_k := \begin{bmatrix}A^{k-1}B & \cdots & B & 0 & \cdots & 0\end{bmatrix},$$

$$\Psi_k := \begin{bmatrix}A^k & A^{k-1}F & \cdots & F & 0 & \cdots & 0\end{bmatrix}.$$

Substituting (12) to Problem 1 yields the following optimization problem

Problem 3:

$$\min_{U(0)} \mathbf{E}\ f(U(0), W(0), x(0)) \quad (14a)$$

subject to

$$\mathbf{E}h_{i,k}(U(0), W(0), x(0)) \le 0, \quad (14b)$$
$$i = 1,\cdots,m,\ k = 0,\cdots,N_c$$

$$\mathbf{E}\ g_l(U(0), W(0), x(0)) \le 0,\ l = 0,\cdots,N_M, \quad (14c)$$
$$N_M := N_c(N_D+1) + N_F.$$

where

$$f(U(0), W(0), x(0)) = U^T(0)\Phi U(0) + U^T(0)\Psi\begin{bmatrix}x(0)\\W(0)\end{bmatrix} + \begin{bmatrix}x(0)\\W(0)\end{bmatrix}^T \Gamma \begin{bmatrix}x(0)\\W(0)\end{bmatrix}$$

$$h_{i,k}(U(0), W(0), x(0)) = U^T(0)H^u_{i,k}U(0) + U^T(0)H^{ux}_{i,k}\begin{bmatrix}x(0)\\W(0)\end{bmatrix} + \begin{bmatrix}x(0)\\W(0)\end{bmatrix}^T H^x_{i,k}\begin{bmatrix}x(0)\\W(0)\end{bmatrix} - \epsilon_i \quad (15)$$

$$g_l(U(0), W(0), x(0)) = U^T(0)G^u_lU(0) + U^T(0)G^{ux}_l\begin{bmatrix}x(0)\\W(0)\end{bmatrix} + \begin{bmatrix}x(0)\\W(0)\end{bmatrix}^T G^x_l\begin{bmatrix}x(0)\\W(0)\end{bmatrix} - \gamma,$$
$$l = 0,\cdots,N_c \quad (16)$$

$$g_l(U(0), W(0), x(0)) = A^x_{l-N_c}\begin{bmatrix}x(0)\\W(0)\end{bmatrix} + A^u_{l-N_c}U(0) - 1,$$
$$l = N_c+1,\cdots,N_M \quad (17)$$

The constraints (14), (15) and (16) respectively correspond to the constraints (9b), (9c), and (9d), (9e) in Problem 1.

The matrices in the above equations are defined by

$$\Phi := \sum_{k=0}^{N_c-1} \Phi_k^T P \Phi_k + \Phi_{N_c}^T P_N \Phi_{N_c}$$

$$\Psi := \sum_{k=0}^{N_c-1} \Psi_k^T P \Psi_k + \Psi_{N_c}^T P_N \Psi_{N_c}$$

$$\Gamma := 2\sum_{k=0}^{N_c-1} \Phi_k^T P \Psi_k + 2\Phi_{N_c}^T P_N \Psi_{N_c}$$

$$H_{i,k}^u = \Phi_k^T Q_i \Phi_k,\ H_{ux}^u i,k = 2\Psi_k^T Q_i \Phi_k,\ H_{i,k}^x = \Psi_k^T Q_i \Psi_k,$$
$$i = 1,\cdots,m,\ k = 0,\cdots,N_c,$$
$$G_l^u = \Phi_l^T Q \Phi_l,\ G_{ux}^u = 2\Psi_l^T Q \Phi_l,\ G_l^x = \Psi_l^T Q \Psi_l,$$
$$l = 0,\cdots,N_c,$$
$$A^u = \begin{bmatrix} M_D\Phi_0 \\ \vdots \\ M_D\Phi_{N_c-1} \\ M_F\Phi_{N_c} \end{bmatrix},\ A^x = \begin{bmatrix} M_D\Psi_0 \\ \vdots \\ M_D\Psi_{N_c-1} \\ M_F\Psi_{N_c} \end{bmatrix},$$

and A_l^x and A_l^u denotes l-th lows of A_x and A_u respectively.

Let us now define the Lagrange function

$$L(U(0),\lambda) = \mathbf{E}f + \sum_{i=1}^{m}\sum_{k=0}^{N_c} \lambda_{(N_c+1)(i-1)+k+1}\mathbf{E}h_{i,k} + \sum_{l=0}^{N_M} \lambda_{l+1+m(N_c+1)}\mathbf{E}g_l,$$

with the Lagrange multipliers $\lambda = (\lambda_1,\cdots,\lambda_{m(N_c+1)+N_M+1})$. We assume that all the elements of λ are nonnegative. More specifically, the function is equivalent to

$$L(U(0),\lambda) = \mathbf{E}U^T(0)\bar{\Phi}(\lambda)U(0) + \mathbf{E}\tilde{\Psi}(\lambda)U(0) + \mathbf{E}\eta(\lambda), \tag{18}$$

where

$$\bar{\Phi}(\lambda) = \Phi + \sum_{i=1}^{m}\sum_{k=0}^{N_c} \lambda_{(N_c+1)(i-1)+k+1}H_{i,k}^u + \sum_{l=0}^{N_c}\lambda_{l+1+m(N_c+1)}G_l^u,$$

$$\tilde{\Psi}(\lambda) = \Psi + \sum_{i=1}^{m}\sum_{k=0}^{N_c} \lambda_{(N_c+1)(i-1)+k+1}\begin{bmatrix} x(0) \\ W(0)\end{bmatrix}^T H_{i,k}^{ux} + \sum_{l=0}^{N_c}\lambda_{l+1+m(N_c+1)}\begin{bmatrix} x(0) \\ W(0)\end{bmatrix}^T G_l^{ux} + \sum_{l=N_c+1}^{N_M}\lambda_{l+1+m(N_c+1)}A_{l-N_c}^u$$

$$\eta(\lambda) = \begin{bmatrix} x(0) \\ W(0)\end{bmatrix}^T \tilde{\Gamma}(\lambda)\begin{bmatrix} x(0) \\ W(0)\end{bmatrix} - \sum_{i=1}^{m}\sum_{k=0}^{N_c}\lambda_{(N_c+1)(i-1)+k+1}\epsilon_i - \gamma\sum_{l=0}^{N_c}\lambda_{l+1+m(N_c+1)} + \sum_{l=N_c+1}^{N_M}\lambda_{l+1+m(N_c+1)}\left(A_{l-N_c}^x\begin{bmatrix} x(0) \\ W(0)\end{bmatrix} - 1\right)$$

$$\tilde{\Gamma}(\lambda) = \Gamma + \sum_{i=1}^{m}\sum_{k=0}^{N_c}\lambda_{(N_c+1)(i-1)+k+1}H_{i,k}^x + \sum_{l=0}^{N_c}\lambda_{l+1+m(N_c+1)}G_l^x$$

Then, we also define $\theta(U(0)) = \sup_{\lambda\geq 0} L(U(0),\lambda)$ and $\omega(\lambda) = \inf_{U(0)} L(U(0),\lambda)$. Note that the problem of $\inf_{U(0)}\theta(U(0))$ is the same as Problem 3. The dual problem of the problem is represented by $\sup_{\lambda\geq 0}\omega(\lambda)$.

In the following, we attempt to solve the dual problem $\sup_{\lambda\geq 0}\omega(\lambda)$. For this purpose, we first consider $\omega(\lambda) = \inf_{U(0)} L(U(0),\lambda)$. From optimality conditions of $\inf_{U(0)} L(U(0),\lambda)$, for any optimal solution $\tilde{U}$, we get

$$2\bar{\Phi}(\lambda)\tilde{U} + \mathbf{E}\tilde{\Psi}(\lambda) = 0,$$

namely $\tilde{U} = -\bar{\Phi}^{-1}(\lambda)\mathbf{E}\tilde{\Psi}(\lambda)/2$ holds true. Notice that $\mathbf{E}\tilde{\Psi}(\lambda)$ is given by

$$\mathbf{E}\tilde{\Psi}(\lambda) = \Psi + \left(\sum_{i=1}^{m}\sum_{k=0}^{N_c}\lambda_{(N_c+1)(i-1)+k+1}H_{i,k}^{ux} + \sum_{l=0}^{N_c}\lambda_{l+1+m(N_c+1)}G_l^{ux}\right)\begin{bmatrix} y_0 \\ 0\end{bmatrix}^T + \sum_{l=N_c+1}^{N_M}\lambda_{l+1+m(N_c+1)}A_{l-N_c}^u \tag{19}$$

For notational simplicity, we define $\bar{\Psi}(\lambda) := \mathbf{E}\tilde{\Psi}(\lambda)/2$. Then, $\tilde{U}$ is represented by $\tilde{U} = -\bar{\Phi}^{-1}(\lambda)\bar{\Psi}(\lambda)$. Hence, we have

$$\begin{aligned}\omega(\lambda) &= \inf_{U(0)} L(U(0),\lambda) = L(\tilde{U},\lambda) \\ &= (\bar{\Phi}^{-1}(\lambda)\bar{\Psi}(\lambda))^T\bar{\Psi}(\lambda) - 2\bar{\Psi}(\lambda)\bar{\Phi}^{-1}(\lambda)\bar{\Psi}(\lambda) + \mathbf{E}\eta(\lambda) \\ &= -\bar{\Psi}(\lambda)\bar{\Phi}^{-1}(\lambda)\bar{\Psi}(\lambda) + \mathbf{E}\eta(\lambda).\end{aligned}$$

Notice that $\mathbf{E}\eta(\lambda)$ (denoted by $\bar{\eta}(\lambda)$) is given by

$$\begin{aligned}\bar{\eta}(\lambda) &= \mathbf{E}\begin{bmatrix} x(0) \\ W(0)\end{bmatrix}^T\tilde{\Gamma}(\lambda)\begin{bmatrix} x(0) \\ W(0)\end{bmatrix} - \sum_{i=1}^{m}\sum_{k=0}^{N_c}\lambda_{(N_c+1)(i-1)+k+1}\epsilon_i \\ &\quad -\gamma\sum_{l=0}^{N_c}\lambda_{l+1+m(N_c+1)} + \sum_{l=N_c+1}^{N_M}\lambda_{l+1+m(N_c+1)}\left(A_{l-N_c}^x\mathbf{E}\begin{bmatrix} x(0) \\ W(0)\end{bmatrix} - 1\right) \\ &= \mathbf{Tr}\tilde{\Gamma}(\lambda)\begin{bmatrix} y_0^T y_0 + 2R_{vv} & 0 \\ 0 & R_{ww}\end{bmatrix} - \sum_{i=1}^{m}\sum_{k=0}^{N_c}\lambda_{(N_c+1)(i-1)+k+1}\epsilon_i \\ &\quad -\gamma\sum_{l=0}^{N_c}\lambda_{l+1+m(N_c+1)} + \sum_{l=N_c+1}^{N_M}\lambda_{l+1+m(N_c+1)}\left(A_{l-N_c}^x\mathbf{E}\begin{bmatrix} y_0 \\ 0\end{bmatrix} - 1\right)\end{aligned}$$

In summary, $\omega(\lambda)$ is given by

$$\omega(\lambda) = -\bar{\Psi}(\lambda)\bar{\Phi}^{-1}(\lambda)\bar{\Psi}(\lambda) + \bar{\eta}(\lambda),$$

which is deterministic. By letting s be an upper bound of $-\omega(\lambda)$ and using Schur Complement [15], the problem $\sup_{\lambda\geq 0}\omega(\lambda)$ is reduced to the problem (11). This completes the proof. ■

Note that since $\bar{\Phi}$ and $\bar{\Psi}$ are linear in terms of λ, Problem 2 is a linear matrix inequality (LMI) optimization problem, which is solvable by some existing solvers.

3.3 Estimation for Systems with Information Structure

In the previous subsection, we presented a state feedback predictive control law for systems with information structures. However, it is difficult to apply it to practical systems because the augmented systems usually include the disturbances as state variables. In this section, we thus present a state estimation scheme for systems with information structure.

Here we employ a moving horizon estimator [16] with variance minimization for state estimation in order to get a state estimate $\hat{x}^i(k)$. In terms of this issue, if we employ the variance minimization for state estimation the objective function to be minimized is given by

$$\mathbf{E}\sum_{i=0}^{N_e}(y(t-i) - C\hat{x}(t-i))^T Q_e (y(t-i) - C\hat{x}(t-i)). \tag{20}$$

By using the same procedure as the previous subsection the minimization problem of (19) under the communication delay constraint (6) is also reduced to an LMI problem and it can be solved in a distributed fashion via dual decomposition techniques. Hence, we can implement an output feedback predictive control scheme at least according to the certainly equivalence principle. Though we guess that separation principle holds for the control and estimation in the absence of power and mean constraints, its theoretical investigations are one of future works of this paper. The separation principal in the presence of constraints does not hold and analysis on the integrated system is also left as a future work.

4. EVALUATION VIA CONTROL OF MICRO GRID

4.1 Dynamics of Micro Grid

The reference [4] presented a modeling method where the communication delays are represented by covariance constraints. Consider a micro grid system connected each other in Fig.2. Then, the available information $\mathcal{Z}_i(t)$ of i-th subsystem at time t for determining $u_i(t)$ is:

$$\begin{aligned} \mathcal{Z}_1(t) &= (\bar{z}_1(t), \bar{z}_2(t-1), \bar{z}_3(t-2)), \\ \mathcal{Z}_2(t) &= (\bar{z}_1(t-1), \bar{z}_2(t), \bar{z}_3(t-1)), \\ \mathcal{Z}_3(t) &= (\bar{z}_1(t-2), \bar{z}_2(t-1), \bar{z}_3(t)), \end{aligned} \tag{21}$$

where $\bar{z}_i(t) := (z_i(t),\ z_i(t-1),\ \cdots,\ z_i(0))$, $i = 1,2,3$. Equations in (20) are rewritten as

$$\begin{aligned} \mathcal{Z}_1(t) &= (\bar{z}(t-2), w_1(t-1), w_1(t-2), w_2(t-2)), \\ \mathcal{Z}_2(t) &= (\bar{z}(t-2), w_1(t-2), w_2(t-1), w_2(t-2), \\ &\qquad , w_3(t-2)) \qquad (22) \\ \mathcal{Z}_3(t) &= (\bar{z}(t-2), w_2(t-2), w_3(t-1), w_3(t-2)), \end{aligned}$$

where $\bar{z}(t) := (z(t),\ z(t-1),\ \cdots,\ z(0))$ and $z(t) := [z_1(t)\ z_2(t)\ z_3(t)]^T$. The above equations mean that communication delays of (5) are reduced to delays of disturbances. Thus, communication delays can be represented by the following covariance constraints between inputs and disturbances [4].

$$\begin{aligned} &\mathbf{E}\ u_1(t)w_2(t-1) = 0 \quad \mathbf{E}\ u_2(t)w_3(t-1) = 0 \\ &\mathbf{E}\ u_1(t)w_3(t-1) = 0 \quad \mathbf{E}\ u_3(t)w_1(t-1) = 0 \\ &\mathbf{E}\ u_1(t)w_3(t-2) = 0 \quad \mathbf{E}\ u_3(t)w_1(t-2) = 0 \\ &\mathbf{E}\ u_2(t)w_1(t-1) = 0 \quad \mathbf{E}\ u_3(t)w_2(t-1) = 0 \end{aligned} \tag{23}$$

If we define the vectors

$$\begin{aligned} u(t) &:= [u_1(t)\ u_2(t)\ u_3(t)]^T, \\ w(t) &:= [w_1(t)\ w_2(t)\ w_3(t)]^T, \\ x(t) &:= [z^T(t)\ w^T(t-1)\ w^T(t-2)]^T, \end{aligned}$$

we have the augmented state equation with the same form as (1a), where

$$A := \begin{bmatrix} \Xi & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \end{bmatrix} \quad B := \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix} \quad F := \begin{bmatrix} I \\ I \\ 0 \end{bmatrix}$$

Namely, state evolutions of the micro grid system are modeled by (1a) with covariance constraints (22).

We next derive a model used for state estimation. Note that though we ignore the control input $u(t)$, all of the following statements are also true in the presence of $u(t)$ as long as it satisfies (22). Under the simplification of $z_i^{ref}(t) = \Delta z_i(t) = 0\ \forall t \in \mathbb{Z}_+$, the output equation is given by

$$\begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = \begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \end{bmatrix} + \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix}. \tag{24a}$$

Let us now denote by $\mathcal{Y}_i(t)$ available measurement information of subsystem Grid i at time t. Then, $\mathcal{Y}_i(t)$ is given by

$$\begin{aligned} \mathcal{Y}_1(t) &= (\bar{y}_1(t), \bar{y}_2(t-1), \bar{y}_3(t-2)), \\ \mathcal{Y}_2(t) &= (\bar{y}_1(t-1), \bar{y}_2(t), \bar{y}_3(t-1)), \\ \mathcal{Y}_3(t) &= (\bar{y}_1(t-2), \bar{y}_2(t-1), \bar{y}_3(t)), \end{aligned} \tag{25}$$

where $\bar{y}_i(t) := (y_i(t),\ y_i(t-1),\ \cdots,\ y_i(0))$. Equation (24) is replaced by

$$\begin{aligned} \mathcal{Y}_1(t) &= (\bar{y}(t-2), y_1(t), y_1(t-1), y_2(t-1)), \\ \mathcal{Y}_2(t) &= (\bar{y}(t-1), y_2(t)), \\ \mathcal{Y}_3(t) &= (\bar{y}(t-2), y_2(t-1), y_3(t), y_3(t-1)), \end{aligned} \tag{26}$$

where $\bar{y}(t) := (y(t),\ y(t-1),\ \cdots,\ y(0))$ and $y(t) := (y_1(t),\ y_2(t),\ y_3(t))$.

This means that available information is different from subsystem to subsystem due to the communication delays. Similarly to the previous subsection, the difference of available measurements is reduced to that of information on the noises $v(s)$, $s \le t$. For example, subsystem : Grid 1 has the following covariance constraints.

$$\begin{aligned} &\mathbf{E}\ y_1(t)v_2^T(t) = 0, \\ &\mathbf{E}\ y_1(t)v_3^T(t) = 0, \qquad (27) \\ &\mathbf{E}\ y_1(t-1)v_3^T(t-1) = 0. \qquad (28) \end{aligned}$$

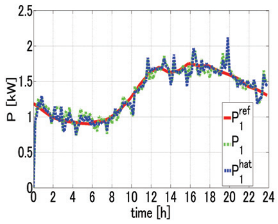
4.2 Simulation Results

Finally the micro grid system has been represented as same with the Problem 1 and the predictive output control law in the Section 3 can be applied to the system.

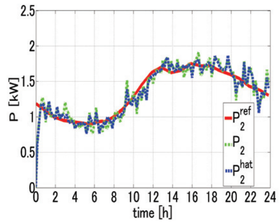
In this section, we demonstrate the effectiveness of the present control law through the system in Fig.2 with $\Xi_{ij} = 0.3\ \forall i, j \in \{1, 2, 3\}$. Let the parameters be $Q = I,\ P_{xx} = I,\ N_c = 2$.

In order to compute the power by photo-voltaic generators, the data of daylight at Tokyo on 30th August, 2008 is employed (See Fig. 3 (d)). We also let the reference power P_i^{ref} be the power consumption pattern [14].

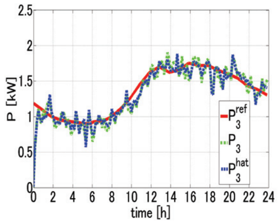
It can be seen from Fig. 3 that the states are successfully estimated and our predictive control scheme achieves a good tracking of $z_i(t)$ to the reference $z_i^{ref}(t)$. This simulation demonstrates the effectiveness of the present control scheme.



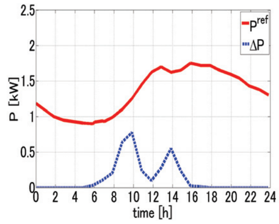
(a) Time response of z_1



(b) Time responses of z_2



(c) Time responses of z_3



(d) Δz

Fig. 3 Simulation Results

5. CONCLUSION

This paper has dealt with a predictive control scheme for systems with information structures.

First, we have proposed a predictive control scheme for a finite-time optimal control problem. Then, we have proved that the constrained finite-time optimal control problem is reduced to a deterministic convex programming problem which means that the optimal control problem can be solved efficiently.

Furthermore an estimation scheme for systems with information structure has been presented. In order to formulate the constrained finite-time optimal control problem, we have used the modeling method due to [4], where the information structures are described by covariance constraints.

Finally the effectiveness of the proposed control law and the estimator have been demonstrated through a numerical simulation of a simplified micro grid.

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