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Load Leveling Control by Real-Time Dynamical Pricing Based on Steepest Descent Method

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Abstract: This paper describes load leveling control by Real-Time Dynamical pricing based on optimization method. First, we propose a profit maximization problem of supply and demand, satisfying the supply constraint and the balance between supply and demand. Then we rewrite them to dual problem which minimizes electricity price. We made algorithm to solve the dual problem by Steepest Descent Method and proof its convergence. Finally some numerical simulations show the effectiveness of the proposed load leveling control.

Keywords: Real-Time Pricing, Smart Grid, Steepest Descent Method, Electricity market

1. INTRODUCTION

The power of demand is increasing because of economic growth, on the other hand, that of supply is decreasing because of environment concern[1]. In this paradigm shift, smart-grid system with Demand Side Management (DSM) is required to use electricity efficiently. One of the DSM technique is to use electricity price, which is called "dynamic pricing". Several dynamic pricing is already proposed, Time-Of-Use Pricing (TOU)[2] and Critical-Peak-Pricing (CPP)[3]. These pricing set fixed price at given times or peak times. It's same to existing pricing in that the price is pre-determined. Consumers use power in pre-determined price plan and generators generates power to balance supply and demand. Left side of Fig.1 shows the fixed price model.

In this paper, we use Real-Time-Pricing (RTP) which sets price one hour period at beginning of the day[4]. Right side of Fig.1 shows the RTP model. This pricing calculates optimal price, hence RTP can control demand more flexible than TOU and CPP. It's useful for unstable renewable energy (like wind power and solar power) and unexpected short supply (like accidents). However, there are little approach for RTP by control theorem and there are needs to verify the system with new smart-grid system.

In the RTP model, there are three types of player: generators, consumers, and Independent System Operator (ISO). The ISO is a non-profit institution and independents from the generators and the consumers. In Japan, the institution corresponds to the load dispatching center.

ISO has an electricity market and calculate price to match balance of supply and demand. There are many electricity markets[5] and they have some system correspond to ISO's market. In this market, generators and consumers don't bit and don't conduct derivatives trading and negotiation transaction.

The information asymmetry is large problem for ISO to calculate price. Generators know their own cost to generate power, hence ISO can get information about generators and formulate supply model. But consumers don't know their own utility to consume power, (in addition consumers composed from many consumers and it's decentralized[6]), hence ISO can't formulate demand model. ISO must calculate optimal price from only supply and demand plan. Roozbehani[7] modeled and analyzed the nonlinear system. The system is updated by a kind of Gauss-Seidel Method[8] and it may be unstable relying on generators or consumers model and the system doesn't consider constrained generation. In this paper, we propose RTP model with Steepest Descent Method and level demand load. Price is updated from supply or demand plan. Assign proper stepsize value and the system will be stable and converge in the constrained generation.

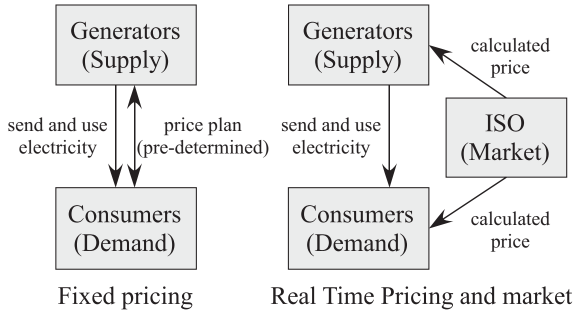

Fig. 1 Fixed Pricing and Real Time Pricing

This paper is organized as follows. The system, supply and demand model is presented in Section 2. We propose maximization problem and its dual problem in Section 3. To solve the problem, we employ Steepest Descent Method as an optimization method in Section 4. In section 5, we add perturbation to extend the model. Simulation result are given in Section 6, finally this paper is concluded in Section 7.

2. PROBLEM FORMULATION

We model an electricity market with three participants: generators, consumers, and ISO. Generators and Consumers plan their supply or demand to maximize their profit. In this model, consumers and generators make plan for next day's 24 hour period every per 1 hour. ISO

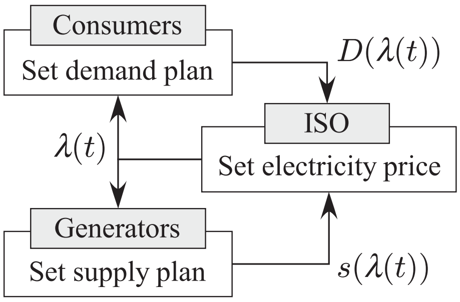

Fig. 2 Schematic views of the model

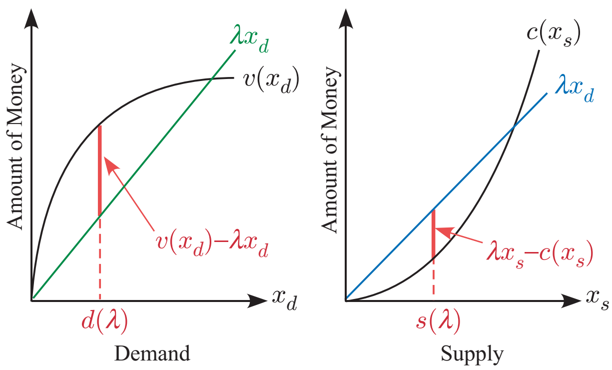

Fig. 3 Utility function $v(x_d)$ and Cost function $c(x_s)$

controls the electric system to match supply and demand and to maximize profits of society as a whole, which is expressed as the sum of welfare of generators and consumers. We consider the maximization problem solved by the ISO.

The market sets the electricity price as follows. Fig.2 is the conceptual diagram.

1. The ISO sets within-day electricity price per hour and convey it to generators and consumers.
2. Generators and consumers create their plan to generate or consume and convey these plan to ISO.
3. If the supply and the demand is not match, ISO resets price and convey again.
4. Repeat step.2 and step.3 N times.

2.1 Supply and demand model

Let $x_s \in \overline{\mathbb{R}}_+$(the set of nonnegative real numbers) and $x_d \in \overline{\mathbb{R}}_+$ denote the power supply of generators and the power demand of consumers. Let $[s_{\min}, s_{\max}]$ denotes the constrained generation. Let $\lambda \in \mathbb{R}_+$(the set of positive real numbers) denotes the electricity price. The monetary cost when the generators generate x_s is denoted by $c(x_s)$ which is generically called "cost function". Similarly, the monetary welfare when the consumers consume x_d is denoted by $v(x_d)$ which is generically called "utility function". These generators and consumers are modeled based on the Representative Agent model[6] and put them as one generator and one consumer. Here we employ assumptions for utility and cost function.

Assumption 1. $v(x_d)$ is a $\mathcal{C}^2[0,\infty)$ function, strictly increasing and strictly convex. $c(x_s)$ is a $\mathcal{C}^2[0,\infty)$ function, strictly increasing and strictly concave.

Let optimal power of supply denotes $s(\lambda) \in \overline{\mathbb{R}}_+$ which are called "supply function" and optimal power of demand denotes $d(\lambda) \in \overline{\mathbb{R}}_+$ which are called "demand function". We define $d(\lambda)$ and $s(\lambda)$ as follows.

$$d(\lambda) = \operatorname*{argmax}_{x_d \geq 0} v(x_d) - \lambda x_d \quad (1)$$

$$= \max_{x_d \geq 0}\{0, \{x | \dot{v}(x_d) = \lambda\}\} \quad (2)$$

$$s(\lambda) = \arg \max_{s_{\min} \leq x_s \leq s_{\max}} \lambda x_s - c(x_s) \quad (3)$$

$$= \max_{s_{\min} \leq x_s \leq s_{\max}} \{0, \{x | \dot{c}(x_s) = \lambda\}\} \quad (4)$$

λx_d is the cost to buy energy x and (1) means that the consumers buy $d(\lambda)$ to maximize their own welfare.

Similarly, λx_s is the sales of energy x and (3) means that the generators sell $s(\lambda)$ to maximize their own welfare. Fig.3 is the conceptual diagram of (1) and (3).

We rewrite supply and demand function to $s(\lambda) = \dot{c}^{-1}(\lambda), \quad d(\lambda) = \dot{v}^{-1}(\lambda)$ for simplification.

3. MAXIMIZATION PROBLEM

3.1 ISO model

In the electricity model, generators and consumers don't bit. So, we introduce the following assumption.

Assumption 2.

1. ISO knows function $c(x_s)$ and $s(\lambda)$.
2. On the other hand, ISO doesn't konw function $v(x_d)$ and $d(\lambda)$.

To simplify the maximization problem for ISO, we add the following assumption.

Assumption 3.

1. Resistive losses in the transmission and distribution lines are negligible.
2. The line capacities are high enough, and congestion will not occur.
3. There are sufficient reserve capacity.

ISO control to maximize the sum of generators and consumers profit, hence the maximization problem is as follows.

$$\max_{x_d \geq 0, s_{\min} \leq x_s \leq s_{\max}} \{v(x_d) - \lambda x_d\} + \{\lambda x_s - c(x_s)\} \quad (5)$$

$$\iff \max_{x_d \geq 0, s_{\min} \leq x_s \leq s_{\max}} v(x_d) - c(x_s) \quad (6)$$

$$\text{s.t. } x_s - x_d = 0$$

Because ISO can't get function $v(x_d)$ and control x_d, x_s, we can't solve (6) directly. With that, we will use a dual problem.

3.2 Dual Problem

Let lagrange coefficient denotes λ_0. (6) is primal problem and the dual problem is as follows.

$$\min_{\lambda} \varphi(\lambda), \quad (7)$$

$$\varphi(\lambda) = \max_{x_d \geq 0, s_{\min} \leq x_s \leq s_{\max}} L(\lambda_0, x_d, x_s), \quad (8)$$

$$L(\lambda_0, x_d, x_s) = (v(x_d) - \lambda_0 x_d) + (\lambda_0 x_s - c(x_s)) \quad (9)$$

Here is obtained a following lemma. It is derived from the Lagrangian.

Lemma 1. If assumptions 1 - 3 are satisfied, optimal solutions $x_d^* = d(\lambda_0)$, $x_s^* = s(\lambda_0)$ is obtained for the primal problem (6). Moreover λ corresponds to λ_0. The proof is written in [9].

We write λ instead of λ_0, and optimal solution write as $x_d^*(\lambda), x_s^*(\lambda)$.

When we describe $f(x_d, x_s) = v(x_d) - c(x_s)$, $h(x_d, x_s) = x_s - x_d$, here is obtained a following lemma.

Lemma 2. Assumptions 1 - 3 are satisfied. If $f(x_d, x_s)$ and $h(x_d, x_s)$ are continuous and the domain of (x_d, x_s) is closed unbounded set, $\varphi(\lambda)$ is convex function. In addition, If optimal solutions of dual problem λ^* is unique, $\varphi(\lambda)$ is differentiable and the gradient is as follows. The proof is written in [9].

$$\nabla\varphi(\lambda) = h(x_d^*(\lambda), x_s^*(\lambda)) \tag{10}$$

4. STEEPEST DESCENT METHOD

$\varphi(\lambda)$ can replace each optimization problem for x_d and x_s. φ_d, φ_s correspond to (1),(3) and these means to maximize welfare of generators or consumers.

$$\varphi(\lambda) = \varphi_d(\lambda) + \varphi_s(\lambda) \tag{11}$$

$$\varphi_d = \max_{x_d \geq 0} (v(x_d) - \lambda x_d) \tag{12}$$

$$\varphi_s = \max_{s_{\min} \leq x_s \leq s_{\max}} (\lambda x_s - c(x_s)) \tag{13}$$

Because (8) satisfies lemma 2, $\varphi(\lambda)$ is differentiable and the gradient is written as (14). The parameter $d(\lambda)$ gets from consumer and ISO can calculate $\nabla\varphi(\lambda)$

$$\nabla\varphi(\lambda) = x_s^*(\lambda) - x_d^*(\lambda) = -(d(\lambda) - s(\lambda)) \tag{14}$$

Steepest Descent Method is well known to solve optimal solution λ^*. Let $t \in \mathbb{Z}_+$(the set of positive integers) denotes iteration count and γ denotes stepsize which is constant and small value. The update formula can be written by (15).

$$\begin{aligned} \lambda(t+1) &= \lambda(t) - \gamma(\nabla\varphi(\lambda)) \\ &= \lambda(t) + \gamma(d(\lambda(t)) - s(\lambda(t))) \end{aligned} \tag{15}$$

We propose the price-update algorithm under the assumptions 1 - 3.

Algorithm 1.

1. The ISO set initial price $\lambda(0)$.
2. Generators and consumers calculate supply and demand from the electricity price. When the number of occurrences is t, they plan $d(\lambda(t))$ and $s(\lambda(t))$.
$$d(\lambda(t)) = \operatorname*{argmax}_{x_d \geq 0} v(x_d) - \lambda(t) x_d \tag{16}$$
$$s(\lambda(t)) = \arg\max_{s_{\min} \leq x_s \leq s_{\max}} \lambda(t) x_s - c(x_s) \tag{17}$$
3. Price is updated by Steepest Descent Method.
$$\lambda(t+1) = \lambda(t) + \gamma(D(\lambda(t)) - s(\lambda(t))) \tag{18}$$
4. Repeat step.2 and step.3 N times.

4.1 The convergence of Steepest Descent Method

We will proof convergence of Steepest Descent Method when the step size is constant[10]. Let $\lambda_1, \lambda_2 (\forall \lambda_1, \lambda_2 \in \mathbb{R})$ denote parameters of λ and we make the following assumption.

Assumption 4.

1. $\varphi(\lambda) \geq 0, \quad \forall\lambda \in \mathbb{R}$
2. $\varphi(\lambda)$ is Lipschitz continuity, which solves following equation where $K_0 \geq 0$. The minimum of K_0 is called Lipschitz constant and denoted as K.
$$\|\nabla\varphi(\lambda_1) - \nabla\varphi(\lambda_2)\| \leq K_0 \|\lambda_1 - \lambda_2\| \tag{19}$$
3. There is α_0 which satisfy following equation. The maximum of α_0 is denoted as α.
$$(\nabla\varphi(\lambda_1) - \nabla\varphi(\lambda_2))(\lambda_1 - \lambda_2) \geq \alpha_0 \|\lambda_1 - \lambda_2\|^2 \tag{20}$$

We have the following theorem.

Theorem 1. Consider assumptions 1 - 3 are satisfied. Then Algorithm 1 converges when γ holds the following inequality.

$$0 < \gamma < \frac{2\alpha}{K^2} \tag{21}$$

Proof. When we write algorithm as following equation, $T(\lambda(t))$ is called a contraction mapping.

$$\lambda(t+1) = T(\lambda(t)), \quad t = 0, 1, \cdots \tag{22}$$

$$\|T(\lambda_1) - T(\lambda_2)\| \leq \zeta \|\lambda_1 - \lambda_2\|, \quad 0 \leq \zeta < 1 \tag{23}$$

From the Algorithm 1, $T(\lambda(t))$ can be set as follows .

$$T(\lambda(t)) = \lambda(t) - \gamma\nabla\varphi(\lambda(t)) \tag{24}$$

For simplifying, we rewrite $\lambda(t)$ as λ. From assumption4,

$$\begin{aligned} &\|T(\lambda_1) - T(\lambda_2)\|^2 \\ &= \|((\lambda_1 - \gamma\nabla\varphi(\lambda_1)) - ((\lambda_2 - \gamma\nabla\varphi(\lambda_2))\|^2 \\ &= \|\lambda_1 - \lambda_2\|^2 + \gamma^2(\nabla\varphi(\lambda_1) - \nabla\varphi(\lambda_2))^2 \\ &\quad -2\gamma(\nabla\varphi(\lambda_1) - \nabla\varphi(\lambda_2))(\lambda_1 - \lambda_2) \\ &\leq \|\lambda_1 - \lambda_2\|^2 + \gamma^2 K \|\lambda_1 - \lambda_2\|^2 - 2\gamma\alpha\|\lambda_1 - \lambda_2\| \\ &= \left(1 - 2\alpha\gamma + K^2\gamma^2\right)\|\lambda_1 - \lambda_2\|^2 \end{aligned} \tag{25}$$

$(1 - 2\alpha\gamma + K^2\gamma^2)$ corresponds to ζ. If (24) is a contraction mapping, γ is given as follows.

$$\begin{aligned} (23) \quad &\Leftrightarrow \quad 0 < \left(1 - 2\alpha\gamma + K^2\gamma^2\right) < 1 \\ &\Leftrightarrow \quad 0 < \gamma < \frac{2\alpha}{K^2} \end{aligned} \tag{26}$$

□

4.2 Calculation of Lipschitz constant K and α

From (19),

$$\frac{\|\nabla\varphi(\lambda_1) - \nabla\varphi(\lambda_2)\|}{\|\lambda_1 - \lambda_2\|} \leq K_0 \tag{27}$$

If λ_1 approximates to λ_2, left-hand side of (27) means derivation. Now therefore (19) rewrite to as follows.

$$K_0 \geq \|\nabla^2\varphi(\lambda)\| = \left\| \frac{1}{\ddot{v}(\dot{v}^{-1}(\lambda))} - \frac{1}{\ddot{c}(\dot{c}^{-1}(\lambda))} \right\| \tag{28}$$

From assumption 1, $\ddot{c} > 0, \ddot{v} < 0$ and we can calculate Lipschitz constant K as follows.

$$K = \max_{\lambda} \left(\frac{1}{\ddot{c}(\dot{c}^{-1}(\lambda))} - \frac{1}{\ddot{v}(\dot{v}^{-1}(\lambda))} \right) \quad (29)$$

In a similar way, from (20),

$$\begin{aligned} \alpha_0 &\leq \frac{(\nabla(\lambda_1) - \nabla(\lambda_2))(\lambda_1 - \lambda_2)}{\|\lambda_1 - \lambda_2\|^2} \\ &\leq \frac{\|\nabla\varphi(\lambda_1) - \nabla\varphi(\lambda_2)\|}{\|\lambda_1 - \lambda_2\|} \end{aligned} \quad (30)$$

Same as (27), left-hand side of (30) means derivation. Now therefore (30) rewrite to as follows.

$$\alpha_0 \leq \|\nabla^2\varphi(\lambda)\| = \left\| \frac{1}{\ddot{v}(\dot{v}^{-1}(\lambda))} - \frac{1}{\ddot{c}(\dot{c}^{-1}(\lambda))} \right\| \quad (31)$$

From assumption 1, $\ddot{c} > 0, \ddot{v} < 0$ and we can α as follows.

$$\alpha = \min_{\lambda} \left(\frac{1}{\ddot{c}(\dot{c}^{-1}(\lambda))} - \frac{1}{\ddot{v}(\dot{v}^{-1}(\lambda))} \right) \quad (32)$$

It must be noted that parameters K and α can't get in real because ISO can't get function $\dot{v}^{-1}(\lambda)$.

5. CONSIDER WITH UNCERTAIN PERTURBATION MODEL

We consider two models of uncertainty in demand model, Additive Perturbation and Aggregative Perturbation. Utility function is modelled as

$$\tilde{v}(x_d) = v(x - d_1) \quad (33)$$

$$\tilde{v}(x_d) = (1+\delta_2) v \left(\frac{x_d}{(1+\delta_2)} \right) \quad (34)$$

$d_1 \in \mathbb{R}_+$ is aggregative parameter and $(1+\delta_2) \in \mathbb{R}_+$ is additive parameter. $\tilde{v}(x_d) : \overline{\mathbb{R}}_+ \mapsto \mathbb{R}_+$ is utility function with perturbation. (35) and (36) are demand function of (33) and (34).

$$\tilde{d}(\lambda) = d_1 + \dot{v}^{-1}(\lambda) \quad (35)$$

$$\tilde{d}(\lambda) = (1+\delta_2)\dot{v}^{-1}(\lambda) \quad (36)$$

Parameter d_1 means minimum demand, which is non-volatility for price. Parameter $(1+\delta_2)$ means model error, especially for aggregating.

Considering these two perturbation, new utility function $\tilde{v}(x_d)$ and demand function $D(\lambda)$ represents as follows.

$$\tilde{v}(x_d) = (1+\delta_2) v \left(\frac{x_d}{1+\delta_2} - d_1 \right) \quad (37)$$

$$D(\lambda) = d_1 + (1+\delta_2)\dot{v}^{-1}(\lambda) \quad (38)$$

From Section 4.2, K and α gets as follows.

$$K = \max_{\lambda} \left(\frac{1}{\ddot{c}(\dot{c}^{-1}(\lambda))} - \frac{(1+\delta_2)}{\ddot{v}(\dot{v}^{-1}(\lambda))} \right) \quad (39)$$

$$\alpha = \min_{\lambda} \left(\frac{1}{\ddot{c}(\dot{c}^{-1}(\lambda))} - \frac{(1+\delta_2)}{\ddot{v}(\dot{v}^{-1}(\lambda))} \right) \quad (40)$$

Rewrite $d(\lambda(t)) \to D(\lambda(t))$, $s(\lambda(t)) \to \dot{c}^{-1}(\lambda(t))$ and we get an algorithm from (15).

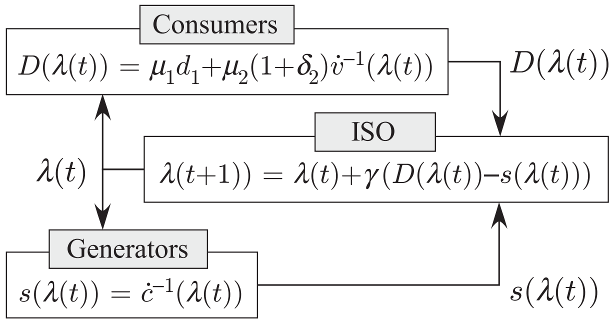

Fig. 4 Schematic views of the model

Algorithm 2.

1. The ISO set initial price $\lambda(0)$.
2. Generators and consumers calculate supply and demand from the electricity price. When the number of occurrences is t, the equation is as follows.
$$D(\lambda(t)) = d_1 + (1+\delta_2)\dot{v}^{-1}(\lambda(t)) \quad (41)$$
$$s(\lambda(t)) = \dot{c}^{-1}(\lambda(t)) \quad (42)$$
3. Price is updated by Steepest Descent Method.
$$\lambda(t+1) = \lambda(t) + \gamma \left(D(\lambda(t)) - s(\lambda(t)) \right) \quad (43)$$
4. Repeat step.2 and step.3 N times.

Fig.4 is the conceptual diagram.

6. NUMERICAL SIMULATION

The simulation goal is to converge demand plan of the next day in the constrained generation $[s_{\min}, s_{\max}]$. This simulation is for 24hour period and price is updated every per 1 hour.

We denoted parameters μ_1, μ_2 and rewrote (41) to following equation.

$$D(\lambda(t)) = \mu_1 d_1 + \mu_2(1+\delta_2)\dot{v}^{-1}(\lambda(t)) \quad (44)$$

$\delta_2 \sim \mathcal{N}(0, 0.01^2)$ is random disturbance. d_1 is electricity demand data of TEPCO in 10th August 2011 [11]. Using μ_1, minimum demand represents $\mu_1 d_1$.

Parameter μ_2 sets considering price elasticity. Parameter μ_1 adjust such that the day-long sum demand of $D(t)$ without constrained generation equal to the day-long sum demand of d_1. Denote iteration counts as N and demand or price at time k as $\bullet_k$, it represents as follows.

$$\sum_{k=0}^{24} D_k(N) \approx \sum_{k=0}^{24} d_{1,k} \quad (45)$$

Set utility function and cost function as follows.

$$v(x_d) = a \log(x_d), \quad c(x_s) = b x_s^3 \quad (46)$$

Parameter a adjusted such that the day-long sum demand of d_1 equal to day-long sum demand calculated by $v(x_d)$ and fixed price of TOU. Similarly, Parameter b adjusted such that the day-long sum demand of d_1 equal to day-long sum supply calculated by $c(x_s)$ and fixed price of

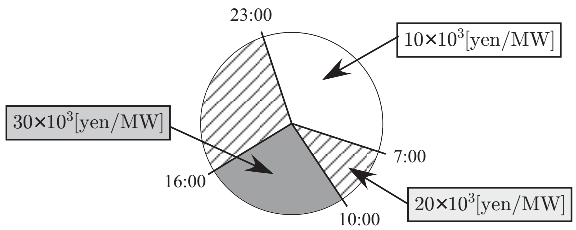

Fig. 5 TOU pricing plan

TOU. Denote fixed price at time k as $\lambda_{f,k}$, these represent as follows.

$$\sum_{k=0}^{24} d_{1,k} \approx \sum_{k=0}^{24} \dot{v}^{-1}(\lambda_{f,k}) \tag{47}$$

$$\sum_{k=0}^{24} d_{1,k} \approx \sum_{k=0}^{24} \dot{c}^{-1}(\lambda_{f,k}) \tag{48}$$

From (46), parameters calculate as follows.

$$a = \frac{\sum_{k=0}^{24} d_{1,k}}{\sum_{k=0}^{24} (1/\lambda_{f,k})} \tag{49}$$

$$b = \frac{1}{3}\left(\frac{\sum_{k=0}^{24} \sqrt{(\lambda_{f,k})}}{\sum_{k=0}^{24} d_{1,k}}\right)^2 \tag{50}$$

Fixed price $\lambda_{f,k}$[yen/MW] set as follows. We used TEPCO price plan "Season-and time-specific lighting (Denka Jozu)" [11] as a reference. (51) shows in Fig.5

$$\lambda_{f,k} = \begin{cases} 10 \times 10^3 & k = 0, \cdots, 6, 23, 24 \\ 20 \times 10^3 & k = 7, \cdots, 9, 16, \cdots, 22 \\ 30 \times 10^3 & k = 10, \cdots, 15 \end{cases} \tag{51}$$

We chose parameters $N = 100$, $s_{\min} = 3.4 \times 10^4$[MW], $s_{\max} = 3.6 \times 10^4$[MW], $\mu_2 = 0.2$, and $\gamma = 0.5$. From above parameters, $a = 4.009 \times 10^7$, $b = 9.993 \times 10^{-4}$, $\mu_1 = 0.67$ were calculated.

The result of simulation shows in Fig.6 - Fig.8. The horizontal axis is time k and the vertical axis is power or price. Yellow line is constrained generation. Red, cyan, green, purple and blue line is iteration count when $t = 1, 2, 3, 4, 5, 6$ and black line is when $t = 100$. From 9:00 to 19:00 in Fig.8, the demand exceed constrained generation at first, but price increase and demand decrease, then the black line converge near constrained generation.

6.1 Comparing existing and proposed method

We compare with previous method[7] and shows in Fig.9. Solid line is proposed method and dot line is existing method when $t = 100$.

Converging at 13:00 and 23:00 shows in Fig.10 and Fig.11. The horizontal axis is iteration count t and the vertical axis is power or price. Red and blue line are demand at 13:00 and 23:00. Solid and dot line are proposed method and existing method. In Fig.11 at 13:00, the demand converges to constrained generation. At 23:00, the convergence value,in the constrained generation, is same between existing method and proposed method. In this simulation at 23:00, proposed method is faster to converge than existing method, but the convergence speed of proposed method depends on the value of step size γ.

7. SUMMARY AND FUTURE WORKS

We proposed a load leveling control in the electricity market with the supply constraints based on the Steepest Descent Method. First, we modeled demand, supply, and electricity market, ISO. We proposed optimization problem for ISO and solved by dual problem and the Steepest Descent Method. We proved the system is stability by contraction mapping and confirmed by simulation. The system predicts the next day's electricity price or demand, hence we will extend to the prediction of one hour or five minutes ahead. Another future work is to derive the model considering variable energy and batteries.

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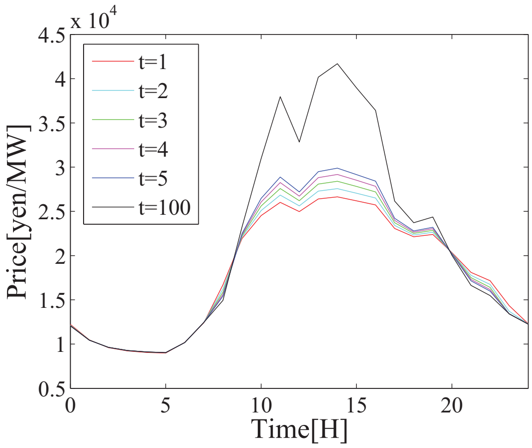

Fig. 6 Electricity price

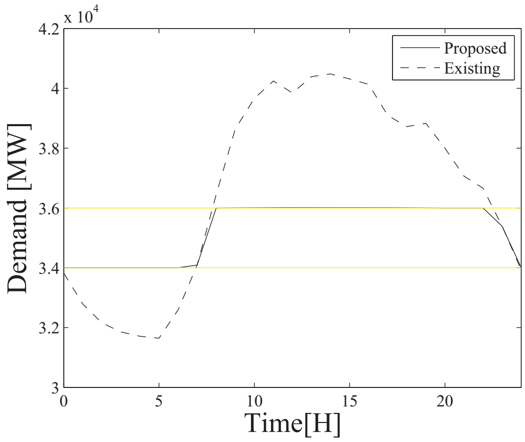

Fig. 9 Comparison between proposed method and existing method $(t = 100)$

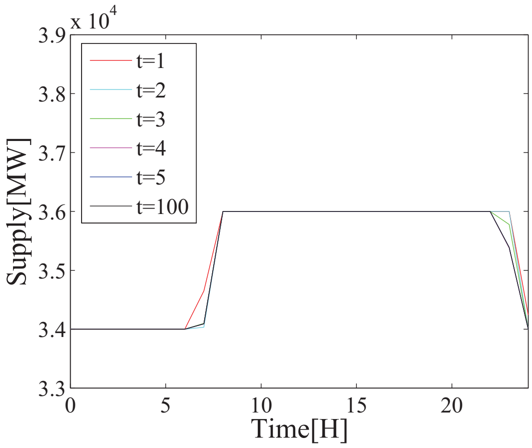

Fig. 7 Power supply

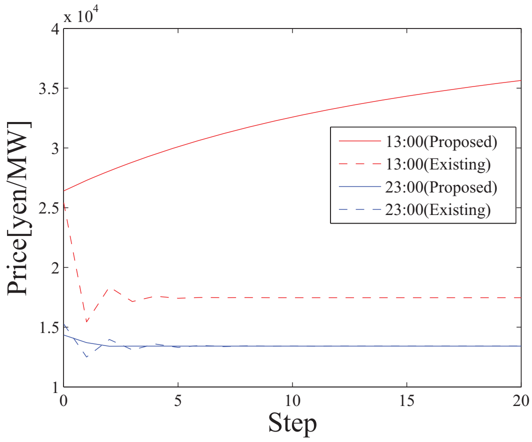

Fig. 10 Price at 13:00 and 23:00

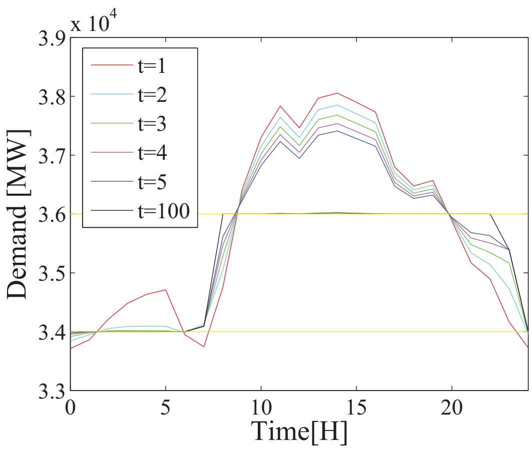

Fig. 8 Power demand

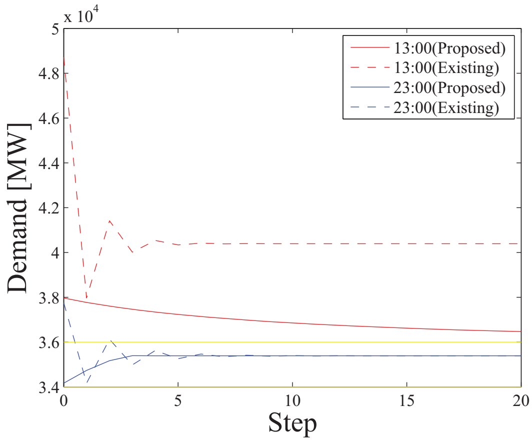

Fig. 11 Demand at 13:00 and 23:00