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Distributed Control for Load Frequency of Power Networks based on Iterative Gradient Methods

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Abstract: This paper deals with a distributed control for load frequency of power networks based on iterative gradient methods. The control objective is to minimize the quadratic cost function of load frequency control problem, and we apply this distributed control methodology to complicated large scale electrical power networks by using iterative gradient methods. Finally, simulation results of distributed large scale power network systems including distributed generations, batteries, and renewable energies in order to show the effectiveness of the load frequency control compared with decentralized control and centralized control.

Keywords: Gradient Methods, Distributed control, Load Frequency, Power Network, Distributed Generator

1. INTRODUCTION

Decision making problems to use different information concerning underlying uncertainties has been studied since the 1950's. Some of typical problems in these fields are called game problem and team problem et.al. Relations with the distributed control and decision making was strengthened in the 1970's, and distributed control was studied actively, and some special types of team problems were solved by [1, 2] . Recently, the concern for the distributed control has risen because many new large scale systems appeared that can apply a distributed control theory. The problem area has become recently gain renewed interest and researched by [3-5].

In recent years, energy problems and global warming have become the hottest worldwide problems. Therefore, a lot of distributed generations such as the photo voltaic and wind power generations, the biomass power generations, and the co-generations, are going to be installed in power systems from viewpoints of energy conservation and the cost reduction. At the same time, they have adverse affects on system frequency and fluctuation of voltage in power system. Hence, it is necessary to control every generation cooperatively and optimally and ensure safety. The optimal control of the electric power system was applied by [6]. Recently, system frequency control in power networks installing photo voltaic and wind power generations, battery energy storage system, and heat pump system, has been studied by [7].

In this paper, we propose the system frequency control method by using distributed control based on the iterative gradient methods to the distributed electric power network systems introducing the distributed generations. The varying state feedback gains are tuned and optimized repeatedly using iterative gradient methods. The advantage of this method is to change controller of new system and its adjacent system when the system configuration is changed.

Finally, The proposed method is applied to the frequency control on the electric power networks with the distributed generations, and effectiveness is shown by the simulation.

2. EXPRESSION OF DISTRIBUTED SYSTEM

We consider electric power networks that consist of more than two subsystems. Assume that there exist $N(\geq 2)$ electric power systems and i th subsystem($i = 1, \cdots, N$) is given by

$$x_i(t+1) = \sum_{j=1}^{N} A_{ij} x_j(t) + B_i u_i(t) + w_i(t) \tag{1}$$

where $\mathbb{Z}_+$is sets of nonnegative integers, $t \in \mathbb{Z}_+$, $x_i(t) \in \mathbb{R}^{n_{xi}}$ is a state of systems i, $A_{ii} \in \mathbb{R}^{n_{xi} \times n_{xi}}$, if $j \neq i$ $A_{ij} \in \mathbb{R}^{n_{xj} \times n_{xj}}$, $u_i(t) \in \mathbb{R}^{n_{ui}}$ is a control signal of system i, $w_i(t) \in \mathbb{R}^{n_{wi}}$ is a white noise with variance W, and $w_i(t)$ is independent of $x_i(s)$ for $s \leq t$. It is written as $(i, j) \in E$ when i th subsystem in the electric power system and j th subsystem are connected. If they are not connected, this topology is expressed by

$$A_{ij} = 0 \qquad \text{if } (i, j) \notin E \tag{2}$$

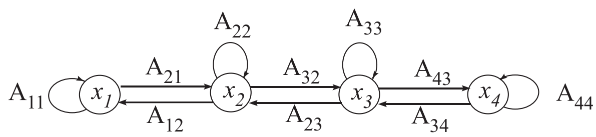

Fig. 1 An example of the distributed systems

A_{ij} shows how electric power system i has an influence on j. The matrix A of power networks whose structure is shown in Fig.1 can be expressed as the following equation.

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 \\ A_{21} & A_{22} & A_{23} & 0 \\ 0 & A_{32} & A_{33} & A_{34} \\ 0 & 0 & A_{43} & A_{44} \end{bmatrix} \tag{3}$$

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The electric power network that consists of N systems can be shown like equation (4).

$$x(t+1) = Ax(t) + Bu(t) + w(t) \tag{4}$$

where $t \in \mathbb{Z}_+$, $x(t) \in \mathbb{R}^{n_x}$ is state, $u(t) \in \mathbb{R}^{n_u}$ is control signal, $w(t) \in \mathbb{R}^{n_w}$ is white noise. It is assumed we can obtain information of all elements of the state vectors. This system uses the following state feedback as a control input.

$$u(t) = -Lx(t) \tag{5}$$

When we consider a distributed setup here, we limit feedback matrices to have a structure that matches the system. Therefore, only the measurement value in the adjoining electric power system is required to calculate control input $u_i(t)$ in electric power system i. It is assumed that the electric power system of N systems has the following feedback matrix.

$$L = \begin{bmatrix} L_{11} & L_{12} & \dots & L_{1N} \\ L_{21} & L_{22} & \dots & L_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ L_{N1} & L_{N2} & \dots & L_{NN} \end{bmatrix} \tag{6}$$

$$L_{ij} = 0 \quad \text{if } (i,j) \notin E \tag{7}$$

With this setup the dynamics closed-loop matrix $A - BL$ has the same structure as A.

3. DISTRIBUTED CONTROL BASED ON ITERATIVE GRADIENT METHODS

To decide feedback matrix L, the cost function is defined

$$J(L) = \boldsymbol{E}(|x|_Q^2 + |u|_R^2) \tag{8}$$

We define $|x|_Q^2 = x^T Qx$ and $|u|_R^2 = u^T Ru$, and assume Q and $R > 0$.

$$L_{k+1} = L_k - \gamma \bigtriangledown_L J \tag{9}$$

It is assumed that γ is small enough. Feedback matrix L can be calculated repeatedly by using gradient method. Next, the gradient of $J(L)$ was given by the following proposition 1.

Proposition 1: Anders Rantzer et.al (2009)
Given matrices A and B, consider L such that $A-BL$ has eigenvalues inside the unit circle. It is assumed that stationary stochastic process satisfies (4)-(**??**) and (5). The gradient of $J(L)$ defined by (8) can be shown by the next expression.

$$\bigtriangledown_L J = 2[RL - B^T P(A - BL)]X \tag{10}$$

where X and P satisfy the Lyapunov equations, respectively.

$$X = (A - BL)X(A - BL)^T + W \tag{11}$$

$$P = (A - BL)^T P(A - BL) + Q + L^T RL \tag{12}$$

What we want to know is not only the gradient of $\bigtriangledown_L J$ but distributed method of calculation. To know distributed method, we introduce the adjoint system and the following was already obtained proposition 2.

Proposition 2: Anders Rantzer et.al (2009)
Stationary stochastic process λ is defined by back wards iteration under the condition of proposition 1.

$$\lambda(t-1) = (A - BL)^T \lambda(t) - (Q + L^T RL)x(t) \tag{13}$$

where $x(t)$ are states of the original system. Then, the gradient of J is expressed as the following equation.

$$\bigtriangledown_L J = 2(RL\boldsymbol{E}xx^T + B^T \boldsymbol{E}\lambda x^T) \tag{14}$$

Proposition 2 gives a way of estimating an update direction for the feedback matrix L. But, we can't calculate in a distributed way. Because the covariance between all states and all adjoint states need to be determined under proposition 2. With appropriate prediction for the gradient, we can calculate feedback matrix L in a distributed way. To do this we restrict Q and R to be block-diagonal, which fit the size of the state space and number of inputs of each agent, respectively.

The system i needs to estimate λ_i by using only local information of adjacent system. Hence, adjoint state of system i is expressed as (15).

$$\lambda_i(t-1) = [A_L^T \lambda(t)]_i - [(Q + L^T RL)x(t)]_i \tag{15}$$

Each system is considered to be able to know both A_{Lij} when system i and system j are adjacent by previous assumptions. The equation (15) can be expressed as in (16).

$$\lambda_i(t-1) = \sum_{j\in E_i} (A - BL)_{ji}^T \lambda_j(t) - \Big(Q_i x_i(t) - \sum_{j\in E_i} L_{ji}^T R_j u_j(t)\Big) \tag{16}$$

This means that the adjoint state equation (15) can be simulated in each agent using only local information. As L satisfies (7), the actual update direction of the feedback matrix also need to satisfy this structure. Therefore, the gradient of $\bigtriangledown_L J$ is considered to be the subspace with the same structure. Letting G be updated, the following equation is obtained.

$$G_{ij} = (\bigtriangledown_L J)_{ij} \quad \text{if } (i,j) \in E \tag{17}$$

$$G_{ij} = 0 \quad \text{otherwise} \tag{18}$$

Assuming the projected gradient G is not 0, $-G$ decent direction of $J(L)$. This means that system i has only to determine the gradient in the blocks corresponding to the neighboring system. Therefore, it is necessary to estimate both $(RL\boldsymbol{E}xx^T)_{ij}$ and $(B^T \boldsymbol{E}\lambda x^T)_{ij}$ from information in the adjoining systems. The first term of the gradient given by proposition 2 can be simplified to the next expression.

$$(RL\boldsymbol{E}xx^T)_{ij} = -R_i \boldsymbol{E}u_i x_j^T \tag{19}$$

The second term can be rewritten as the next equation with the assumption of the structure on B.

$$(B^T \boldsymbol{E}\lambda x^T)_{ij} = B_i^T \boldsymbol{E}\lambda_i x_j^T \tag{20}$$

We can also estimate this term from the adjoining systems. With this analysis, it shows that we can estimate the update of a feedback matrix using only local information. The method is summarised in the following update scheme.

Algorithm 1: At time t_k, let the state feedback $u(t) = L^{(k)}x(t)$. To update the feedback matrix in system i .
1)Simulate the state $x_i(t)$ of the (4) for $t = t_k, \dots, t_k + N$ by exchanging states information with states of neighboring system.

$$x_i(t+1) = \sum_{j \in E_i} (A - BL)_{ij} x_j(t) + w_j(t) \tag{21}$$

2)Simulate adjoint state $\lambda_i(t)$ of the (15) for $t = t_k, \dots, t_k + N$ in back wards direction by exchanging states information with states of neighboring system.

$$\lambda_i(t-1) = \sum_{j \in E_i} (A - BL)^T_{ji} \lambda_j(t) - \Big(Q_i x_i(t) - \sum_{j \in E_i} L^T_{ji} R_j u_j(t)\Big) \tag{22}$$

3) $\boldsymbol{E} u_i x_j^T$ and $\boldsymbol{E} \lambda_i x_j^T$ are calculated by using all states of adjoining systems j.

$$(\boldsymbol{E} u_i x_j^T)_{est} = \frac{1}{N+1} \sum_{t=t_k}^{t_k+N} u_i(t) x_j(t)^T \tag{23}$$

$$(\boldsymbol{E} \lambda_i x_j^T)_{est} = \frac{1}{N+1} \sum_{t=t_k}^{t_k+N} \lambda_i(t) x_j(t)^T \tag{24}$$

4)The estimate of i, j block of the gradient becomes

$$G_{ij} = -2\big[R_i(\boldsymbol{E} u_i x_j^T)_{est} + B_i^T(\boldsymbol{E} \lambda_i x_j^T)_{est}\big] \tag{25}$$

5)When assuming some step γ , update of each neighboring system j is expressed as $L_{ij}^{(k+1)} = L_{ij}^{(k)} - \gamma G_{ij}$
6) $t_{k+1} = t_k + 1$, increase k by one and go to 1)

4. SYSTEM FREQUENCY CONTROL ON ELECTRIC POWER NETWORK

4.1 Electric power system model

We consider electric power network shown in Fig.2. It is assumed that the composition of four electric power systems are same, there are gas turbine generators and wind power generations in the system, and the power supply is done to the electric power demand with these power generating machines. To bring frequency deviation Δf of the system frequency close to 0 by using the TBC method as a frequency control in the electric power system in consideration of tie-line flow. The gas turbine output is controllable. We apply the electric power network in Fig.2 to the frequency analysis model as in Fig.4. Each system capacity is assumed to be same 40[MW] in this paper. In this paper, we consider distributed mass loads, which are battery electric storage systems (electric vehicle etc.) and electric water heaters (heat pump etc.), are controllable. We assume battery electric storage systems around 5% of the system capacity and electric water heaters are around 15% respectively. We assume the heat pump group and battery electric storage systems as first-order systems, and don't think of the capacity of the heat

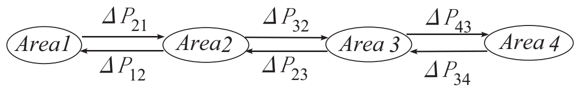

Fig. 2 Power networks of the system

pump group and battery electric storage system, and all system operate a constant characteristic. Here time constant T_H is assumed to be 4 [s], and T_E is assumed to be 0.2 s.

ΔP_{gi} is an output of gas turbine generator, Δx_{gi} is a governor input of gas turbine generator, ΔP_{Wi} is an output of wind power generation, ΔP_{Li} is a load fluctuation except controllable load, ΔP_{Ei} is an output of battery electric storage system, ΔP_{Hi} is an output of electric water heater, ΔP_{tie_i} is an tie-line power flow deviation in Fig.4. ΔP_i of the (26) shows the power generation electric power of area i and the supply error margin of power consumption. Frequency deviation Δf_i can be calculated in the block shown from the supply error margin that occurs in the system in Fig.3. when it is assumed that all generators in area is completely synchronous driving, system can be expressed as one equivalent model like Fig.3.

$$\Delta P_i = \Delta P_{gi} + \Delta P_{Wi} - \Delta P_{Li} + \Delta P_{tie\ i} + \Delta P_{Ei} - \Delta P_{Hi} \tag{26}$$

As the tie-line power flow deviation of are i is expressed $\Delta P_{tie_i} = T_{ij}(\Delta f_j - f_i)$ when the adjoining area is j. AR which is a regional demand is written as $AR_i = \Delta P_{tie_i} - B_i \Delta f_i$, and we defined as $U_{AR_i} = \int AR_i\, dt$. Moreover, the LFC signal is made by the PI control, and in the ratio of each generator. a_g, a_E, and a_H satisfy the ratio of the system capacity of each gas turbine, battery electric storage system, and electric water heater. They also satisfy following assumption $a_g + a_E + a_H = 1$. B_i is frequency bias, T_{ij} is synchronising coefficient, and R_g is regulation constant.

4.2 Expression of state space on electric power network

When the electric power system that consists of N system ($1 \leq i \leq N$) is expressed by the state space equation like (27). where $x_c(t) = [x_1^T(t), \cdots, x_N^T(t)]^T \in R^{7N}$, $u_c(t) = [u_1^T(t), \cdots, u_N^T(t)]^T \in R^{7N}$, $A_c = \begin{bmatrix} A_{11} & \cdots & A_{1N} \\ \vdots & \ddots & \vdots \\ A_{N1} & \cdots & A_{NN} \end{bmatrix}$,

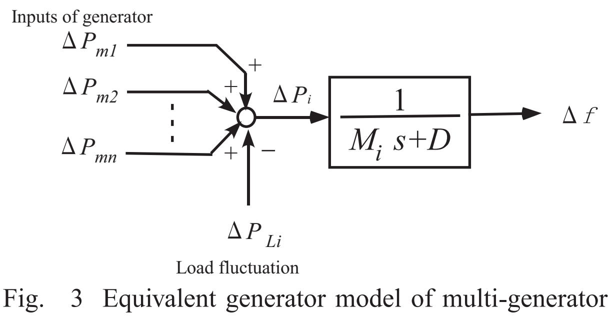

Fig. 3 Equivalent generator model of multi-generator system

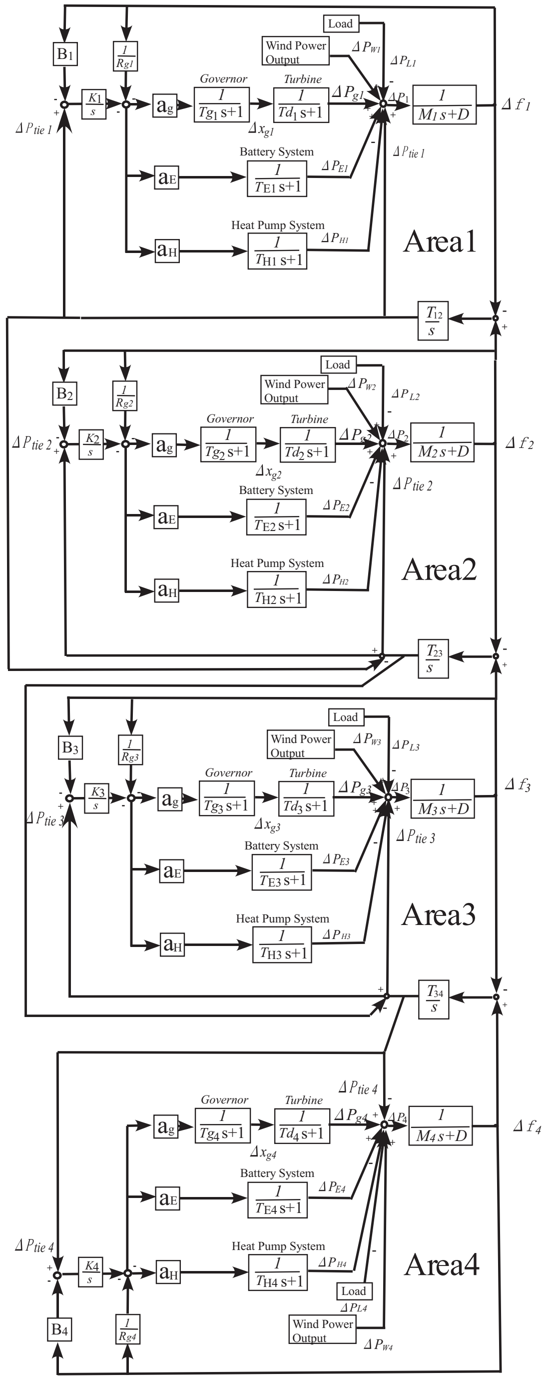

Fig. 4 Frequency analysis model of Power network

$$B_c = \mathrm{diag}[B_{11}, \cdots, B_{NN}].$$

$$\dot{x_c} = A_c x(t) + B_c u_c(t) + w(t) \tag{27}$$

The states and system matrices are given as the following equations.

Table 1 Parameters of Powernetwork

Parameters	Symbol	Value	Unit
inertia constant	M	0.20	puMWEs/Hz
damping constant	D	0.26	puMW/Hz
governer time constant	T_g	0.20	s
gas turbine constant	T_d	5.0	s
BESS time constant	T_E	0.20	s
HP time constant	T_H	4.5	s
Regulation constant	R_g	2.5	Hz/pu MW
Synchronising coefficient	T_{ij}	0.50	pu MW

$$x_i = \begin{bmatrix} \Delta P_{tie_i} \\ \Delta f_i \\ \Delta P_{g_i} \\ \Delta x_{vg_i} \\ \Delta P_{E_i} \\ \Delta P_{H_i} \\ U_{AR_i} \end{bmatrix}, \ B_{ii} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{T_{gi}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{T_{Ei}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{T_{Hi}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{28}$$

$$A_{ii} = \begin{bmatrix} 0 & -\sum_{j \in E_i} T_{ij} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{M_i} & -\frac{D}{M_i} & \frac{1}{M_i} & 0 & \frac{1}{M_i} & -\frac{1}{M_i} & 0 \\ 0 & 0 & -\frac{1}{T_{di}} & \frac{1}{T_{di}} & 0 & 0 & 0 \\ 0 & -\frac{1}{T_{gi}R_{gi}} & 0 & -\frac{1}{T_{gi}} & 0 & 0 & \frac{a_g K_i}{T_g} \\ 0 & -\frac{1}{T_{Ei}R_{gi}} & 0 & 0 & -\frac{1}{T_{Ei}} & 0 & \frac{a_E K_i}{T_E} \\ 0 & -\frac{1}{T_{Hi}R_{gi}} & 0 & 0 & 0 & -\frac{1}{T_{Hi}} & \frac{a_H K_i}{T_H} \\ 1 & -B_i & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{29}$$

$$A_{ij} = A_{ji} = \begin{bmatrix} 0 & T_{ij} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \ \text{if } (i,j) \in E \tag{30}$$

$$A_{ij} = \mathbf{0} \ \text{ if } (i,j) \notin E \tag{31}$$

To treat this system (27) as discrete system, it is converted by the sampling time $T_s = 1.0$[s]. $A = \exp(A_c T_s)$, $B = \int_0^{T_s} \exp(A\tau) d\tau B_c$.

$$x(t+1) = Ax(t) + Bu(t) + w(t) \tag{32}$$

where $w(t)$ is assumed a white noise, and given by the size of $10^{-4} \times I$. We assume a white noise is added as the state as turbulence, and assume load fluctuation and wind power generation and other white noises turbulence. The control objective is to minimize the following cost function. The control input is assumed as the $u = -Lx$, $Q = 0.1 \times I$, and $R = I$.

$$J(L) = \boldsymbol{E}(|x|_Q^2 + |u|_R^2) \tag{33}$$

4.3 Verification by simulation

We simulate the case all generators operate normally and the case one generator break down. Here, we only think optimal operating, so we neglect the subjection of power network. In this paper, we simulate and compare with the case of centralized control which can use all information in the system, the case of distributed control which can exchange information in the adjacent systems, and the case of decentralized control which can't exchange information. Fig.5 is a decentralized control with no information exchanges. On the other hand, distributed control have the structure like Fig.6 to exchange plant information by among controllers. In this paper,decentralized control and distributed control repeatedly calculate the state feedback gain, and update the gain here one after another. centralized control uses fixed optimal feedback gain. Moreover, iteration time N is 5. The numerical values of parameters are given in Table 1.

4.3.1 In case all generators operate normally

In this section, we assume that all generators operate normally. Case that all generators operate normally means system matrix is stable. We compare proposed distributed control with conventional method of PI control. We give the 0.2 disturbance as load fluctuation in 450[s] when we can regard feedbackgain as converged. This reason is from Fig.9. Fig.9 express feedback gains without disturbance at the same situation, and we understand it converge at 200[s]. Fig.7-8 show results. The folmer shows fluctuation of frequency with white noize, the latter shows fluctuation of frequency without white noize. Fig.7-8 show proposed methods improve the speed of covergence.

4.3.2 In case one generator break down

In this section, we assume that one generator break down at 0[s]. This case means that system matrix is unstable. Fig.10 shows fluctuation of frequency in area1. We compare proposed distributed control with conventional method of PI control. Huge blackout will happened if the case happen, but proposed method stabilize the system using distributed generators. Fig.11 shows the comparative result of the frequency fluctuation of centralized control, decentralized control, and distributed control. Fig.12 shows the comparative result of the cost function of centralized control, decentralized control, and distributed control. We can understand the cost function has lowered in order of centralized control, distributed control, and decentralized control. Centralized control have improved the performances mostly. However, we wishes to control in a distributed way because large scale system have difficulty to obtaining same information and control centralized way. Compared with distributed control and decentralized control, the former performance has improved. However, actually we should consider that distributed control has the cost in which information is exchanged.

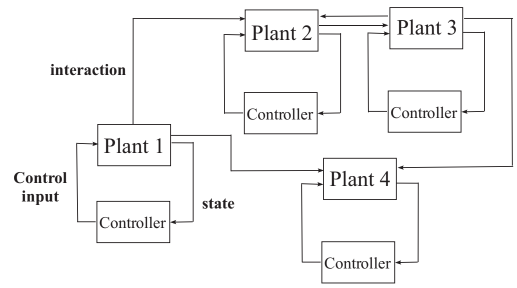

Fig. 5 Decentralized control

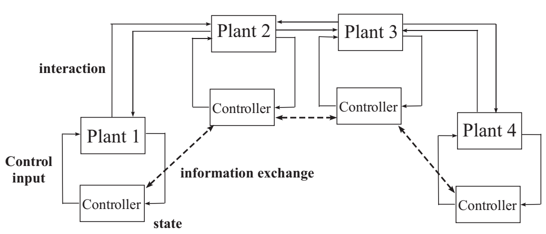

Fig. 6 Distributed control

5. CONCLUSION

In this paper, we applied the distributed control based on the gradient methods iteratively to the system frequency control of the electric power network system that introduced the distributed generations. We showed that the state feedback gain is updated one after another by repeatedly by using the gradient method, and system is stabilized to a white noise that assumed the output of load fluctuation and wind power generation. Then, we compared with centralized control, decentralized control, and distributed control, and show the effectiveness of the distributed control that used the gradient method iteratively.

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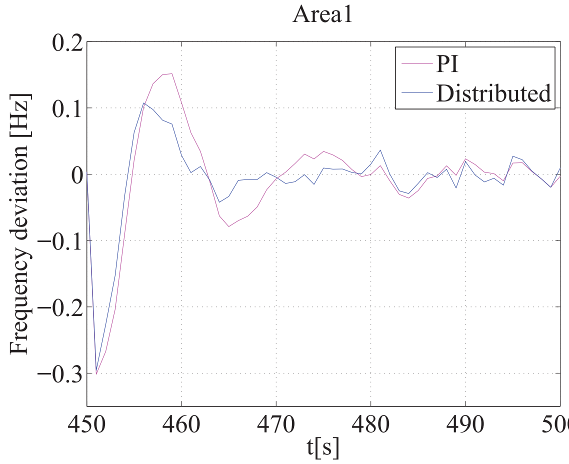

Fig. 7 Comparison between conventional method and proposed method in case1 with white noize

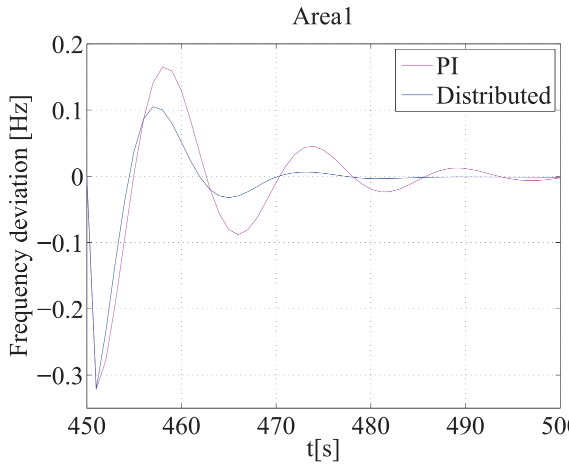

Fig. 8 Comparison between conventional method and proposed method in case1 without white noize

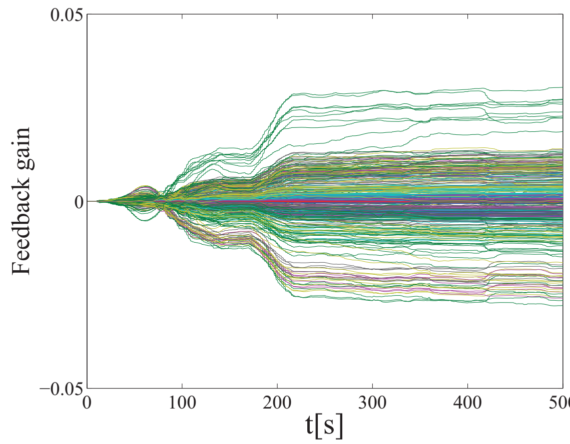

Fig. 9 Feedback gain of distributed control

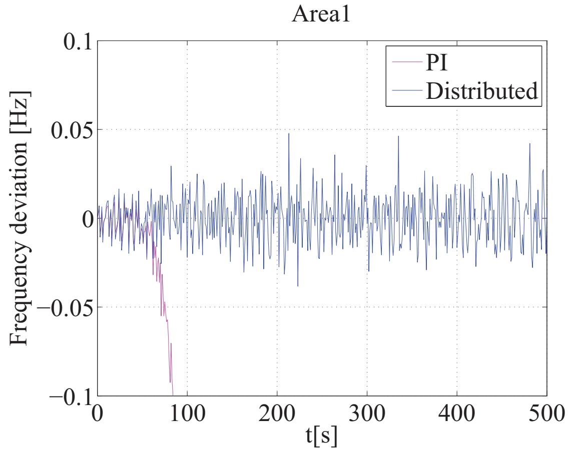

Fig. 10 Comparison between conventional method and proposed method in case2

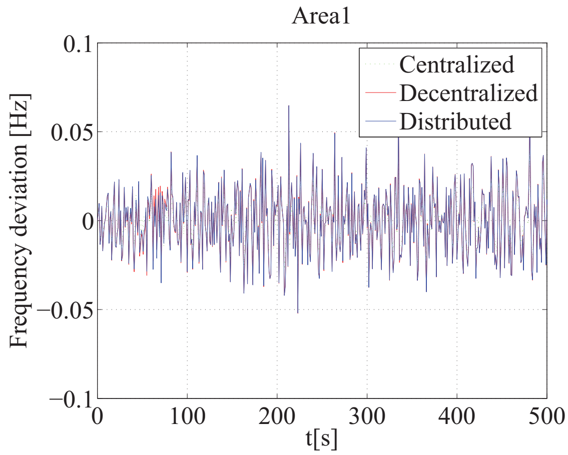

Fig. 11 Comparison between centralized control and decentralized control and distributed control in case2

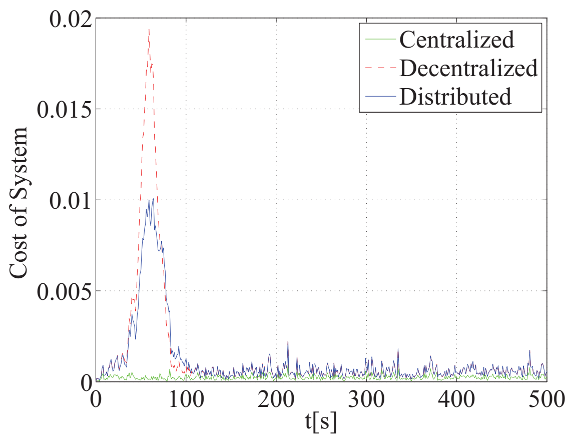

Fig. 12 Cost of centralized control and decentralized control and distributed control