Cooperative Conveyance by Vehicle Swarms with Dynamic Network Topology

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Abstract: This paper deals with cooperative conveyance control by vehicle swarms with dynamic network topology. First, we introduce a dynamic network topology that depends on relative distance between the vehicles. Secondly, we propose a novel conveyance strategy based on consensus seeking with dynamic network topology. The proposed strategy needs at least one vehicle which can acquire the information of the target-object and network topology among vehicles is time-varying but always connected. Furthermore the strategy is composed of Surround and. Transport mode. To analyze the convergence of conveyance with dynamic network topology, algebraic graph theory and matrix theory are utilized. Finally, numerical simulation results demonstrate the effectiveness of the proposed method.

Keywords: Multi-Vehicle System, Cooperative Conveyance, Dynamic Network Topology, Surround, Transport

1. INTRODUCTION

In recent years, there have been increasing research interests in the distributed cooperative control of networked multi-vehicle systems. Several research groups developed the coordination control strategies that achieve a capturing formation around a target-object and conveyance by multiple mobile vehicles using neighbor information [1]-[6].

kobayashi et al. [1] proposed a decentralized control of multiple mobile robots for the capturing task of a target object based on the gradient descent. The proposed strategy was applied to the case with convex but unknown shape of oject. So local approximation of the object shape was introduced in [1]. An approach to multi-robot manipulation: Dynamic Object Closure, a closure condition that guarantees to trap a moving object in a predefined future timing, was addressed in [2]. The target-capturing strategy for networked multiple vehicles with dynamic network topology is proposed in [3]. The strategy controlled to converge to the formation while they are tracking the target-object moving in 3 dimensional space. Decentralized control policies for a group of robots to move toward a goal position while maintaining a condition of object closure is presented in [4]. A strategy which using simple vector fields to enable a team of vehicles approaching an object is proposed in [5]. Vehicles surround object to cage it, and transport it to a destination, while avoiding inter-robot collisions. V. Kumar et al.[6] presented multi-robot manipulation of non-circular objects, cooperative manipulation in environments with obstacles. Furthermore, their strategy enables vehicles avoiding collision and obstacle by sensing the positions/velocities of relative neighbors.

In our work, we build on [3], [6] and present multirobot manipulation, cooperative conveyance with dynamic network topology, and via the composition of surrounding behaviors with feedback controllers. Furthermore, our strategy enables vehicles to avoid terminating network connection from masking by sliding the surrounding positions of vehicles for target-object. While we assume that vehicles are holonomic, it is possible to extend our methodology to include non-holonomic vehicles using feedback linearization techniques. Finally, the effectiveness of the proposal method is verified by the numerical simulations.

This paper is organized as follows. Section 2 introduces the multi-vehicle systems and target-object, network topology that depends on relative distance between the vehicles and control objectives. Section 3 describes the proposed method which enables cooperative conveyance in various initial positions and relaxing assumption for number of vehicles which recognizing targetobject. Section 4 describes the results by numerical simulations. Finally, we summarize the obtained results in Section 5.

2. PROBLEM STATEMENT

2.1 Multi-vehicle Systems

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We consider the following $N \geq 1$ mobile vehicles in Cartesian coordinates

$$\mathbf{r}_i = u_i, \qquad i = 1, \cdots, N. \tag{1}$$

where $r_i = [x_i \ y_i]^T \in \mathbb{R}^2$ is the position of i^{th} vehicle and $u_i \in \mathbb{R}^2$ is the control input of i^{th} vehicle.





Fig. 2 Surrounding

where $\xi \in \mathbb{R}$ is the capturing radius. Moreover, the object is assumed to be always geostationary. The vehicle which are disk-shaped with radius ℓ ables to sense the position of their teammates.

2.2 Network Topology

Information exchange between vehicles or between vehicle and target-object can be represented as a graph. We give here some basic some terminology and definitions from graph theory. Let $\mathscr{G} = (\mathscr{V}, \mathscr{E})$ denoted a graph with the set of vertices $\mathscr{V} = \{1, 2, \ldots, N\}$ and the set of edges $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$.

The adjacency matrix $\mathcal{A}(\mathscr{G}) = [a_{ij}] \in \mathbb{R}^{N \times N}$ is defined as $a_{ij} = 0$ and $a_{ij} = 1$ if $(j,i) \in \mathscr{E}$ where $i \neq j$. The adjacency matrix of a undirected graph is defined accordingly except that $a_{ij} = a_{ji}, \forall i = j$, since $(j,i) \in E$ implies $(i,j) \in E$. The degree of vertex i is the number of its neighbors and is denoted by deg(i). The degree matrix of graph \mathscr{G} is diagonal matrix defined as $\mathcal{D}(\mathscr{G}) = [d_{ij}] \in \mathbb{R}^{N \times N}$ where

$$d_{ij} = \begin{cases} \deg(i) = \sum_{j=1, j \neq i}^{N} a_{ij} & , i = j \\ 0 & , i \neq j \end{cases}$$
(2)

graph Laplacian of the graph \mathcal{G} is defined by

$$\mathcal{L}(\mathscr{G}) = \mathcal{D}(\mathscr{G}) - \mathcal{A}(\mathscr{G}) = [l_{ij}]$$
(3)

It is well known that the graph Laplacian has some fundamental properties as follows.

Property 1 All the row sums of \mathcal{L} are zero and thus $\mathbf{1} = [1 \ 1 \ \dots 1]^T \in \mathbb{R}^N$ is eigenvector of \mathcal{L} associated with the eigenvalue $\lambda(\mathcal{L}) = 0$.

Property 2 For a connected graph, the graph Laplacian \mathcal{L} is symmetric positive semi-definite and the eigenvalue $\lambda(\mathcal{L}) = 0$ is unique.

And we assume that the following Assumptions for vehicles is satisfied.

Assumption 1 Network topology among the vehicles is time-varying but always connected.

Assumption 2 *At least one vehicle can get the information of target-object.*

2.3 Behaviors

Our approach to caging and manipulation of objects can be summarized by the behavior architecture in Fig.3. The architecture relies on two behaviors.



Fig. 3 Behaviors Architecture

Surround: The *N* vehicles are spaced out around the target-object at intervals of the assigned angles and maintain these angles and each vehicle approaches to the target-object and finally keeps the distance ξ .

Transport: The vehicle moves toward the goal configuration or tracks a reference trajectory derived from the object's reference trajectory.

As shown in Fig.3, transitions between behaviors are based on simple conditions derived from information among vehicles. When *Number* and *Caging* flags are both set, the vehicle enters Transport mode and starts transporting the object. Thus, reseting the flags can cause the vehicles to regress into a different mode.

3. PROPOSAL TECHNIQUE

3.1 Setting of mode

The conveyance of the target-object is composed of Surround and Transport mode. We introduce the control law proposed by [3] in Surround modes. It allows vehicles to cooperative capturing target-object with dynamic network topology. In Transport mode, vehicles continue to follow and surround virtual target-object in order to transport a target-object to goal.

[Control Objectives]

The control objectives for the Surround mode can be formulated as follows.

C1)
$$\lim_{t \to \infty} || r_i(t) - r_{obj}(t) || = \xi,$$

C2) $\lim_{t \to \infty} || \dot{r}_i(t) - \dot{r}_{obj}(t) || = 0,$

C3)
$$\lim_{t \to \infty} \|\phi_{i+1}(t) - \phi_i(t)\| = \frac{2\pi}{N} [rad], \ i = 1, 2, \dots, N.$$

Let $\phi_i = \tan^{-1}(y_i/x_i)$ denotes the counterclockwise angle of ith vehicle and the center is the target-object. In control objective C3), if i = N, then N + 1 = 1. In the next section, the target-capturing strategy which achieves the control objectives C1)-C3) is developed.

3.2 Surrounding of object

The surrounding of the object utilizes the control law given in (4) \sim (9) [1].

$$u_{i} = \kappa_{i} \left[a_{iobj} \{ -k(\hat{r}_{i} - r_{obj}) + (\dot{R}_{i} - \dot{R}_{obj}) + \dot{r}_{obj} \} + \sum_{j=1}^{N} a_{ij} \{ -k(\hat{r}_{i} - \hat{r}_{j}) + (\dot{R}_{i} - \dot{R}_{j}) + \dot{\hat{r}}_{j} \} \right]$$
(4)

$$a_{ij} = \begin{cases} 1, \ (\|r_i - r_j\| \le \rho_j) \\ 0, \ (\|r_i - r_j\| > \rho_j) \end{cases}$$

(j = 1, ..., N, obj(N + 1)) (5)

$$\kappa_i = \frac{1}{\sum\limits_{j=1}^{N+1} a_{ij}} \tag{6}$$

$$R_i = \sigma_{-}(\omega) * goal_1 + \sigma_{+}(\omega) * goal_2 \tag{7}$$

$$\sigma_{\pm}(\omega) = \frac{1}{e^{\pm t_1(\omega + t_2)}} \tag{8}$$

$$\omega = \frac{-1}{|\hat{r}_i - \hat{r}_{obj}|^p} \tag{9}$$

where $k > 0 \in \mathbb{R}$ is constant gain, $R_i \in \mathbb{R}^2$ is surrounding position for each vehicle, $r_{ij} = r_i - r_j$, $\hat{r}_i = r_i - R_i$, $\rho_j \in R$, $\alpha_i \in [0, 2\pi)$ are the desired capturing angles. a_{ij} is a variable that represents whether vehicles can recognize the target-object. Actually, ρ_j is a sensor range. $goal_1, goal_2$ is initial and final capturing position. $t_1, t_2 > 0$ defined as constant scalars. p is a positive even number. Fig.4 and 5 show the graph of $\sigma_{\pm}(\omega)$ which enable sliding surround position. and Fig.6 shows that the surround positions depend on distance of vehicles and target.





Fig. 6 Surround position change

The object closure is used as a surrounding method. We introduce N_{min} and N_{max} for a given ξ and $D_{min}(obj)$, minimum radius of vehicles R. N_{min} and N_{max} allow agents to ensure object closure which are given as

$$N_{min} = \frac{2\pi\xi}{2\ell + D_{min}(obj)}, \quad N_{max} = \frac{\pi\xi}{\ell} \qquad (10)$$

Theorem 1 : Consider the system of N vehicles (1) and . We apply the capturing control laws (4)-(9) to the system. If the system satisfies k > 0 and [Assumptions 1-2], then the system asymptotically achieves the control objective C1)-C3).

Proof: Substituting Eqs. (4)-(9) into (1).

$$\dot{r}_{i} = \kappa_{i} \left[a_{iobj} \{ -k(\hat{r}_{i} - r_{obj}) + (\dot{R}_{i} - \dot{R}_{obj}) + \dot{r}_{obj} \} + \Sigma_{j=1}^{N} a_{ij} \{ -k(\hat{r}_{i} - \hat{r}_{j}) + (\dot{R}_{i} - \dot{R}_{j}) + \dot{\hat{r}}_{j} \} \right] (11)$$

We assume the target-object as N+Ith vehicle. $(r_{obj} = \hat{r}_{N+1}, a_{iobj} = a_{iN+1})$. Here, $\dot{R}_i \to 0$, $as \ t \to \infty$, so we can get the following closed loop system.

$$= -k \begin{bmatrix} \sum_{j=1}^{N+1} a_{1j} & 0 & \dots & 0 \\ 0 & \sum_{j=1}^{N+1} a_{2j} & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & 1 \end{bmatrix} \otimes I_{2} \begin{bmatrix} \dot{r}_{1} \\ \dot{r}_{2} \\ \vdots \\ \dot{r}_{N+1} \end{bmatrix}$$
$$= -k \begin{bmatrix} \sum_{j=1}^{N+1} a_{1j} & -a_{12} & \dots & -a_{1(N+1)} \\ -a_{21} & \sum_{j=1}^{N+1} a_{2j} & \dots & -a_{2(N+1)} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix} \otimes I_{2} \begin{bmatrix} \dot{r}_{1} \\ \dot{r}_{2} \\ \vdots \\ \dot{r}_{N+1} \end{bmatrix}$$
$$+ \begin{bmatrix} 0 & a_{12} & \dots & a_{1(N+1)} \\ a_{21} & \sum_{j=1}^{N+1} a_{2j} & \dots & -a_{2(N+1)} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix} \otimes I_{2} \begin{bmatrix} \dot{r}_{1} \\ \dot{r}_{2} \\ \vdots \\ \dot{r}_{N+1} \end{bmatrix}$$
(12)

Here I_2 means $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is shown.

Eq. (12) can be rewritten as

$$(\mathcal{L}_{\sigma} \otimes I_2)\dot{\hat{r}}_e = -k(\mathcal{L}_{\sigma} \otimes I_2)\hat{r}_e \tag{13}$$

where \mathcal{L}_{σ} , \mathcal{D}_{σ} and \mathcal{A} are graph Laplacian, degree matrix and adjacency matrix, \otimes is Kronecker product and $\hat{r} \in \mathbb{R}^{2(N+1)}$ is $\hat{r} = [\hat{r}_1^T \hat{r}_2^T \dots \hat{r}_{N+1}^T (= r_{obj}^T)]^T$. Next, we introduce error variables $\hat{r}_{ei} = \hat{r}_i - \hat{r}_{N+1}(r_{obj})$. Thus the following error system is obtained.

$$\begin{bmatrix} \sum_{j=1}^{N+1} a_{1j} & -a_{12} & \dots & -a_{1N} \\ -a_{21} & \sum_{j=1}^{N+1} a_{2j} & \dots & -a_{2N} \\ \vdots & \ddots & \ddots & \vdots \\ -a_{N1} & \dots & -a_{N(N-1)} & \sum_{j=1}^{N+1} a_{1j} \end{bmatrix} \otimes I_2 \begin{bmatrix} \dot{r}_1 \\ \dot{r}_2 \\ \vdots \\ \dot{r}_{N+1} \end{bmatrix}$$
$$= -k \begin{bmatrix} \sum_{j=1}^{N+1} a_{1j} & -a_{12} & \dots & -a_{1N} \\ -a_{21} & \sum_{j=1}^{N+1} a_{2j} & \dots & -a_{2N} \\ \vdots & \ddots & \ddots & \vdots \\ -a_{N1} & \dots & -a_{N(N-1)} & \sum_{j=1}^{N+1} a_{1j} \end{bmatrix} \otimes I_2 \begin{bmatrix} \hat{r}_{e1} \\ \hat{r}_{e2} \\ \vdots \\ \hat{r}_{eN} \end{bmatrix}$$
(14)

Eq. (14) can be rewritten as

$$(\mathcal{M}_{\sigma} \otimes I_2)\dot{\hat{r}}_e = -k(\mathcal{M}_{\sigma} \otimes I_2)\hat{r}_e \tag{15}$$

where $\hat{r}_e = [(\hat{r}_1 - \hat{r}_{N+1})^T \dots (\hat{r}_N - \hat{r}_{N+1})^T]^T \in \mathbb{R}^{2N}$ and $\mathcal{M}_{\sigma} \in \mathbb{R}^{N \times N}$ is new matrix to represent the network topology.

Here, Eq. (13) and Eq. (15) are equivalent equations. If Assumptions 1, 2 is satisfied, then $(\mathcal{M}_{\sigma} \otimes I_2)\hat{r}_e = 0$ has an evident solution $\hat{r}_e = 0$ and \mathcal{M}_{σ} is nonsingular matrix.

From the following relationship, we can verify whether \mathcal{M}_{σ} is nonsingular matrix.

$$\mathcal{M}_{\sigma}$$
 is positive definite $\Longrightarrow \mathcal{M}_{\sigma}$ is nonsingular. (16)

Here, we introduce a nonzero vector $x = [x_1 \ x_2 \ \dots \ x_n] \in \mathbb{R}^N$ to prove it. If the network of system is connected graph $(a_{ij} = a_{ji}, \forall_i, j(i, j \neq N + 1))$, then $x^T \mathcal{M}_{\sigma} x$ can be expressed as follows.

$$x^{T} \mathcal{M}_{\sigma} x = \sum_{l=1}^{N} a_{lN+1} x_{l}^{2} + \sum_{j=1}^{N} a_{ij} (x_{i} - x_{j})^{2} \quad (17)$$

And furthermore, if at least one vehicle can get the information of target-object $(a_{iN+1} = 1)$ and $x \neq \alpha \mathbf{1}$, then we can get the following condition.

$$\sum_{l=1}^{N} a_{lN+1} x_l^2 > 0 \tag{18}$$

where $\alpha \in \mathbb{R}$ is any scalar constant. From property 1 of graph Laplacian, the second term of Eq. (17) is represented as

$$\sum_{j=1}^{N} a_{ij} (x_i - x_j)^2 = x^T \mathcal{L}_\sigma x \ge 0$$
(19)

From *property 2* of graph Laplacian, when the eigenvector of graph Laplacian and $\mathbf{1} = [1 \ 1 \ \dots \ 1]^T$ are only linearly dependent, the eigenvalue of graph Laplacian is zero.

If the second term of the right side of Eq. (17) is zero, then the first term is strictly positive. Conversely, the first term of the right side of Eq. (17) is zero, then the second term is always positive. Here, we can get the following condition.

$$x^{T} \mathcal{M}_{\sigma} x = \sum_{l=1}^{N} a_{lN+1} x_{l}^{2} + \sum_{j=1}^{N} a_{ij} (x_{i} - x_{j})^{2} > 0$$
(20)

Accordingly, if assumption 2 is satisfied, M_{σ} is strictly positive definite and nonsingular.

Multiply the both sides of Eq. (15) by $(\mathcal{M}_{\sigma} \otimes I_2)^{-1}$. Thus, we can get the following equations. $(\mathcal{M}_{\sigma} \otimes I_2)^{-1}$ is put from the left side on both sides of the expression.

$$\dot{\hat{r}}_e = -k(\mathcal{M}_\sigma \otimes I_2)^{-1}(\mathcal{M}_\sigma \otimes I_2)\hat{r}_e$$

$$\dot{\hat{r}}_e = -k\hat{r}_e$$
(21)

This closed loop system does not depend on the network topology. Therefore if the network topology is the time-varying, then the closed loop system representation does not change. Convergence speed of vehicles is decided only by controller gain k and choice of the gain is easy. From the control gain is positive (k > 0), we can get:

$$\begin{aligned} \hat{r}_e &\to 0, \ \dot{\hat{r}}_e \to 0 \ as \ t \to \infty, \\ \hat{r} &\to 1 \otimes \hat{r}_{N+1}, \ \dot{r} \to 1 \otimes \dot{\hat{r}}_{N+1} \ as \ t \to \infty. \end{aligned}$$
(22)

Hence, the states of the system converge to the targetobject.

$$\begin{aligned} \hat{r}_i &\to r_{obj}, \, \dot{r}_i \to \dot{r}_{obj} \, as \ t \to \infty, \, i \in \mathscr{V}, (24) \\ \hat{r}_i &\to \hat{r}_i, \, \dot{r}_i \to \dot{r}_j \, as \ t \to \infty, \, i, j \in \mathscr{V}. \end{aligned}$$

In other words, the control objectives C1)-C3) are achieved. \Box

3.3 Transport

In Transport mode, vehicles continue to follow and sirround virtual target-object so as to transport real targetobject to goal. Then, the control input is given by the following control law (26)-(28).

Given the workspace \mathcal{W} and a goal configuration r_{goal} ,

$$\beta_0 = s_{\mathcal{W}_0}^2 - s_{\mathcal{W}(x,y)}^2 \tag{26}$$

$$\Phi(r) = \frac{\|r - r_{goal}\|}{\left[\|r - r_{goal}\|^{2\kappa} + \beta_0\right]^{1/\kappa}}$$
(27)

$$u_{obj} = -\nabla_{obj} \Phi(r_{obj}) \tag{28}$$

where $\kappa > 0 \in \mathbb{R}$.

3.4 Achievement of surrounding and conversion

We propose an algorithm for local estimation of closure.

1. Number

To locally define quorum, we introduce the concept of a *forward* and *backward* neighborhood with respect to the manipulated object and the Surround mode. For vehicle *i*, the Surround mode introduces an approach component, i.e. $\nabla_i \varphi_i$ and rotation component, i.e. $\nabla_i \times \psi_i$, so that we can define some set of vehicles to be in front of agent *i* and another sets behinds.

If a neighborhood $\overline{\Gamma}_i$ represents the vehicles within a distance $D_{min}(obj)$, then

$$\hat{\Gamma}_{i}^{\pm} = \{ j \in \hat{\Gamma}_{i} \mid 0 < \pm (r_{j} - r_{i})^{T} (\nabla_{i} \times \psi_{i}) \}.$$
(29)

$$\psi = \left[0, 0, \frac{\gamma}{\sqrt{\gamma^2 + \beta_0}}\right]^T \tag{30}$$

where $\gamma = s(x, y), s(x, y) = 0$ is shape boundary for surrounding target-object.

In addition, the vehicle of $\hat{\Gamma}^+_i$ can be defined as follows.

$$i^{+} = \operatorname{argmax}_{k \in \hat{\Gamma}_{i}^{+}} \frac{(r_{k} - r_{i})^{T} \nabla_{i} \varphi_{i}}{\parallel r_{k} - r_{i} \parallel}$$
(31)

$$i^{-} = \operatorname{argmax}_{k \in \hat{\Gamma}_{i}^{-}} \frac{-(r_{k} - r_{i})^{T} \nabla_{i} \varphi_{i}}{\parallel r_{k} - r_{i} \parallel}$$
(32)

$$\varphi(r) = \frac{\gamma^2}{[\gamma^2 + \beta_0]} \tag{33}$$

We consider the following update rule for $number_i$

$$\operatorname{number}_{i} = \begin{cases} 0 & \text{if } (\hat{\Gamma}_{i}^{+} = \emptyset) \lor (\Gamma_{i}^{-} = \emptyset), \\ N_{min} & \text{if } f(i^{+}, i^{-}) > N_{min}, \\ f(i^{+}, i^{-}) & \text{otherwize} \end{cases}$$
(34)

with $f(j,k) = \min(\text{number}_j, \text{number}_k) + 1$. We use number_i and $\text{number}_{i^{\pm}}$ to determine whether object closure is achieved.

2. Caging

We define local target-capturing as follows.

$$\operatorname{caging}_{i} = (\operatorname{number}_{i} \ge N_{min}) \land$$
$$(\operatorname{number}_{i} = \operatorname{number}_{i^{+}}) \land$$
$$(\operatorname{number}_{i} = \operatorname{number}_{i^{-}})$$
(35)

When an agent estimates that local closure has been attained, it switches to the Transport mode and begin manipulation of the object. If closure was lost during manipulation, each agent in the system returns to the Surround mode to achieve object closure.



Fig. 7 Vehicle *i*'s neighborhoods $\hat{\Gamma}_i^{\pm}$ with i^{\pm}

4. NUMERICAL SIMULATIONS

The simulation results are shown in Fig.8 - 11. Figures illustrate the trajectories of four vehicles and the targetobject in Surround and Transport mode. In each figure, ' Red circle' shows vehicles with radius, 'Blue circle' shows target-object with radius, 'Blue circle on green circle'shows surround position for each vehicle. Small blue and red circles show center position of vehicles and target-position. In initial position, only one vehicle recognizes the target-object, and network topology among the vehicles is always connected.

Fig.8 shows initial position of vehicles and targetobject. In Fig.9, vehicles achieve surrounding the targetobject. Making a comparison between Fig.8 and Fig.9, surround positions slide from *goal1* to *goal2*. Fig.10 shows vehicles transporting the target-object. In Fig.11, vehicles achieve transporting the target-object to the goal.



Fig. 8 Initial position of vehicles and target-object



Fig. 9 Achieving surround the target-object



Fig. 10 Transport the target-object



Fig. 11 Achieving transport the target-object to goal

5. CONCLUSION

In this paper, we proposed multi-robot manipulation, cooperative conveyance with dynamic network topology, and via the composition of surrounding behaviors with feedback controllers. Furthermore, their strategy enables vehicles avoiding collision and obstacle by sensing the positions/velocities of relative neighbors. The effectiveness of the proposal method was verified by the numerical simulations.

REFERENCES

- Yuichi Kobayashi, Kyouji Otsubo and Shigeyuki Hosoe "Design of Decentralized Capturing Behavior by Multiple Mobile Robots", *International Transactions on Systems Science and Applications*, vol. 3, no. 3, pp.203-210, 2007.
- [2] ZhiDong Wang, Hidenori Matsumoto, Yasuhisa Hirata and Kazuhiro Kosuge, "A Path Planning Method for Dynamic object closure by Using Random caging Formation Testing" *IEEE/RSJ International Conference on Intelligent Robots and Systems*, St. Louis, October 11-15, 2009 2009.
- [3] Hiroki Kawakami and Toru Namerikawa "Cooperative Target-capturing Strategy for Multi-vehicle Systems with ynamic Network Topology," *Proc. of American Control Conference* pp.635 - 638, 2009.
- [4] G. A. S. Pereira, V. Kumar, and M. F. Campos, "Decentralized algorithms for multi-robot manipulation via caging", The Int. Journal of Robotics Research, vol. 23, no. 7/8, pp. 783-795, 2004.
- [5] J. Fink, N. Michael, and V. Kumar, "Composition of vector fields for multi-robot manipulation via caging", in Proc. 2007 Robotics: Science and Systems III, Atlanta, GA, 2007.
- [6] Jonathan Fink, M. Ani Hsieh, and Vijay Kumar "Multi-Robot Manipulation via caging in Environments with Obstacles", *Robotics and Automation*, pp.1471 - 1476, 2008.
- [7] Daniel E. Koditschek, Elon Rimon "Robot Navigation Functions on Manifolds with Boundary" Advanced Appl. Math., vol.11, pp.412 - 442, 1990.
- [8] E. Rimon and D. E. Koditschek. "Exact robot navigation using arti cial potential functions," *IEEE Transactions on Robotics and Automation*, vol.8, no.5, pp.501-518, October 1992.
- [9] V. de Silva and R. Ghrist, "Homological sensor networks," *Notices of the American Mathematical Society*, vol. 54, no. 1, pp. 10-17, 2007.
- [10] A. Muhammad and A. Jadbabaie, "Decentralized computation of homology groups in networks by gossip," in Proc. of the American Control Conf., New York, July 2007.