Robust Stabilization of Running Self-Sustaining Two-wheeled Vehicle

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***Abstract*— This paper deals with robust stabilization of running self-sustaining two-wheeled vehicle. Recently, some researches about stabilization of two-wheeled vehicle have been reported. These researches have achieved the stabilization running only by the steering control. However, an actual two-wheeled vehicle is running while accompanying stabilization by the rider. We have proposed the stabilization of two-wheeled vehicle in the state of stillness, and have shown the effectiveness. In this research, we compose the control system that aims at the running stabilization of two-wheeled vehicle. We use $\mathcal{H}_\infty$ mixed sensitivity problem to design the controller to achieve stability running even if the mass of two-wheeled vehicle changes. The experimental results show stability running even if the mass of two-wheeled vehicle changed.**

I. Introduction

Recently, some researches about stabilization of two-wheeled vehicle have been reported. Murakami et al.[1] have utilized a bicycle on the market as an experiment model. They install the actuator for the steering control and for running in the model, and have achieved the running stabilization on the running roller. Yoshida et al.[2] have achieved the running stabilization by using the model considering the sideslip of the tire. Yi et al.[3] utilize the model considering the motorcycle trail and caster angle. They designed the control system aiming trajectory tracking and balance stabilization. But, experimental verifications have not been carried out. These models are stabilized only by the handle operation.

On the other hand, Yamakita et al.[4] have achieved stabilization in the state of stillness. They use the model with the handle operation and the stabilization device which is the center of gravity movement. They have achieved stabilization in the state of running, but the numerical model were not considered in the state of running. Sumida et al.[5] use the model which is similar to Yamakita's and they are researching concerning the gyrating control, but an experimental verification have not been carried out. Murata boy[6] have achieved stabilization in the state of stillness by controlling the rotor installed internally. It has achieved running stabilization at low speed too. These models do not consider that the two-wheeled vehicles are running, and an experimental verifications are insufficient.

We have developed a stationary self-sustaining two-wheeled vehicle which is a two-wheeled vehicle equipped with a cart system to move a center-of-gravity of the vehicle for stabilizing the system. We have derived a linear model of two-wheeled vehicle via Lagrange method. As attitude control experiment results, the effectiveness of the independent self-sustaining two-wheeled vehicle and the derived two-wheeled vehicle model was able to be proven[7].

In this paper, we constitute a control system for stabilization of running self-sustaining two-wheeled vehicle. And we achieve stabilization of running even if the mass of two-wheeled vehicle changes by using $\mathcal{H}_\infty$ mixed sensitivity problem to design the controller. Because the two-wheeled vehicle is difficult to run actually, the running experiments are carried out on the running roller. Experimental results show effectiveness of the designed robust attitude controller.

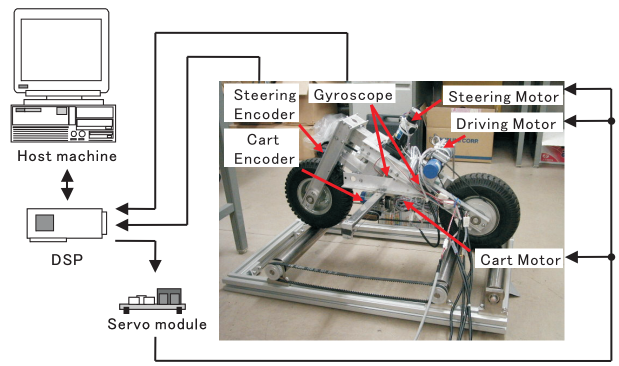
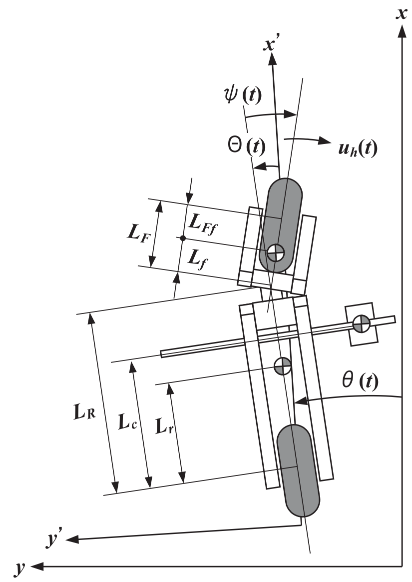


Fig. 1. Composition of experimental system

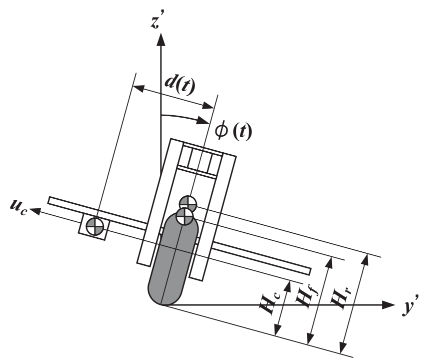
II. Composition and Modeling of Experimental System

A. Composition of Experimental System

Figure 1 shows a composition of the experimental system. The two-wheeled vehicle consists of three parts. There are a cart system that corresponds to the rider's center-of-gravity movement, a steering system (a front part) for steering, and a body (a rear part). The front part and the rear part are structures that finish being movable through a steering axis. A cart system and a steering system are driven by DC servo motor, and DC motors are controlled by servo amplifier which contains the velocity control system. Handle angle and cart position are measured by encoders. Attitude angles of the two-wheeled vehicle (roll angle and yaw angle) are measured by gyroscopes.



(a) Top view



(b) Front view

Fig. 2. Two-wheeled vehicle model

A system's length is about 70 [cm], width is about 57 [cm], height is about 40 [cm], and weight is about 10 [kg]. Movable ranges of a cart system is ± 25 [cm], and a steering system is ± 0.5 [rad] respectively.

We used MATLAB, Simulink for a controller design and used dSPACE DSP-CIT for control experiments. The running experiments are carried out on the running roller in Figure 1.

B. Preliminary

Figure 2 shows a model of two-wheeled vehicle. We assume that two-wheeled vehicle is stabilized by the cart movement $d(t)$ and the handle operation $\psi(t)$. The control inputs are the voltage $u_c(t)$, $u_h(t)$ to add to an amplifier. We assume that cart position $d(t)$, handle angle $\psi(t)$, bike angle $\phi(t)$, and yaw angle of a rear part $\theta(t) + \Theta(t)$ can be measured directly.

For modeling, we consider the following assumptions.

1) x - y - z axis are set as absolute coordinates.
2) x' - y' - z' axis are set as relative coordinates on two-wheeled vehicle. The contact points on the ground of a front wheel and a rear wheel are set as the x' axis, the y' axis is orthogonal to the x' axis, the z' axis is vertical upward.
3) The bike angle $\phi(t)$, cart position $d(t)$, handle angle $\psi(t)$, and yaw angle of a rear part $\theta(t) + \Theta(t)$ can be measured directly.
4) The tire does not slip horizontally.
5) Two-wheeled vehicle runs in the straight line on the running roller.
6) The bike angle, cart position, handle angle, and bike yaw angle are small enough.
7) The center-of-gravity movement x axially and z axially by the handle operating are omitted.
8) Two-wheeled vehicle is a rigid body, and the twist is not occurred.
9) A cart system and a steering system are driven by DC servo motor, and DC motors are controlled by servo amplifier which include the velocity control system.
10) The state variables which are differentiated twice are small enough. They can be omitted.

Table I shows the definition of the symbols in the expressions. The modeling for the state of stillness are only considering by the coordinates on the two-wheeled vehicle.

C. Modeling[7]

1) Center-of-gravity coordinate of each parts: The center-of gravity coordinates of a front part (y'_f, z'_f), a rear part (y'_r, z'_r), and a cart system (y'_c, z'_c) on the relative coordinates are obtained as next expressions.

$$\begin{cases} y'_f = H_f \sin\phi(t) + L_{Ff} \sin\{\psi(t) - \Theta(t)\} \cos\phi(t) \\ z'_f = H_f \cos\phi(t) - L_{Ff} \sin\{\psi(t) - \Theta(t)\} \sin\phi(t) \end{cases} \tag{1}$$

$$\begin{cases} y'_r = H_r \sin\phi(t) + L_r \sin\Theta(t) \cos\phi(t) \\ z'_r = H_r \cos\phi(t) - L_r \sin\Theta(t) \sin\phi(t) \end{cases} \tag{2}$$

$$\begin{cases} y'_c = H_c \sin\phi(t) + \{L_c \sin\Theta(t) - d(t)\cos\Theta(t)\} \cos\phi(t) \\ z'_c = H_c \cos\phi(t) - \{L_c \sin\Theta(t) - d(t)\cos\Theta(t)\} \sin\phi(t) \end{cases} \tag{3}$$

$x' - y' - z'$ coordinates were rotated from $x - y - z$ coordinates by only $\theta(t)$ around z axis. Here, because we assumed that two-wheeled vehicle is a straight running, Yaw angle $\theta(t)$ is small enough. Therefore we can consider that the two coordinates is equivalent[8].

$$\begin{cases} y = y' \\ z = z' \end{cases} \tag{4}$$

2) Translation by Vehicle running: Figure 3 shows the model that a stabilized two-wheeled vehicle is running. The two-wheeled vehicle is running in the direction where $\theta(t)$ shifts from x axis. There is a center-of-gravity of a stabilized two-wheeled vehicle on the x' axis. Here, l is a horizontal length from a rear wheel rotation axis to a center-of-gravity of two-wheeled vehicle, and L is a wheelbase.

TABLE I

DEFINITION OF SYMBOLS

M_f, M_r, M_c	Mass of each part
H_f, H_r, H_c	Vertical length from a floor to a center-of-gravity of each part
L_{Ff}, L_F	Horizontal length from a front wheel rotation axis to a center-of-gravity of part of front wheel and steering axis.
L_r, L_R	Horizontal length from a rear wheel rotation axis to a center-of-gravity of part of rear wheel and steering axis.
L_c	Horizontal length from a rear wheel rotation axis to a center-of-gravity of the cart system.
l	Horizontal length from a rear wheel rotation axis to a center-of-gravity of two-wheeled vehicle.
J_x	Moment of inertia around center-of-gravity x axially.
J_z	Moment of inertia around center-of-gravity z axially.
μ_x	Viscous coefficient around x axis.
μ_z	Viscous coefficient around z axis.
V	Velocity of two-wheeled vehicle.
subscript f, r, c	Part of front wheel, rear wheel, and cart system respectively

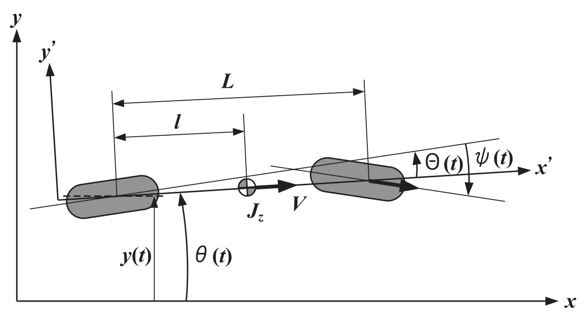


Fig. 3. The two-wheeled vehicle shifted from the course

When the handle angle $\psi(t) = 0$, two-wheeled vehicle runs speed V in the direction of x' axis. Then, the translation at $V \sin\theta(t)$ occurs in the direction of y axis. On the other hand, because we assumed that the tire does not sideslip, the gyration motion satisfies the relation of geometrical of Ackerman. However, because two-wheeled vehicle is running on the running roller, the translation will be done in the direction of y axis where the handle was turned by $\frac{l}{L} V \sin\{\psi(t) - \Theta(t)\}$. Therefore, the two-wheeled vehicle that runs speed V follows the next translation in the direction of y axis.

$$\dot{y}(t) = V \sin\theta(t) - \frac{l}{L} V \sin\{\psi(t) - \Theta(t)\} \tag{5}$$

3) Derivation of the state space model: From (1) $\sim$ (5), we derive the motion equation via Lagrange method. The derived motion equation via Lagrange method are non-linear equations. Therefore, we use the Taylor expansion in the equilibrium ($d(t) = \phi(t) = \psi(t) = \theta(t) = 0$) neighborhood and the variable differentiated twice or more are omitted. Here, $\Theta(t)$ is replaced with the following approximation expression.

$$\Theta(t) = \frac{L_F}{L_F + L_R} \psi(t) \tag{6}$$

And, from the assumptions, the motion equation of a cart system and a handle system are expressed by the following equations[9].

$$\begin{cases} \ddot{d}(t) + \alpha \dot{d}(t) = \beta u_c(t) \\ \ddot{\psi}(t) + \gamma \dot{\psi}(t) = \delta u_h(t) \end{cases} \tag{7}$$

Where, α, β, γ and δ are physical parameters of the motor systems. We obtained the final state space linear model shown in the following expressions.

$$\begin{cases} \dot{x} &= Ax + Bu \\ y &= Cx \end{cases} \tag{8}$$

where,

$$x = \begin{bmatrix} d(t) & \phi(t) & \psi(t) & \theta(t) & \dot{d}(t) & \dot{\phi}(t) & \dot{\psi}(t) & \dot{\theta}(t) \end{bmatrix}^T$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\alpha & 0 & 0 & 0 \\ a_{61} & a_{62} & a_{63} & 0 & a_{65} & a_{66} & a_{67} & a_{68} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\gamma & 0 \\ 0 & 0 & a_{83} & a_{84} & a_{85} & a_{86} & a_{87} & a_{88} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & \beta & b_{61} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & b_{62} & \delta & 0 \end{bmatrix}^T,$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{43} & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad u = \begin{bmatrix} u_c(t) \\ u_h(t) \end{bmatrix}$$

$$den = M_f H_f^2 + M_r H_r^2 + M_c H_c^2 + J_x$$

$$a_{61} = -\frac{M_c g}{den}, \quad a_{62} = \frac{(M_f H_f + M_r H_r + M_c H_c) g}{den}$$

$$a_{63} = -\frac{(M_f L_{Ff} L_R + M_r L_r L_F + M_c L_c L_F) g}{(L_F + L_R) den}$$

$$a_{65} = -\frac{M_c H_c \alpha}{den}, \quad a_{66} = -\frac{\mu_x}{den}$$

$$a_{67} = \frac{M_f H_f L_{Ff} \gamma}{den} + \frac{l L_R (M_f H_f + M_r H_r + M_c H_c) V}{(L_F + L_R)^2 den}$$

$$a_{68} = -\frac{(M_f H_f + M_r H_r + M_c H_c) V}{den}$$

$$a_{83} = -\frac{l L_R (M_f + M_r + M_c) V^2}{(L_F + L_R)^2 J_z}$$

$$a_{84} = \frac{(M_f + M_r + M_c) V^2}{J_z}, \quad a_{85} = -\frac{M_c V}{J_z}$$

TABLE II

PHYSICAL PARAMETERS OF TWO-WHEELED VEHICLE

Parameter	Value	Parameter	Value
M_f [kg]	2.14	H_f [m]	0.0800
M_r [kg]	5.91	H_r [m]	0.161
M_c [kg]	1.74	H_c [m]	0.0980
L_{Ff} [m]	0.0390	L_F [m]	0.133
L_r [m]	0.128	L_R [m]	0.308
L_c [m]	0.259	l [m]	0.2112
J_x [kgm^2]	0.2	J_z [kgm^2]	0.3218
μ_x [kgm^2/s]	0.333	μ_z [kgm^2/s]	0.333
α	735	β	64
γ	111	δ	253
V [m/s]	1.0		

$$\begin{aligned} a_{86} &= \frac{(M_f H_f + M_r H_r + M_c H_c)V}{J_z} \\ a_{87} &= \frac{(M_f L_{Ff} L_R + M_r L_r L_F + M_c L_c L_F)V}{(L_F + L_R) J_z} \\ a_{88} &= -\frac{\mu_z}{J_z} \\ b_{61} &= \frac{M_c H_c \beta}{den}, \quad b_{62} = -\frac{M_f H_f L_{Ff} \delta}{den} \\ c_{43} &= \frac{L_F}{L_F + L_R} \end{aligned} \quad (9)$$

In the case of the state of stillness, there are no terms related to the yaw angle of the two-wheeled vehicle. And there are no terms related to the running speed because the running speed $V = 0$. The state space model which was considered above conditions coincides with the state space model of the state of stillness.

Unknown parameters in above expressions were identified by control experiments. Table II shows physical parameters of the two-wheeled vehicle.

III. CONTROLLER DESIGN

A. Composition of generalized plant and problem setting

First, we consider the disturbance to the two-wheeled vehicle. The behavior of the cart system and the handle system affect the attitude of the two-wheeled vehicle. Therefore, the large control values occur the large attitude changes. Then, we define the disturbance w_1 as the disturbance concerned with the control values.

The uncertainty of model concerned with simplification etc. for modeling and the external factors concerned with the rider's boarding and installing of luggage affect the attitude of the two-wheeled vehicle. Then we define them as the disturbance w_2.

Next, we set the controlled values. Because the stability running of the two-wheeled vehicle is a basic specification of the control, we define the control values z_1, z_2 who weighted the attitude of the two-wheeled vehicle ($d(t)$, $\phi(t)$, $\psi(t)$, $\theta(t)$). Similarly, we define the control value z_3 who weighted the controlled inputs u, because a large control inputs want to be suppressed.

Finally, we collect the plant and the weight matrices together and composed the generalized plant as shown in Figure 4. Then, W_1, W_2, W_3 and W_4 are the weight matrices concerning the sensitivity function, the multiplicative uncertainty, the controlled inputs and the input disturbances, respectively.

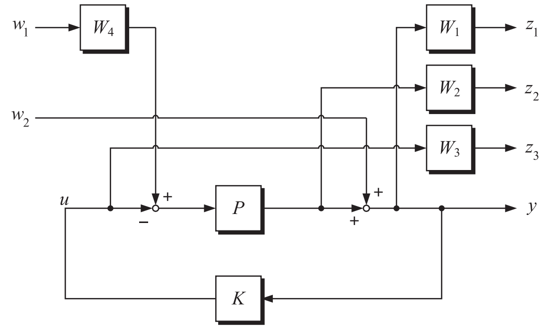


Fig. 4. Generalized plant

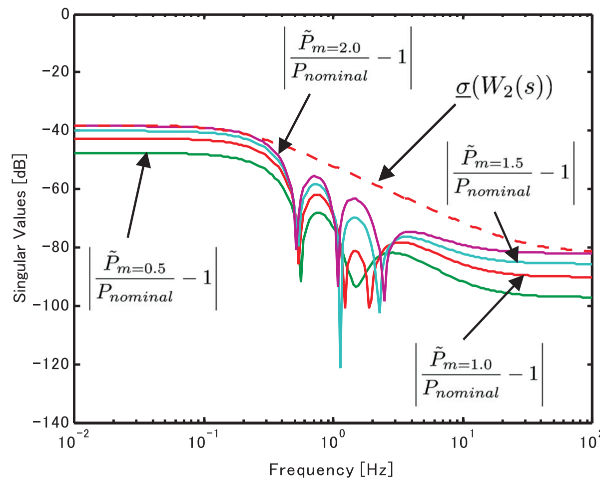


Fig. 5. Multiplicative uncertainties

B. Controller design

First, we calculate the multiplicative uncertainly of the two-wheeled vehicle. Here, we set the two-wheeled vehicle with no additional mass as the nominal model. Figure 5 shows the multiplicative uncertainty due to mass variations, where, $P_{nominal}$ is the nominal model, $\tilde{P}_{m=**}$ are the model for mass variations.

The weighting matrix $W_2(s)$ has to be set to include this uncertainty. Then, we set the weighting matrix $W_2(s)$ as the following expressions.

$$W_2(s) = diag\,(W_{21}, W_{22}, W_{23}, W_{24}) \quad (10)$$

$$\begin{aligned} W_{21} = W_{22} = W_{23} = W_{24} \\ = \frac{1}{80} \times \frac{0.2 \times 2\pi}{s + 0.2 \times 2\pi} \times \frac{s + 30 \times 2\pi}{30 \times 2\pi} \end{aligned}$$

A dashed line in Figure 5 shows the weighting matrix $W_2(s)$ which covers the multiplicative uncertainties.

We used $\mathcal{H}_\infty$ mixed sensitivity problem for the stabilization controller design. Here, we used the other weight

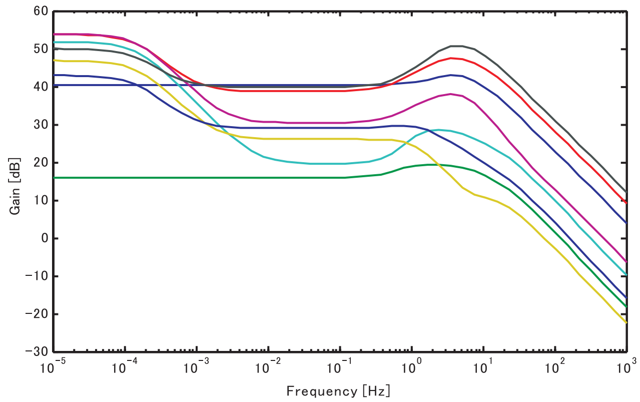


Fig. 6. Bode diagram of Controller

matrices shown in the following expressions.

$$W_1(s) = diag\,(W_{11}, W_{12}, W_{13}, W_{14}) \qquad (11)$$
$$W_3 = diag\,(75, 110)\ ,\ W_4 = diag\,(50, 10) \qquad (12)$$

$$W_{11} = \frac{35000}{s+50}\,, \quad W_{12} = W_{13} = W_{14} = \frac{20}{s+0.001}$$

We designed the running speed of the two-wheeled vehicle as $V = 1.0$ [m/s]. Figure 6 shows the gain diagram of a designed controller.

IV. Attitude control experiments

We measured on the impulse disturbance responses for the two-wheeled vehicle without additional mass (nominal model) and with additional mass (perturbed model). Figure 1 shows the situation of the attitude control experiments. The attitude control experiments were executed when the two-wheeled vehicle run at $V = 1.0$ [m/s] on the running roller. This running roller diameter is 50 [mm] and width is 420 [mm].

As a result, the stability running was achieved for the nominal model. Moreover, even if the disturbance was input to the perturbed model, the stability running was also achieved.

A. Impulse disturbance response for nominal model

Figure 7 shows the impulse disturbance response results for the nominal model. The impulse disturbance voltage of 4.76 [V] was added to the operation voltage of the cart for 0.1 [sec]. The impulse disturbance voltage was inputted at 1 [sec] on the graphs. The above graphs show cart position, bike angle, and handle angle, respectively.

Before disturbance was inputted, the two-wheeled vehicle was able to achieve a better stabilization of running. A cart position, a bike angle and a handle angle were within $\pm$ 9 [mm], $\pm$ 0.4 [deg] and $\pm$ 0.8 [deg] respectively. After disturbance was inputted, the large attitude change occurred. However, the two-wheeled vehicle didn't get into unstable, and the stabilization of running was achieved.

B. Impulse disturbance responses for perturbed models

The two-wheeled vehicle equipped with the additional mass was used in this experiments. Figure 8 $\sim$ 10 show the impulse disturbance response results for the additional mass which are changed to 0.99 [kg] (weight ratio 10.1 %), 1.49 [kg] (weight ratio 15.2 %), and 1.98 [kg] (weight ratio 20.2 %). As well as the nominal model, cart position, bike angle and handle angle are shown respectively. The impulse disturbance voltage was inputted at 1 [sec] on the graphs.

In the case of additional mass is 0.99 [kg], the stabilization of running can be achieved before disturbance was inputted. In the case of additional mass are 1.49 [kg] and 1.98 [kg], the two-wheeled vehicle was running by the handle operation little by little before the disturbance was inputted. The attitude changes after the disturbance becames larger when the additional mass increases, and the convergency was deteriorated. However, the stability was still kept sufficiently.

V. Conclusions

In this paper, we constituted a control system for stabilization of running self-sustaining two-wheeled vehicle. A motion equation of the running model was derived by the Lagrange method. The controller was designed by the $\mathcal{H}_\infty$ mixed sensitivity problem. This controller stabilized two-wheeled vehicle even if the mass of two-wheeled vehicle changes. Experimental results have shown effectiveness of the designed robust attitude controller. The control system design for varying running speed are underway, we are designing the robust controller who can stabilize to the varying running speed and the mass variations. We will improve the robust performance for the wide perturbations.

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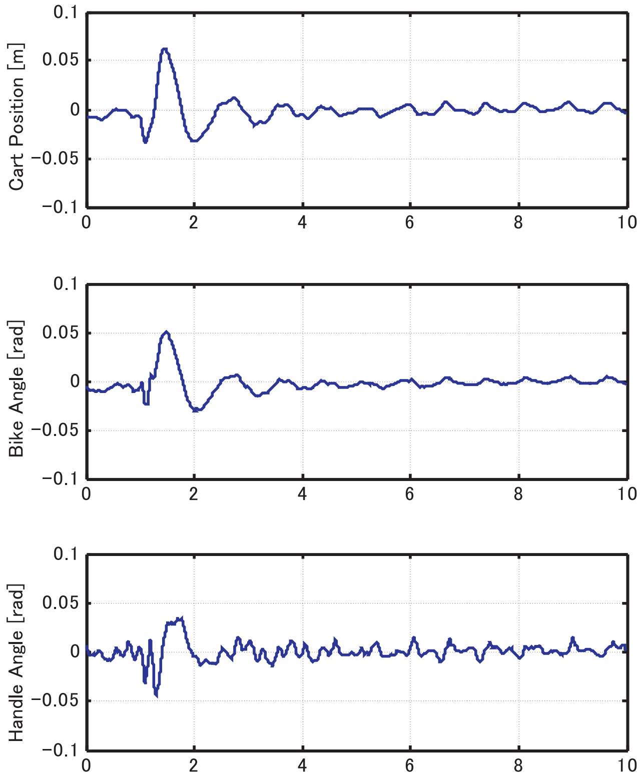

Fig. 7. Experimental results for running vehicle (nominal model)

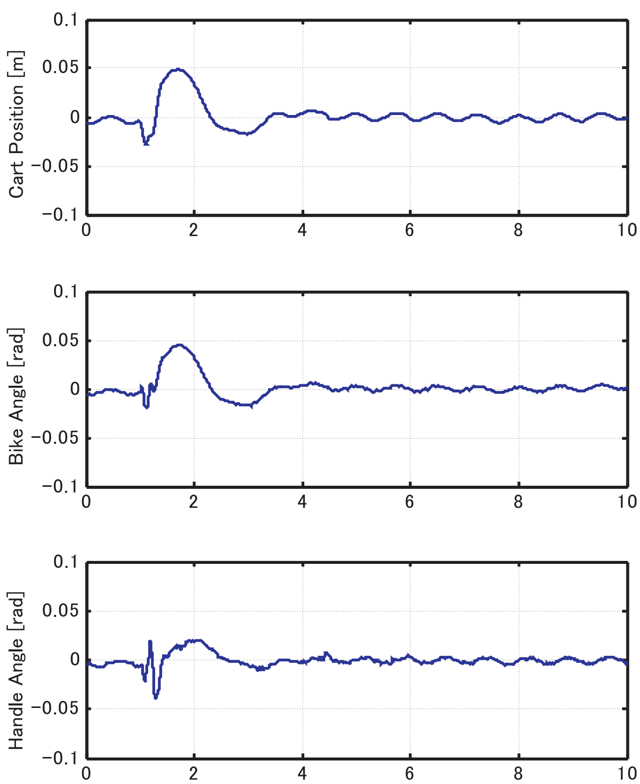

Fig. 8. Experimental results for running vehicle on additional mass (0.99 [kg])

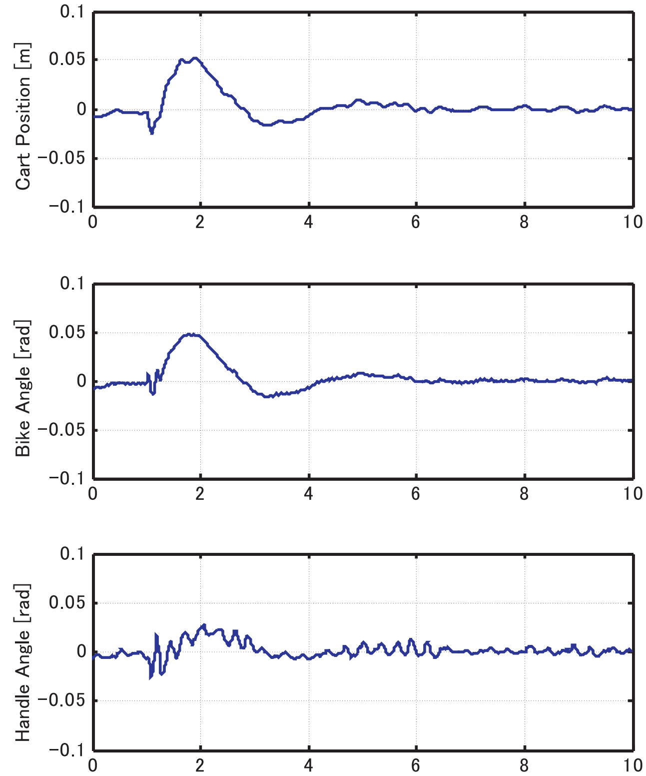

Fig. 9. Experimental results for running vehicle on additional mass (1.49 [kg])

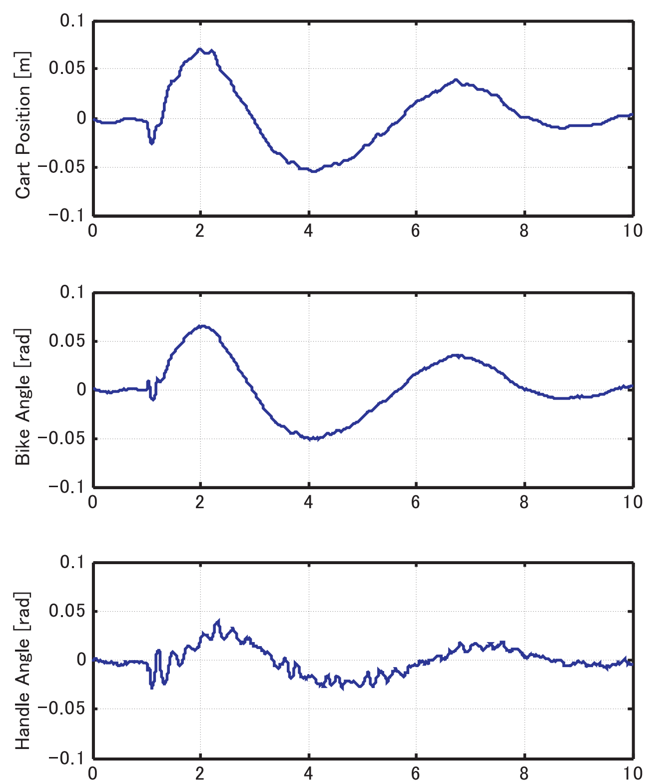

Fig. 10. Experimental results for running vehicle on additional mass (1.98 [kg])