Fourth International Conference on Computational Intelligence, Robotics and Autonomous Systems
November 28-30, 2007, Palmerston North, New Zealand

Formation Control of Nonholonomic Multi-Vehicle Systems via Virtual Structure

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Abstract

This paper deals with formation control strategies based on Virtual Structure (VS) for multi-vehicle systems. We propose several control laws for networked multi-nonholonomic vehicle systems in order to achieve *VS consensus*, *VS Flocking* and *VS Flocking* with collision avoidance. First, Virtual Vehicle for the feedback linearization is considered, and we propose VS consensus and Flocking control laws based on a virtual structure and consensus algorithms. Then, *VS Flocking* control law considering collision avoidance is proposed and its asymptotical stability is proven. Finally, simulation and experimental results show the effectiveness of the proposed approaches.

Keywords: Formation Control, Networked Multi-Vehicle System, Nonholonomic System, Virtual Structure

1 Introduction

Recently there have been a lot of progress for new theories that create fusions of graph theories and control theories for cooperative control problems of distributed networked systems[1]. A multi-agent control problem is one of significant topics where each agent works autonomously by using information of other agents over the communication network. In the multi-agent systems, consensus means that shared information of agents converge to a common constant value and the problem is main topic of the systems. Consensus algorithm using graph theory is studied as a control problem of multi-agent systems in [2, 3]. Formation control problems are expected at various fields, e.g. satellites, airship, intelligent transport systems and load carriage. The consensus problems can be applied to formation control for multiple vehicles which are essential for high-efficiency [4, 5, 6, 7]. A vehicle is generally a nonholonomic system and it has velocity constraint that its wheels cannot move side-away. Many research results for formation control of nonholonomic systems have been reported [4, 5, 8]. Consensus problems with collision avoidance for multi-agent systems have been discussed in [4, 5, 6]. However the control law could not achieve desired formation because it dose not consider control of relative position. In [7], a control law which can construct any formations, was proposed for multi-agent systems. However it is difficult to apply it for general nonholonomic vehicle control systems. On the other hand, a control law that makes any formation using deviation model was proposed in leader-follower types, but it had no information exchange among agents [8].
In this paper, we construct multi-agent systems by using virtual structure and propose a formation control law using information of other agents. Finally, we extend a formation control law with collision avoidance and the effect of the proposed control laws are evaluated via control simulations and experiments.

2 Multi-vehicle systems

A plant is the networked multi-vehicle systems which consists N vehicles as N agents under following assumption.

Assumption 1 *There are an information network between any ith vehicle and jth vehicle ($i \neq j$).*

Graph theory is a useful to represent network structures. The network structure with Assumption 1 is connected graph if it has bidirectional communications, or strongly connected digraph if it has unidirectional communications. In this paper, we use graph Laplacian for network structures expressed mathematically. Graph Laplacian $L = [l_{ij}]$ consists of $l_{ii} = \sum_{j \neq i} a_{ij}$, $l_{ij} = -a_{ij}$, $i \neq j$ if $a_{ij} = 1$ that means jth vehicle send some information to ith vehicle, otherwise $a_{ij} = 0$.

2.1 Vehicle Model

The vehicle model that is considered in this paper is a two-wheeled vehicle as shown in Fig.1 (lower left). We assume that N vehicles can be identical models and friction force is ignored in the models. The kinematics model of ith vehicle is described as

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}, \qquad (1)$$

where (x_i, y_i) are the positions of center of gravity of ith vehicle, θ_i is a heading angle of ith vehicle and v_i and ω_i are the control inputs. It is well known that above vehicle models have constraint on its velocity as

$$\dot{x}_i \sin\theta_i - \dot{y}_i \cos\theta_i = 0. \qquad (2)$$

Therefore these vehicles are nonholonomic systems.

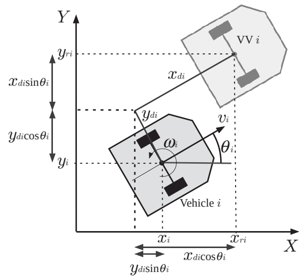

Figure 1: ith Real Vehicle and its Virtual Vehicle

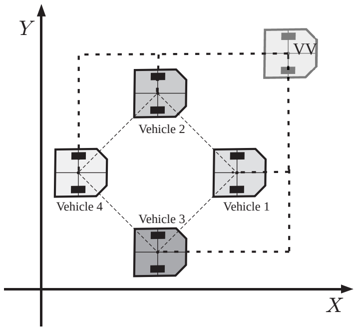

Figure 2: Position of VSs

2.2 Virtual Structure (VS)

We consider Virtual Structure (VS) using Virtual Vehicle (VV) [8] for each vehicle as shown in Fig.1 (upper right). By the positional relationship between vehicle and VV in Fig.1, the kinematics model of ith VV is described as

$$\begin{bmatrix} x_{ri} \\ y_{ri} \\ \theta_{ri} \end{bmatrix} = \begin{bmatrix} x_i + x_{di}\cos\theta_i - y_{di}\sin\theta_i \\ y_i + x_{di}\sin\theta_i + y_{di}\cos\theta_i \\ \theta_i \end{bmatrix}. \quad (3)$$

where $(x_{ri},\ y_{ri})$ are positions of center of gravity of ith VV, θ_{ri} is heading angle of ith VV and x_{di}, y_{di} are distance between VVs and vehicles. The derivative of (3) are given by

$$\begin{bmatrix} \dot{x}_{ri} \\ \dot{y}_{ri} \\ \dot{\theta}_{ri} \end{bmatrix} = \begin{bmatrix} B_i \\ B_\theta \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix},$$

where

$$B_i = \begin{bmatrix} \cos\theta_i & -x_{di}\sin\theta_i - y_{di}\cos\theta_i \\ \sin\theta_i & x_{di}\cos\theta_i - y_{di}\sin\theta_i \end{bmatrix}, \quad (4)$$

$$B_\theta = \begin{bmatrix} 0 & 1 \end{bmatrix}. \quad (5)$$

In this kinematics model, B_i is nonsingular matrix if $x_{di} \neq 0$. In this paper, we consider formation control problems for these VS systems (4).

3 VS Consensus Problems

The goal of formation control problems is that N vehicles preserve any formation based on information exchange between them over the network. To maintain any formations, the VVs of each vehicle has to converge to a common position as shown in Fig.2.

3.1 Control Objectives

To converge to a common value for VV of each vehicle, It is necessary to guarantee consensus for positions of center of gravity and heading angle of VVs as

$$x_{ri} \to x_{rj},\ y_{ri} \to y_{rj},\ \theta_{ri} \to \theta_{rj}\ \ (t \to \infty). \quad (6)$$

This consensus is called *VS consensus*.

Lemma 1 *Consider the $N \times N$ graph Laplacian L with strongly connected digraph. If the systems can be described as*

$$\dot{x} = -L_{.m}x \quad (7)$$

where $x = [x_1^T x_2^T \cdots x_N^T]^T \in \mathbb{R}^{Nm}$ are the state of all systems and $L_{.m} = L \otimes I_m$, the state x converge as

$$x \to (x_{r1}x_{l1}^T \otimes I_m)x(0) = \mathbf{1} \otimes \alpha\ \ (t \to \infty), \quad (8)$$

where x_{r1}, x_{l1} are right and left eigenvector of zero eigenvalue of L with $x_{l1}^T x_{r1} = 1$ and $x_{l1}^T \mathbf{1} = 1$, $\otimes$ denotes Kronecker product, $\alpha \in \mathbb{R}^m$ is consensus value and $\mathbf{1} = [1\ 1 \cdots\ 1]^T \in \mathbb{R}^N$ [3].

Proof 1 *See [3] for proof.*

From Lemma 1, the all of states converge to a common value α as

$$x_1 = x_2 = \cdots = x_N = \alpha. \quad (9)$$

3.2 Control Law for VS Consensus

To achieve *VS consensus*, we propose the following control law 1 for the vehicle i as

Control law 1

$$u_i = B_i^{-1}\Big(-k \sum_{j \in \mathcal{N}_i} (r_i - r_j) + \dot{r}_d\Big), \quad (10)$$

where $u_i = [v_i\ \omega_i]^T$, $r_i = [x_{ri}\ y_{ri}]^T$, $\mathcal{N}_i$ is ith neighbor set, $\dot{r}_d \in \mathbb{R}^2$ is constant reference velocity and $k > 0$ is controller gain.

Theorem 1 *Consider a system of the N vehicles with kinematics (4) and Control Law 1 (10). If the Assumption 1 and $\dot{r}_d \neq 0$ are satisfied, then VS consensus achieves asymptotically.*

Proof 2 *All of the VS systems (4) without its angle θ_{ri} can be written as*

$$\dot{r} = \oplus \sum_{i=1}^{N} B_i u, \quad (11)$$

where $r = [r_1^T \ r_2^T \ \cdots \ r_N^T]^T$, $u = [u_1^T \ u_2^T \ \cdots \ u_N^T]^T$, $\oplus \sum_{i=1}^{N} B_i$ *is matrix that diagonal block elements are* B_i. *The Control law 1 (10) can be written as*

$$u = \oplus \sum_{i=1}^{N} B_i^{-1} \left(-kL_{.2} r + \mathbf{1} \otimes \dot{r}_d\right). \tag{12}$$

Let $r_e = r - \mathbf{1} \otimes r_d$, *then we get the following from (11) and (12),*

$$\dot{r_e} = -kL_{.2} r_e. \tag{13}$$

By Lemma 1, the systems (13) achieve consensus as $r_e \to \mathbf{1} \otimes \alpha \ \ (t \to \infty)$. *Hence, we can conclude that the positions of VVs converge to a common value as*

$$r \to \mathbf{1} \otimes (\alpha + r_d) \quad (t \to \infty). \tag{14}$$

The consensus for r *is achieved as* $r_i \to r_j \to \alpha + r_d$. *Next, we consider heading angles* θ_{ri} *of VVs. Substituting Control law 1 (10) into* $\dot{\theta}_{ri}$ *in (4) and considering* $\dot{r}_d = [v_d \cos\theta_d \ v_d \sin\theta_d]^T$, *we get*

$$\dot{\theta}_{ri} = -\frac{v_d}{x_{di}} \sin(\theta_{ri} - \theta_d). \tag{15}$$

Hence, we have that $\theta_{ri} \to \theta_d$ $(t \to \infty)$. *Therefore VS consensus is achieved asymptomatically. Furthermore, the any formations can be shaped.*

B_i is nonsingular matrix, B_i^{-1} exists and Control Law 1 can be applied. Then, the vehicles can make any formations when VVs converge to a common value. By selecting the distance of VVs ($x_{di}, \ y_{di}$) appropriately as shown in Fig.2, the vehicles achieve any formation shapes. The Control law 1 can be extended and the vehicles can achieve any formations even if distances for VVs are same as

$$x_{d1} = x_{d2} = \cdots = x_{dN}, \ y_{d1} = y_{d2} = \cdots = y_{dN} \tag{16}$$

We propose the new control law for the ith vehicle as

Control law 2

$$u_i = B_i^{-1} \left(-k \sum_{j \in \mathcal{N}_i} \Big((r_i - r_{ri}) - (r_j - r_{rj}) \Big) + \dot{r}_d \right) \tag{17}$$

where r_{ri} *is reference relative position to* r_i.

Theorem 2 *Consider a system of the* N *vehicles with kinematics (4) and Control Law 2 (17). Under assumption 1 and* $\dot{r}_d \neq 0$, *VS consensus achieves asymptotically.*

Proof 3 *This can be proven in a same way with Theorem 1.*

3.3 Control Law with Velocity Tracking for VS Consensus

The Control laws 1 and 2 include feedforward terms which are reference signals $\dot{r}_d$. In case of physical vehicles, the motion of vehicles are not exactly same between them. Therefore, the error of velocities ($\dot{r}_d - \dot{r}_i$) do not converge to 0. Consequently we propose new control law with velocity control for ith vehicle as

Control law 3

$$\begin{aligned} \dot{v}_{ri} &= \dot{v}^* - k_{vr}(v_{ri} - v^*) \\ u_i &= B_i^{-1} \left(-k \sum_{j \in \mathcal{N}_i} \Big((r_i - r_{ri}) - (r_j - r_{rj}) \Big) + v_{ri} \right) \end{aligned} \tag{18}$$

where v^* *is constant reference velocity and* $k_{vr} > 0$ *is controller gain.*

Theorem 3 *Consider a system of the* N *vehicles with kinematics (4), and Control law 3 (18). If Assumption 1 and* $v^* \neq 0$ *are satisfied, then VS consensus achieves asymptotically.*

Proof 4 *Substituting Control law 3 (18) into the* i*th vehicle kinematics (4), we get that*

$$\begin{aligned} \dot{v}_r &= \mathbf{1} \otimes \dot{v}^* - k_v(v_r - \mathbf{1} \otimes v^*) \\ \dot{\hat{r}} &= -kL_{.2}\hat{r} + v_r. \end{aligned} \tag{19}$$

Using $v_{re} = v_r - \mathbf{1} \otimes v^*$, $r_e = \hat{r} - \int_0^t \mathbf{1} \otimes v^* d\tau$,

$$\begin{bmatrix} \dot{r}_e \\ \dot{v}_{re} \end{bmatrix} = \begin{bmatrix} -kL_{.2} & I_{2N} \\ 0 & -k_{vr} I_{2N} \end{bmatrix} \begin{bmatrix} r_e \\ v_{re} \end{bmatrix}. \tag{20}$$

By Lemma 1, the systems (20) achieve consensus and velocity errors r_e *converge to* 0 *as*

$$r_e \to \mathbf{1} \otimes \alpha \ \ v_{re} \to 0 \tag{21}$$

Therefore any formation shape is guaranteed.

4 VS Flocking Problems

4.1 Control Objectives

Flocking is defined that velocity and inter-vehicle distances converge to common value. It could be as

$$\dot{r}_i \to \dot{r}_j \tag{22}$$

VS consensus problem considers only relative positions between vehicles. Here, we discuss *VS Flocking* problems that is considered both relative positions and relative velocities between VVs. The velocities is defined as $v_{ri} = [v_{xi}\ v_{yi}]^T$. Then it is expressed as

$$\dot{v}_{ri} = a_i,\ \dot{r}_i = v_{ri}, \tag{23}$$

where a_i is control input.

4.2 Control Law for VS Flocking

The following control law is proposed

Control law 4

$$\begin{aligned} \dot{v}_{ri} &= -\sum_{j\in\mathcal{N}_i} k_i\big((\hat{r}_i - \hat{r}_j) + k_v(v_{ri} - v_{rj})\big) \\ u_i &= B_i^{-1} v_{ri}, \end{aligned} \tag{24}$$

where $k_v,\ k_i > 0$ are controller gains.

Theorem 4 *Consider a system of the N vehicles with kinematics (4) and Control law 4 (18). Then VS Flocking achieves asymptotically if assumption 1 and $1 > |1 + 4/(k_v^2\lambda_i)|$ are satisfied, where λ_i are eigenvalues of weighted graph Laplacian L_w including k_i, and $v_i \to v_j \neq 0$.*

Proof 5 *The control input $\dot{v}_r$ for multi-vehicle systems can be written as*

$$\dot{v}_r = -L_{w.2}\hat{r} - k_v L_{w.2} v_r. \tag{25}$$

By B_i^{-1}, the position coordinate of VS system (4) can be also described as (23). Therefore, if flocking problem achieve in second order system (23), VS systems with (4) achieve VS flocking problem. By (23) and (25), we have following result

$$\begin{bmatrix} \dot{\hat{r}} \\ \dot{v}_r \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & I_N \\ -L_w & -k_v L_w \end{bmatrix}}_{\Sigma} \otimes I_2 \begin{bmatrix} \hat{r} \\ v_r \end{bmatrix} \tag{26}$$

Σ has 2 zero eigenvalues. Selecting k_v to satisfy as

$$1 > |1 + 4/(k_v^2\lambda_i)|, \tag{27}$$

where $\lambda_i$is ith eigenvalue of $-L_w$, All of eigenvalues without zero have negative real parts [7]. Finally, we consider time response of (26) and transform Σ to $\Sigma = SJS^{-1}$ where J is Jordan form composed of any vector as $S = [\omega_1\ \omega_2\ \cdots \omega_{2N}]$, $S^{-1} = [\nu_1\ \nu_2\ \cdots \nu_{2N}]^T$. $\omega_1,\ \nu_2$ are right and left eigenvector of Σ to $\lambda(\Sigma) = 0$. $\omega_2,\ \nu_1$ are vectors that $\Sigma\omega_2 = \omega_1, \nu_1^T\Sigma = \nu_2^T$. The state of multi-vehicle at $t \to \infty$ is expressed as,

$$\begin{aligned} \begin{bmatrix} \hat{r} \\ v_r \end{bmatrix} &= \lim_{t\to\infty} S\exp(Jt)S^{-1} \otimes I_2 \begin{bmatrix} \hat{r}(0) \\ v_r(0) \end{bmatrix} \\ &\to (\omega_1\nu_1^T + \omega_1\nu_2^T t + \omega_2\nu_2^T)_{.2} \begin{bmatrix} \hat{r}(0) \\ v_r(0) \end{bmatrix}. \end{aligned} \tag{28}$$

The each vector is written as

$$\omega_1 = \begin{bmatrix} \mathbf{1} \\ \mathbf{0} \end{bmatrix},\ \omega_2 = \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \end{bmatrix},\ \nu_1 = \begin{bmatrix} p \\ \mathbf{0} \end{bmatrix},\ \nu_2 = \begin{bmatrix} \mathbf{0} \\ p \end{bmatrix},$$

where $\mathbf{0} = [0\ 0 \cdots 0]^T \in \mathbb{R}^N$, p is eigenvector of $\lambda(-L_w) = 0$ and $p^T\mathbf{1} = 1$. Then, we get

$$\begin{bmatrix} \hat{r} \\ v_r \end{bmatrix} \to \begin{bmatrix} (\mathbf{1}p^T)_{.2}\hat{r}(0) + (\mathbf{1}p^T)_{.2}v(0)t \\ (\mathbf{1}p^T)_{.2}v(0) \end{bmatrix} \tag{29}$$

Therefore VS Flocking is achieved asymptotically.

4.3 Control Law with Collision Avoidance for VS Flocking

From Theorem 4, the formation shape was guaranteed in *VS Flocking* problem. However, in case of physical vehicles, the collision avoidance is also important problem. It is well known that artificial potential approach is effective to avoid collision[4]. The artificial potential gives repulsive force to other vehicles if a vehicle come close to other vehicles. Here, we use following artificial potential function [4]

$$U_i = \sum_{j\in\mathcal{N}_i} U_{ij},\ U_{ij} = \frac{d}{\|r_{ij}\|} + \log\|r_{ij}\|, \tag{30}$$

where $r_{ij} = r_i - r_j$ and d is controller gain. We have to select d that satisfies $d > 2(\sqrt{x_{di}^2 + y_{di}^2} + R_v)$ where R_v is the largest radius of the vehicles. Then we propose following control law with collision avoidance as

Control law 5

$$\begin{aligned} \dot{v}_{ri} &= u_i^{co} + u_i^{ca} \\ u_i &= B_i^{-1} v_{ri} \end{aligned} \tag{31}$$

$where$

$$\begin{aligned} u_i^{co} &= \dot{v}^* - k_{vr}(v_{ri} - v^*) \\ &\quad - \sum_{j\in\mathcal{N}_i} k_i\Big((\hat{r}_i - \hat{r}_j) + k_v(v_{ri} - v_{rj})\Big) \end{aligned} \tag{32}$$

$$u_i^{ca} = -\nabla_{r_i} U_i | \sum_{j\in\mathcal{N}_i} k_i(v_{ri} - v_{rj})| \tag{33}$$

where $k_{vr},\ k_v,\ k_i > 0$ are controller gain. (32) is the control low to achieve consensus and (33) is the control law to achieve collision avoidance.

Theorem 5 *Consider a system of the N vehicles with kinematics (4) and Control law 5 (31). Then VS Flocking achieves asymptotically if assumption 1 and assumption of the bidirectional communication for the network and $k_{vr} + k_v\lambda_2 - f_{max}\|L_{w.2}\| > 0$ are satisfied, where λ_2 is the smallest eigenvalue of L_w without zero eigenvalue and f_{max} is maximum potential force of and $v^* \neq 0$.*

Proof 6 *Let $v_e = v_r - \mathbf{1} \otimes v^*$, then the control input $\dot{v}_e$ for multi-vehicle systems is written as*

$$\begin{aligned} \dot{v}_e &= -k_{vr}v_e - L_{w.2}\hat{r} - k_v L_{w.2}v_e \\ &\quad - \oplus \sum_i \nabla_{r_i} U_i |L_{w.2}v_e| \end{aligned} \quad (34)$$

where $\oplus \sum_i \nabla_{r_i} U_i$ is matrix that the diagonal block element are $\nabla_{r_i} U_i$. Now, we define the function V for the system as

$$V(x) = \frac{1}{2}(v_e^T v_e + \hat{r}^T L_{w.2}\hat{r}) \geq 0. \quad (35)$$

where $x = \begin{bmatrix} v_e, \hat{r} \end{bmatrix}^T$. Because of network structure of multi-vehicle systems with bidirectional communication can be represented undirected graph. Then we have that $L_{w.2} = L_{w.2}^T$. The derivative of this function along trajectories of the $\dot{V}$ are given by

$$\begin{aligned} \dot{V} &= \hat{r}^T L_{w.2}\dot{\hat{r}} + v_e^T \dot{v}_e \\ &\leq -(k_{vr} + k_v\lambda_2 - f_{max}\|L_{w.2}\|)\|v_e\|^2, \end{aligned} \quad (36)$$

where λ_2 is smallest eigenvalue of L_w without zero eigenvalue and f_{max} is maximum potential force. Choosing

$$k_{vr} + k_v\lambda_2 - f_{max}\|L_{w.2}\| > 0, \quad (37)$$

the $\dot{V}$ is negative semi-definite. Furthermore, $\dot{V} = 0$ is satisfied by only $v_e = 0$. Applying LaSalle's invariant principle, we can see that v_e converge to 0 asymptotically. Therefore, the consensus is achieved as $v_{ri} \to v^*$. Furthermore, we can see that

$$\dot{v}_r = -L_{w.2}\hat{r} = 0 \quad (38)$$

Therefore, $\hat{r}_i \to \hat{r}_j$. Thus, VS Flocking with collision avoidance is achieved asymptotically.

5 Simulations

Consider a group of 5 vehicles that has network structure as shown in Fig.3 (upper). Fig.4 shows the desired formation and distances of VS.

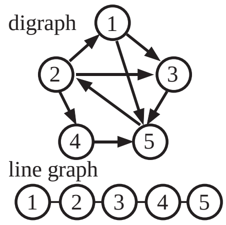

Figure 3: Graph structure

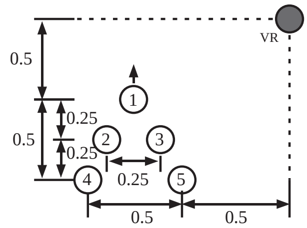

Figure 4: Formation

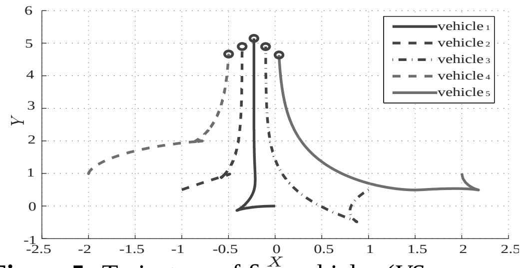

Figure 5: Trajectory of five vehicles (*VS consensus*)

5.1 VS Consensus Problems

We verify the Control law 2 (10). The parameter for VS and control law are selected as $k = 0.5$. The reference velocities are $\dot{r}_d = [0.1\cos(\pi/2)\ 0.1\sin(\pi/2)]^T$. Fig.5 shows the trajectory of the vehicles. From this result, the vehicles achieve desired formation and the position of VVs converge to a common value.

5.2 VS Flocking Problems

The Control law 4 (24) is examined. The parameters for VS and control law are selected as $k_i = 0.1$ and $k_v = 1$. The reference velocities are $\dot{r}_d = [0.1\cos(\pi/2)\ 0.1\sin(\pi/2)]^T$. Fig.6 shows the trajectory of the vehicles and Fig.7 shows the velocity errors between VVs. From these results, the vehicles achieve formation and the position and velocity of VVs converge to a common value.

5.3 VS Flocking Problems with C. A.

We verify the proposed Control law 5 (31). A group of 5 vehicles that has the network structure of line graph is considered as shown in Fig.3(lower). The parameter for VS are selected as $x_{di} = 0.05,\ y_{di} = 0$, i.e. the distances of VVs is a common value. The parameter for control low are selected as $k_{vr} = 1,\ k_v = 2,\ k_i = 0.3$. The parameter for collision avoidance function is selected as $d = 0.3$ by reason of the largest radius of the physical vehicles is $R_v = 0.08$. The reference velocities are $v^* = [0.1\ \frac{\pi}{2}]^T$. The desired formation structure is shown in Fig.4.Fig.8 shows simulation results in case with collision avoidance and without collision avoidance as $u_i^{ca} = 0$. This shows that vehicles achieve formation with collision avoidance.

6 Experiments

We verify the efficacy of the proposed control laws in experiments for *VS consensus* problem and *VS Flocking* problem. The experiments were carried out

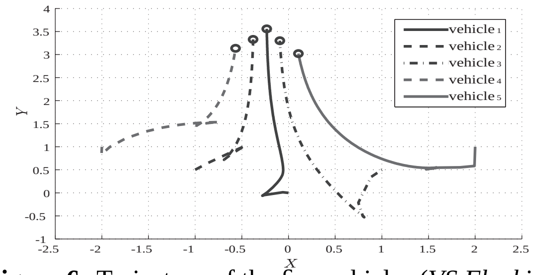

Figure 6: Trajectory of the five vehicles (*VS Flocking*)

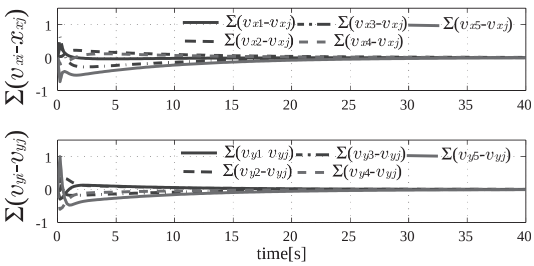

Figure 7: Error of VSs' velocity (*VS Flocking*)

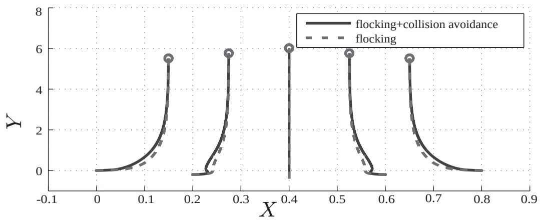

Figure 8: Trajectory

on 2 vehicles as shown in Fig.9. We use the dSPACE as real-time calculating machine and 0.2 [s] sampling rate is obtained because of the delay in wireless network.

6.1 VS Consensus Problem

The proposed Control law 3 (18) for *VS consensus* is verified first. The parameter for VS and control law are selected as $x_{d1} = x_{d2} = 0.5$, $y_{d1} = y_{d2} = 0$, $r_{r1} = [0\ 0.15]^T, r_{r2} = [0\ -0.15]^T$, $k_{vr} = 0.02$, $k = 1$. The initial conditions are $R_1(0) = [0.27\ 0.18\ 0]^T$, $R_2(0) = [0.27\ -0.18\ 0]^T$. The reference velocities are $v_d = [0.07\ 0]^T$. Fig.10 shows the trajectory of the positions of the 2 vehicles in the field. The VVs achieve consensus.

6.2 VS Flocking Problem

We verify proposed Control law 5 (31) for *VS Flocking* problems. However we $u_i^{ca} = 0$ because of initial velocities is 0, The parameter for VS and control law are selected as $x_{d1} = x_{d2} = 0.1$, $y_{d1} = y_{d2} = 0$, $r_{r1} = [0\ 0.15]^T, r_{r2} = [0\ -0.15]^T$, $k_{vr} = 0.5$, $k_i = 0.05$, $k_v = 0.1$. The initial conditions are $R_1(0) = [0.3\ 0.2\ 0]^T$, $R_2(0) = [0.3\ -0.2\ 0]^T$. The reference velocities are $v_d = [0.07\ 0]^T$. Fig.11 shows the trajectory of the positions of the vehicles in the field and this shows the VVs achieve flocking.

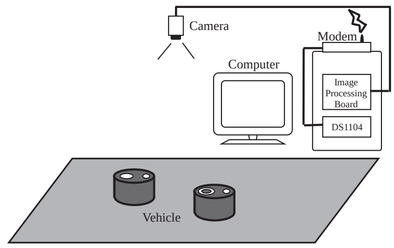

Figure 9: Experimental setup

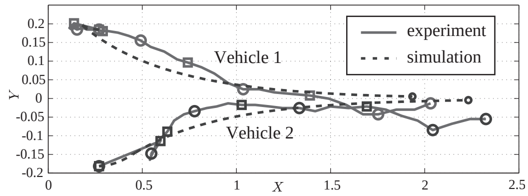

Figure 10: Trajectory of two vehicles (*VS consensus*)

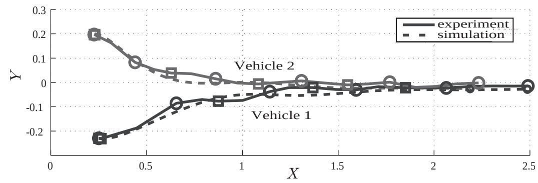

Figure 11: Trajectory of two vehicles (VS Flocking)

7 Conclusions

In this paper, we proposed the formation control methods for networked multi-vehicle systems using virtual structure. Our proposed control laws can achieve desired formations for nonholonomic systems. We proved asymptotic stability for these control strategies. Experimental and simulation results demonstrated the effectiveness of our approaches.

8 References

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