Symmetric Impedance Matched Teleoperation with Position Tracking

Toru Namerikawa and Hisanosuke Kawada

Abstract—In this paper, we propose a novel passivity-based teleoperation architecture for bilateral force and position tracking control problem. It has the passivity-based symmetric impedance matched architecture with a virtual damping. The novel teleoperation can solve the problems of position tracking. Lyapunov stability methods are used to establish the range of position control gains on the master and slave side. We show the asymptotical stability of the system. Then the controller is designed considering a trade-off between an operationability and a position tracking performance. Experimental results show the effectiveness of our proposed symmetric impedance matched teleoperation compared with the conventional one.

I. INTRODUCTION

Teleoperation systems and their control problems have been extensively studied, motivated by a large variety of applications ranging from nuclear operations and space exploration to forestry-related tasks and medical applications. Teleoperation can extend a human’s reach to a remote site or can enhance a person’s capability to handle both the macro and the micro world. A typical teleoperation system consists of the master robot, the slave robot, the human operator, the remote environment and the communication line. If only the master motion and/or forces are transmitted to the slave, the teleoperation system is called unilateral. If, in addition, slave motion and/or force are transmitted to the master, the teleoperation system is called bilateral [1], [2], [3].

In bilateral teleoperation, the master and slave robots are coupled via communication lines and the communication delay is incurred in transmission of data between the master and slave site. It is well known that the delay in a closed loop system may destabilize the system [4], [5].

Stabilization for teleoperation with the constant communication delay was achieved by the scattering transformation based on the idea of passivity [6] (or equivalent wave variable formulation [7]). In addition, the additional structure for a position control to improve the position tracking performance was proposed in [8].

In [8], however, there are two problems as follows.

1) The desired velocity derived from the scattering transformation at the slave side is as follows

\[ \dot{x}_{sd}(t) = \dot{x}_m(t - T) + \frac{1}{2} \ddot{x}_s(t) - \frac{1}{2} \ddot{x}_s(t - 2T), \]

where \( \dot{x}_m \) is the master velocity and \( \dot{x}_s \) is the slave robot velocity. The above equation is selected \( K_v = b \) as impedance matching. Thus, the slave robot’s control input \( F_s \) is given as

\[ F_s(t) = b(\dot{x}_m(t - T) - \frac{1}{2} \ddot{x}_s(t) - \frac{1}{2} \ddot{x}_s(t - 2T)). \]  

The underlined part is the slave robot velocity with the delay of \( 2T \). Therefore, there is a possibility that deterioration is caused in the position tracking performance.

The second problem is a limitation of position control gains. In [8], the upper bound of the positional control gain

In this paper, we propose a novel passivity-based teleoperation architecture for bilateral force and position tracking control problem based on [8]. It has the passivity-based symmetric impedance matched architecture with a virtual damping. The novel teleoperation can solve the above problems of conventional teleoperation. Lyapunov stability methods are used to establish the range of position control gains on the master and slave side. Then the controller is designed considering trade-off between operationability and position tracking performance. Experimental results show the effectiveness of our proposed symmetric impedance matched teleoperation compared with the conventional one.

II. CONVENTIONAL TELEOPERATION

The conventional teleoperation of [8] is shown in Fig. 1. The scattering transformation approach guarantees passivity of the communication block for the constant communication delay. The position controller was added to the master and slave side for position tracking. However, there are the following two problems.

The first problem is wave reflection. The desired velocity \( \dot{x}_{sd} \) derived from the scattering transformation at the slave side is as follows

\[ \dot{x}_{sd}(t) = \dot{x}_m(t - T) + \frac{1}{2} \ddot{x}_s(t) - \frac{1}{2} \ddot{x}_s(t - 2T), \]

where \( \dot{x}_m \) is the master velocity and \( \dot{x}_s \) is the slave robot velocity. The above equation is selected \( K_v = b \) as impedance matching. Thus, the slave robot’s control input \( F_s \) is given as

\[ F_s(t) = b(\dot{x}_m(t - T) - \frac{1}{2} \ddot{x}_s(t) - \frac{1}{2} \ddot{x}_s(t - 2T)). \]  

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Fig. 1. Conventional teleoperation
$K_p$ is as follows.

$$K_p^2 < \frac{B_m B_s}{T^2},$$  \hspace{1cm} (2)

where $B_m$ and $B_s$ represent the master robot and the slave robot damping respectively and $T$ is the communication delay. The position control gain $K_p$ is dependent on the damping and the stability condition of the system. Thus, it is impossible to design arbitrarily.

In the next section, we propose a novel architecture that solves the above problem in the previous teleoperation.

### III. SYMMETRIC IMPEDANCE MATCHED TELEOPERATION

The proposed architecture is shown in Fig. 2 where the teleoperation is symmetric, and impedance matched and has the position control loops.

For simplicity, the master and slave robots have been modeled as mass-damper systems. The system dynamics are given by

$$\begin{align*}
M_m \ddot{x}_m(t) + B_m \dot{x}_m(t) + F_m(t) &= F_{op}(t) - F_{mr}(t) + F_{back}(t) \\
M_s \ddot{x}_s(t) + B_s \dot{x}_s(t) &= F_s(t) - F_{env}(t) + F_{feed}(t),
\end{align*}$$  \hspace{1cm} (3)

where the subscript "m" and "s" show the master and slave indexes respectively, $M_m$ and $M_s$ are inertias and $B_m$ and $B_s$ are damping respectively. $F_{op}$ is an operational force applied to the master robot by human operator, $F_{env}$ is an environmental force applied to the environment by the slave robot.

The velocity controllers are as follows

$$\begin{align*}
F_{mr}(t) &= K_v(\dot{x}_m(t) - \dot{x}_{md}(t)) + D_m \dot{x}_m(t) \\
F_m(t) &= K_v(\dot{x}_m(t) - \dot{x}_{md}(t)), \\
F_{sr}(t) &= K_v(\dot{x}_s(t) - \dot{x}_{sd}(t)) - D_s \dot{x}_s(t) \\
F_s(t) &= K_v(\dot{x}_s(t) - \dot{x}_{sd}(t)),
\end{align*}$$  \hspace{1cm} (4)

where $\dot{x}_{md}$ represent the master and slave robots desired velocities from the scattering transformation at the master side, $K_v$ is a velocity control gain, $D_m$ and $D_s$ are virtual damping.

The position controllers are as follows

$$\begin{align*}
F_{back}(t) &= K_p(x_s(t - T) - x_m(t)) \\
F_{feed}(t) &= K_p(x_m(t - T) - x_s(t)),
\end{align*}$$  \hspace{1cm} (6)

The notation of this transformation is given in [7] as

$$\begin{align*}
u_m(t) &= \frac{1}{\sqrt{2b}}(F_m(t) + b \dot{x}_{md}(t)) \\
u_s(t) &= \frac{1}{\sqrt{2b}}(F_s(t) - b \dot{x}_{sd}(t)),
\end{align*}$$  \hspace{1cm} (7)

and therefore, the system is passive independent of the magnitude of the communication delay $T$.

The difference between the proposed teleoperation and the conventional teleoperation is a velocity controller $K_v$ at the master side by using the symmetric scattering transformation. Moreover, virtual damping $D_m$ and $D_s$ are added to both of the master and slave side respectively.

The symmetric scattering transformation is shown in Fig. 3.

$$\int_0^t y^T(\tau)u(\tau)d\tau = \frac{1}{2} \int_{-T}^T \{u_m^2(\tau) + v_s^2(\tau)\}d\tau \geq 0,$$  \hspace{1cm} (9)

where $b$ is a characteristic wave impedance.

Assuming that the initial energy is zero, it is easily computed that the total energy stored in the communication during the signal transmission between the master and slave robots is given by

$$\dot{x}_{md}(t) = \frac{b - K_v}{K_v + b} \dot{x}_{sd}(t - T) + \frac{K_v}{K_v + b} \dot{x}_m(t),$$  \hspace{1cm} (10)

$$\dot{x}_{sd}(t) = \frac{b - K_v}{K_v + b} \dot{x}_{md}(t - T) + \frac{K_v}{K_v + b} \dot{x}_s(t).$$  \hspace{1cm} (11)

We find that the signal $\dot{x}_{sd}$ is dependent on the signal $\dot{x}_{md}$. The signal $\dot{x}_{md}$ is spurious signal which appears on the master side due to a phenomenon described intuitively in [7] as the wave reflections. To improve the transient performance, which entails choosing $K_v = b$ as impedance matching. The above equations simplifies as
\[
\begin{aligned}
\dot{x}_{md}(t) &= \frac{1}{2}\dot{x}_s(t-T) + \frac{1}{2}\dot{x}_m(t) \\
\dot{x}_{sd}(t) &= \frac{1}{2}\dot{x}_m(t-T) + \frac{1}{2}\dot{x}_s(t).
\end{aligned}
\]  

(12)

The velocity control inputs \( F_m, F_s \) can be derived as

\[
\begin{aligned}
F_{mr}(t) &= \frac{b}{2}(\dot{x}_m(t) - \dot{x}_s(t-T)) + D_m\dot{x}_m(t) \\
F_{sr}(t) &= \frac{b}{2}(\dot{x}_m(t-T) - \dot{x}_s(t)) - D_s\dot{x}_s(t).
\end{aligned}
\]

(13)

Compared with (1), the above equations are not dependent on velocities with the delay of 2\( T \).

IV. STABILITY ANALYSIS

In the stability analysis that follows, we assume that

1) The operator and environment can be modeled as passive systems with \( \dot{x}_m \) and \( \dot{x}_s \) as input respectively.

2) The operational and environmental force are bounded by known function of the master and slave robots velocities \( \dot{x}_m \) and \( \dot{x}_s \) respectively.

3) All signal belong to \( \mathcal{L}_2 \).

4) The velocities \( \dot{x}_m \) and \( \dot{x}_s \) equal zero for \( t < 0 \).

We define the position tracking errors as

\[
\begin{aligned}
e_m(t) &= x_m(t-T) - x_s(t) \\
e_s(t) &= x_s(t-T) - x_m(t),
\end{aligned}
\]

(14)

where \( x_m(t-T) \) is the delayed master robot’s position received on the slave side and \( x_m(t-T) \) is the delayed slave robot’s position received on the master side.

Then we have the following theorem.

**Theorem 1:** Consider the system described by (3), (4), (5), (7), (8), (12) and Fig. 2. Then for range of gain \( 0 < K_p < K_p^0 \), the signals \( \dot{x}_m, \dot{x}_s, \dot{e}_m, \dot{e}_s \) are asymptotically stable.

**Proof:** Define a positive definite function \( V(x) \) for the system as

\[
V(x) = \frac{1}{2}\{M_m\dot{x}_m^2(t) + M_s\dot{x}_s^2(t) + K_p(x_m(t) - x_s(t))^2\} + \int_0^t F_{env}(z)\dot{x}_s(z)dz + \int_0^t F_{op}(z)\dot{x}_m(z)dz + \int_0^t \{F_m(z)\dot{x}_{md}(z) - F_s(z)\dot{x}_{sd}(z)\}dz.
\]

(15)

By assumption 1), the operator and environment are passive. Hence

\[
\int_0^t F_{env}(z)\dot{x}_s(z)dz \geq 0, \quad \int_0^t -F_{op}(z)\dot{x}_m(z)dz \geq 0.
\]

Using the scattering transformation in (9), the communication lines are passive

\[
\int_0^t \{F_m(z)\dot{x}_{md}(z) - F_s(z)\dot{x}_{sd}(z)\}dz \geq 0.
\]

Thus the candidate of Lyapunov function \( V(x) \) is a positive definite. The derivative of (15) along trajectories of the system is given by

\[
\dot{V}(x) = M_m\dot{x}_m\dot{x}_m + M_s\dot{x}_s\dot{x}_s + K_p(x_m - x_s)(\dot{x}_m - \dot{x}_s) + F_{env}\dot{x}_s - F_{op}\dot{x}_m + M_m\dot{x}_{md} - F_s\dot{x}_{sd}.
\]

Substituting (4) and (5), we get

\[
\dot{V}(x) = -(B_m + D_m)\dot{x}_m^2(t) - (B_s + D_s)\dot{x}_s^2(t) - K_v(\dot{x}_m(t) - \dot{x}_{md}(t))^2 - K_v(\dot{x}_s(t) - \dot{x}_{sd}(t))^2 + K_p(x_m(t) - x_s(t)) + K_p(x_s(t) - x_m(t))\dot{x}_s.
\]

Choosing \( K_v = b \) as impedance matching, we get

\[
\dot{V}(x) = -(B_m + D_m)\dot{x}_m^2(t) - (B_s + D_s)\dot{x}_s^2(t) - \frac{b}{4}\dot{q}^2_m - \frac{b}{4}\dot{q}^2_s + K_p(x_s(t) - x_m(t))\dot{x}_s + K_p(x_m(t) - x_s(t))\dot{x}_m.
\]

(16)

Using the fact that

\[
x_i(t-T) = x_i(t) = -\int_0^T \dot{x}_i(t-T)d\tau \quad ; i = m, s,
\]

and integrating the above equation, we get

\[
\int_0^{t_f} \dot{V}(x)dt \leq -(B_m + D_m)||\dot{x}_m||^2_2 - (B_s + D_s)||\dot{x}_s||^2_2 - \frac{b}{4}||\dot{q}^2_m||^2 - \frac{b}{4}||\dot{q}^2_s||^2 - K_p\int_0^{t_f} \{\dot{x}_m\int_0^T \dot{x}_s(t-\tau)d\tau \}dt - K_p\int_0^{t_f} \{\dot{x}_s\int_0^T \dot{x}_m(t-\tau)d\tau \}dt,
\]

(18)

where the notation \( || \cdot ||_2 \) denote the \( \mathcal{L}_2 \) norm of signal on interval \([0,t_f]\). Using Young’s inequality, it is easily seen that, for any \( \alpha_1 > 0 \)

\[
\int_0^{t_f} \{\dot{x}_m\int_0^T \dot{x}_s(t-\tau)d\tau \}dt = \int_0^{t_f} \{\dot{x}_m\sqrt{\alpha_1} \frac{1}{\sqrt{\alpha_1}} \int_0^T \dot{x}_s(t-\tau)d\tau \}dt \leq \frac{\alpha_1}{2}||\dot{x}_m||^2_2 + \frac{1}{2\alpha_1} \int_0^{t_f} \{\dot{x}_s(t-\tau)d\tau \}^2 dt.
\]
In addition, we use Schwartz inequality as

\[
\int_0^{t_f} \left\{ \dot{x}_m \int_0^T \dot{x}_s(t-\tau) d\tau \right\} dt \\
\leq \frac{\alpha_1}{2} \| \dot{x}_m \|^2_2 + \frac{1}{2\alpha_1} \int_0^{t_f} \left\{ \int_0^T \| \dot{x}_s(t-\tau) \|^2_2 d\tau \right\} dt \\
\leq \frac{\alpha_1}{2} \| \dot{x}_m \|^2_2 + \frac{T^3}{2\alpha_1} \| \dot{x}_s \|^2_2.
\]  

(19)

Using the fact that

\[
\int_0^{t_f-\tau} \dot{x}_s^2 dt \leq \int_0^{t_f} \dot{x}_s^2 dt = \| x_s \|^2_2,
\]

the above equation we get

\[
\int_0^{t_f} \{ \dot{x}_m \int_0^T \dot{x}_s(t-\tau) d\tau \} dt \\
\leq \frac{\alpha_1}{2} \| \dot{x}_m \|^2_2 + \frac{T^3}{2\alpha_1} \| \dot{x}_s \|^2_2.
\]

Similarly, it can be shown, for any \( \alpha_2 > 0 \)

\[
\int_0^{t_f} \{ \dot{x}_s \int_0^T \dot{x}_s(t-\tau) d\tau \} dt \\
\leq \frac{\alpha_2}{2} \| \dot{x}_s \|^2_2 + \frac{T^3}{2\alpha_1} \| \dot{x}_m \|^2_2.
\]

Therefore the integral inequality reduces to

\[
\int_0^{t_f} \dot{V}(x) dt \leq - \{ \dot{D}_m - K_p(\alpha_1 + \frac{T^3}{2\alpha_1}) \} \| \dot{x}_m \|^2_2 \\
- \{ \dot{D}_s - K_p(\alpha_2 + \frac{T^3}{2\alpha_1}) \} \| \dot{x}_s \|^2_2 \\
- \frac{b}{4} \| \dot{\dot{e}}_m \|^2_2 - \frac{b}{4} \| \dot{\dot{e}}_s \|^2_2,
\]

(20)

where \( \dot{D}_m = B_m + D_m \dot{S}_s = B_s + D_s \).

So in order for \( \dot{\dot{e}}_m, \dot{\dot{e}}_s \in L_2 \), the following inequalities are sufficient to be satisfied

\[
K_p(\alpha_1 + \frac{T^3}{2\alpha_1}) < \dot{D}_m
\]

(21)

\[
K_p(\alpha_2 + \frac{T^3}{2\alpha_1}) < \dot{D}_s.
\]

(22)

The above two inequalities multiply each other as

\[
\frac{K_p^2T^2}{4} \left( \frac{\alpha_1}{T} + \frac{T}{\alpha_2} \right) \left( \frac{\alpha_2}{T} + \frac{T}{\alpha_1} \right) < \dot{D}_m \dot{D}_s,
\]

\[
\frac{K_p^2T^2}{4} \left( 2 \frac{b}{a} + \frac{a}{b} \right) < \dot{D}_m \dot{D}_s,
\]

where \( \frac{\alpha_1}{T} = \frac{\alpha_2}{T} = b \). Using Young’s inequality as

\[
\frac{K_p^2T^2}{4} \left( 2 + 2 \right) \leq \frac{K_p^2T^2}{4} \left( 2 \frac{b}{a} + \frac{a}{b} \right) < \dot{D}_m \dot{D}_s
\]

(23)

The above inequality has solutions for any constant value of the communication delay. As the derivative of the Lyapunov function is negative-semidefinite, the system is stable in the sense of Lyapunov. \( \dot{V}(x) \) is lower bounded, negative-semidefinite and its derivative (20) is uniformly continuous in time. Applying Barbalat’s Lemma[9], we see that \( \dot{V}(x,t) \to 0 \) as \( t \to \infty \). Therefore the signals \( \dot{x}_m, \dot{e}_m \) and \( \dot{e}_s \) are asymptotically stable. ■

**Proposition 1:** The tracking error defined in (14) remains bounded.

**Proof:** The tracking error defined in (14) can be rewritten as

\[
\left\{ \begin{array}{l}
\dot{e}_m = x_m(t) - x_s(t) - \int_{t_T}^t \dot{x}_m(\tau) d\tau \\
\dot{e}_s = x_s(t) - x_m(t) - \int_{t_{t_T}}^t \dot{x}_s(\tau) d\tau.
\end{array} \right.
\]

(24)

Thus the position tracking error is bounded. ■

**Proposition 2:** The following steady states are assumed as follows

\[
\dot{x}_i(t), \dot{x}_i(t) = 0, x_i(t) = x_i, i = m, s.
\]

(25)

We obtain that the environmental force is accurately transmitted to the master side as follows

\[
F_{op} = K_p(x_m - x_s) = F_{env}.
\]

(26)

**Proof:** In the steady state given by (25), \( F_{sr}, F_{env} \to 0 \) and the master and slave robots dynamics (3) reduce to

\[
\left\{ \begin{array}{l}
F_{op} = -F_{back} = -K_p(x_s - x_m) \\
F_{env} = F_{fcd} = K_p(x_m - x_s).
\end{array} \right.
\]

(27)

The above equation is as

\[
F_{op} = K_p(x_m - x_s) = F_{env}.
\]

(28)

The environmental force is accurately transmitted to the master side. ■

In the steady state, we have \( F_{op} = F_{env} \) which guarantees good force tracking on the master side. The force between slave robot and the environment is proportional to \( K_p \) and a position error.

The above results show abilities of teleoperation when the slave robot is contact with the environment. Next, we discuss the position tracking abilities of the teleoperation in free space as \( F_{env} = 0 \) and/or \( F_{op} = 0 \).

**Proposition 3:** In the steady states given by (25), \( F_{env} = 0 \) and/or \( F_{op} = 0 \), the position tracking error defined in (14) goes to zero.

**Proof:** This result follows easily from (26). ■

**Remark 1:** The proposed architecture can be expected to have a better position tracking performance because \( K_p \) has a wide range. However the virtual damping deteriorates the operationability. Then the controller should be designed with considering trade-off between operationability performance and position tracking performance.
In this section, we verify the efficacy of the proposed architecture. Fixing the 2nd joints, the experiments were carried out on the 1DOF master and slave robots as shown in Fig. 4. Fig. 5 shows the hard environment on the slave side. The parameters of the master and slave robots in (3) are as follows

\[ M_m = M_s = 0.45 \text{ [kgm}^2\text{]} \]
\[ B_m = B_s = 0.317 \text{ [Nms]} \]

As a real-time operating system, we use RT-Linux and 1 [ms] sampling rate is obtained. All experiments have been done with the constant communication delay of 0.5[s].

The controller parameters of conventional and proposed symmetric teleoperation are selected in Table I and Table II, respectively.

<table>
<thead>
<tr>
<th>TABLE I</th>
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<tbody>
<tr>
<td>CONTROLLER GAINS OF CONVENTIONAL TELEOPERATION</td>
</tr>
<tr>
<td>( b ) &amp; ( K_v ) &amp; ( K_p )</td>
</tr>
<tr>
<td>2 &amp; 2 &amp; 0.63</td>
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</table>

In the conventional teleoperation, the position gain \( K_p \) has to be a small value. Because it is limited by the value of \( B_m \) and \( B_s \). However in symmetric teleoperation, the position gain \( K_p \) can be selected as an appropriate value.

Two kinds of experimental conditions are given as follows.

- Case 1: The slave robot moves without any contact
- Case 2: The slave robot moves in contact with the environment

In all experiment results, the slave robot responses are sifted to 0.5[s] to cancel the communication delay.

Fig. 6 shows the results of Case 1 via the symmetric impedance matched teleoperation (a) and via the conventional teleoperation (b). They show time responses of position signals of the master and slave robots, where solid line indicates the master robot signal, and dashed line shows the slave robot signal. The positions of the slave robot accurately track those of the master robot in (a) and (b). However we can see that there is the overshoot in a response of the conventional teleoperation (b). This was caused by the wave reflection (1). This problem was solved by the proposed method in (a).

Fig. 7 shows the results of Case 2 via the symmetric impedance matched teleoperation (a) and via the conventional teleoperation (b). They show time responses of position and force signals of the master and slave robots. This figure shows that the stability with both of two teleoperation is guaranteed. As shown in Fig. 7, when the slave robot is pushing the environment (2-10[sec]), the contact force is faithfully reflected to the master side. The operator can perceive the environment through the force reflection. When the slave robot dose not contact with the environment and the operator dose not apply the force to the master robot (10-20[s]), the position error is getting smaller respectively.

In the conventional teleoperation (b), the operational force is smaller and the information on the environment cannot accurately be known. The convergence of the position error due to the environment contact is slower. On the other hand in the proposed teleoperation (a), the operational force is bigger and has better magnitude. Then the information on the environment can accurately be known. The convergence of position error is faster and smaller than the conventional method.

In Figs. 6 and 7, there are the steady state errors in the position signals, but it seems to be due to physical coulomb friction of robots.
VI. CONCLUSION

In this paper, we proposed a novel passivity-based teleoperation architecture for force and position tracking control problem. Lyapunov stability methods were used to establish the range of position control gains on the master and slave side. We have proven the asymptotical stability of the system. Then the controller was designed considering trade-off between operationability and position tracking performance. Experimental results showed the effectiveness of our proposed symmetric impedance matched teleoperation compared with the conventional one.

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