Robust Control of Master-Slave Robot System  
Considering Environmental Uncertainties

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ABSTRACT

In this paper, a linear robust control system design based on $\mu$-synthesis is proposed for impedance shaped 2-DOF direct-drive robot manipulators in bilateral master-slave system with environmental uncertainties and communication delay. A general condition based on the structured singular value $\mu$ for robustness of a bilateral manipulator is derived. The proposed control methodology can guarantee the robust stability and the robust performance for environmental uncertainty, perturbation of operator dynamics, perturbation of master and slave robot manipulator dynamics and constant communication delay of the master-slave system. Several experimental results show the effectiveness of our proposed approach for various environmental uncertainties and constant communication delay.

Keywords: Robust Control, Master-Slave Robotic System, $\mu$-Synthesis, Environmental Uncertainties, Constant Communication Delay

1. INTRODUCTION

Master-slave system has been well known as one of the extension of a person’s sensing and manipulation capability to a remote location.\(^1,2\) In bilateral master-slave system, the slave manipulator is controlled by an operator through the master manipulator, while the contact force of the slave manipulator with the environment is reflected back to the operator through the master manipulator.\(^3,4\)

The goal of master-slave system control is to achieve transparency while maintaining stability under any operating conditions and for any environment including communication delay.\(^5,6\) Several previous studies for this robust stability problem in teleoperation have been done\(^1,7\) et. al.. However the teleoperation problem including multi-degree of freedom robot dynamics and structure uncertainties have not been fully dealt with.\(^3\)

In this paper, a robust controller based on $\mu$-synthesis is proposed for impedance shaped 2-DOF direct-drive robot manipulators in bilateral master-slave system with environmental uncertainties and communication delay. A general condition based on the structured singular value $\mu$ for robustness of a bilateral manipulator is derived. The proposed control methodology can guarantee the robust stability and the robust performance for all these uncertainties of the master-slave system.

Several experimental results show the effectiveness of our proposed approach for various environmental uncertainties and constant communication delay.

2. DYNAMICAL PROPERTY OF MASTER-SLAVE SYSTEM CONSTRUCTED WITH 2-DOF ROBOT MANIPULATORS

Our Master-Slave teleoperation system is illustrated in Fig.1. It is constructed with 2-DOF Direct-Drive robot master manipulator, 2-DOF DD robot slave manipulator, an operator and an Environment. We derive a mathematical model for the Master-Slave teleoperation system with various uncertainties below in this section.

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2.1. Robot Dynamics of 2-DOF Robot Manipulator

Let us consider the robot dynamics first. The robot dynamics of master and slave manipulators are given in these equations respectively.

\[ M(\theta)\ddot{\theta} + C(\theta, \dot{\theta}) + G(\theta) = \tau_m + J^T(\theta)f_m \]  
\[ M(\theta)\ddot{\theta} + C(\theta, \dot{\theta}) + G(\theta) = \tau_s - J^T(\theta)f_s \]

where \( \theta, \dot{\theta}, \ddot{\theta} \) denote the link position, velocity and acceleration vectors, respectively, \( M(\theta) \) represents the link inertia matrix, \( C(\theta, \dot{\theta}) \) represents the centrifugal-Coriolis matrix, \( G(\theta) \) represents the gravity effects, \( \tau_m, \tau_s \) represent the torque input vectors of master and slave manipulators, respectively, \( f_m, f_s \) are the operator forces to the master manipulator and the slave force to the environment, respectively, and \( J \) is the geometric Jacobian matrix.

Let \( X(t) = [x(t) \ y(t)]^T \) denote the vector of the position of the end-effector frame, then the kinematic model of the manipulator gives the relationship between \( \theta = [\theta_1 \ \theta_2]^T \) and \( X \), and the differential kinematic model gives the relationship between \( \dot{\theta} \) and \( \dot{X} \), respectively

\[ X = P(\theta) \]  
\[ \dot{X} = J(\theta)\dot{\theta} \]  

where \( J(\theta) \) is the Jacobian matrix.

The time derivative of (4) gives the acceleration relationship in the form

\[ \ddot{\theta} = J^{-1}(\theta)(\ddot{X} - J(\theta)\dot{\theta}). \]  

2.2. Linearization of Robot Dynamics by Impedance Shaping

Impedance shaped representation of the master and the slave robot dynamics should be given as in \(^3\)

\[ M_m\ddot{X}_m + D_m\dot{X}_m + K_mX_m =-Z_m\ddot{X}_m = f_{km} + f_m \]  
\[ M_s\ddot{X}_s + D_s\dot{X}_s + K_sX_s = Z_s\ddot{X}_s = f_{ks} - f_s \]

where the desired master and slave impedances \( Z_m \) and \( Z_s \) are defined, respectively as follows.

\[ Z_m = M_ms + D_m + \frac{K_m}{s} \]  
\[ Z_s = M_ss + D_s + \frac{K_s}{s} \]  

Here \( M_m, M_s, D_m, D_s, K_m \) and \( K_s \) are all \( 2 \times 2 \) real matrices and they are, respectively, the master and slave masses, damping and stiffness matrices in the desired impedance model where we apply the standard assumption \( K_m = K_s = 0 \) in the following.
$f_{km}$ and $f_{ks}$ denote the master and slave actuated forces, and they are defined as $f_{km} = [f_{kxm}, f_{kym}]^T$ and $f_{ks} = [f_{ksx}, f_{ksy}]^T$ in the task space, respectively.

Substitute the equations (6) and (5) to the equation (1), then the impedance shaping law for the master manipulator is given by

$$
\tau_m = M(\theta)\{M_m J(\theta)\}^{-1}\{-M_m \dot{J}(\theta) \dot{\theta} - D_m J(\theta) \dot{\theta} + f_m + f_{km}\} + C(\theta, \dot{\theta}) + G(\theta) - J^T(\theta)f_m.
$$

(10)

The impedance shaping law for the slave manipulator is also given as

$$
\tau_s = M(\theta)\{M_s J(\theta)\}^{-1}\{-M_s \dot{J}(\theta) \dot{\theta} - D_s J(\theta) \dot{\theta} - f_s + f_{ks}\} + C(\theta, \dot{\theta}) + G(\theta) + J^T(\theta)f_s.
$$

(11)

These impedance shaping laws can realize the ideal mechanical impedance (6)-(9).

2.3. Operator and Environmental Dynamics

The dynamics of the operator interacting with the master and the dynamics of the environmental interacting with the slave are modeled by the following equations, respectively,

$$
Z_{op}(s) \dot{X}_m = f_{op} - f_m
$$

(12)

$$
Z_{env}(s) \dot{X}_s = f_s
$$

(13)

where $f_{op}$ is an external force supplied by the operator hand, and the operator hand impedance $Z_{op}$ and the environment impedance $Z_{env}$ are defined, respectively as follows,

$$
Z_{op}(s) = M_{op}s + D_{op} + \frac{K_{op}}{s}
$$

(14)

$$
Z_{env}(s) = M_{env}s + D_{env} + \frac{K_{env}}{s}
$$

(15)

Here $M_{op}$, $M_{env}$, $D_{op}$, $D_{env}$, $K_{op}$ and $K_{env}$ are all $2 \times 2$ real matrices and they are, respectively, the masses, damping and stiffness matrices in the impedance models.

3. CONTROL SYSTEM DESIGN

3.1. Configuration of Feedback Control System

For the feedback linearized and decoupled 2-DOF master and slave robot dynamics, the control system is constructed as shown in Fig.2.

The feedback controller $K_x(s)$ and $K_y(s)$ are independently designed for each $X$ and $Y$ axis in the task space. The all behavior in both $X$ and $Y$ dynamics are same in this problem setup. Then we design the feedback control system for only $X$ axis and apply the obtained controller $K_x(s) = K_y(s)$ to the $Y$ axis control system.

The following treats only the control system design for $X$ axis.

3.2. Regulation Performance for position and velocity reference

The control performance for position and velocity control problem in the master-slave system is defined in this section.

First we denote the intervening impedance model of the master-slave system in Fig.3 and its mathematical model is given in (16).

$$
f_{mx} - f_{sx} = Z_i(s)v_{ms}
$$
Figure 2. Feedback control system

\[ Z_i(s) = m_i s + d_i + \frac{c_i}{s} \quad v_{ms} = \frac{v_m + v_s}{2} \quad (16) \]

where \( Z_i(s) \) is a desired intervening impedance and \( m_i, d_i, c_i \) are the mass, damping and stiffness scalar in the impedance models. \( v_{ms} \) is an average of the master velocity and the slave velocity.

Consider the position error \( e \) and the desired error of the master-slave system

\[ e = x_m - x_s, \quad e_d = \lambda \frac{f_{mx} + f_{sx}}{2}, \quad (17) \]

where \( x_m \) and \( x_s \) are the master position and the slave position, respectively.

Equation (17) represents a behavior of the error dynamics of the master and slave manipulators and \( \lambda (\geq 0) \) is a compliance parameter which can adjust a relative position of the master and slave manipulator.\(^4\)

Equation (16) shows a behavior of the average velocity against external relative force, which define a characteristic of the desired intervening impedance \( Z_i(s) \).

Figure 3. Intervening impedance model
Let \( v_{msd} \) denote the desired velocity as
\[
v_{msd} = \frac{1}{Z_i} (f_{mx} - f_{sx}).
\] (18)

Consider the control performance indexes \( e_{rel}, e_{abs} \) as below. Then the final control performance problem is to find a controller \( K_x(s) \) which minimize these two indexes \( e_{rel}, e_{abs} \) simultaneously.
\[
e_{rel} = e_d - e, \quad e_{abs} = v_{msd} - v_{ms}
\] (19)

3.3. Robustness for perturbation of the impedance model

Let \( Z_{mx}(s) \) and \( Z_{sx}(s) \) denote the X-axis components of master and slave impedance model \( Z_m(s) \) and \( Z_s(s) \), respectively.

\( Z_{mx}(s) \) and \( Z_{sx}(s) \) are defined as
\[
Z_{mx}(s) = \hat{Z}_{mx}(s) + \delta Z_{mx}(s) \tag{20}
\]
\[
Z_{sx}(s) = \hat{Z}_{sx}(s) + \delta Z_{sx}(s), \tag{21}
\]
where \( \hat{Z}_{mx}(s) \) are \( \hat{Z}_{sx}(s) \) nominal values and \( \delta Z_{mx}(s) \) are perturbations caused by neglected nonlinearities and exogenous disturbances.

Further, \( \delta Z_{mx}, \delta Z_{sx} \) are defined as
\[
\delta Z_{mx}(s) = \delta m_m s + \delta b_m \tag{22}
\]
\[
\delta Z_{sx}(s) = \delta m_s s + \delta b_s \tag{23}
\]
where \( \delta m_m, \delta m_s, \delta b_m, \delta b_s \) are the mass, damping and stiffness scalar in the impedance models. Assume these values of impedance perturbation are bounded as \( |\delta m_m| \leq \Delta m_m, |\delta m_s| \leq \Delta m_s, |\delta b_m| \leq \Delta b_m, |\delta b_s| \leq \Delta b_s \).

Let \( W_m(s) \) and \( W_s(s) \) denote the weighting functions as
\[
W_m(s) = \Delta m_m s + \Delta b_m, \tag{24} \\
W_s(s) = \Delta m_s s + \Delta b_s.
\]

Finally we have
\[
|\delta Z_{mx}(j\omega)| = \sqrt{\delta m_m^2 \omega^2 + \delta b_m^2} \leq \sqrt{\Delta m_m^2 \omega^2 + \Delta b_m^2} = |W_m(j\omega)|, \quad \forall \omega \in R \tag{25}
\]
\[
|\delta Z_{sx}(j\omega)| = \sqrt{\delta m_s^2 \omega^2 + \delta b_s^2} \leq \sqrt{\Delta m_s^2 \omega^2 + \Delta b_s^2} = |W_s(j\omega)|, \quad \forall \omega \in R \tag{26}
\]
These equations can be represented by using \( \Delta m, \Delta s (\|\Delta m\|_\infty \leq 1, \|\Delta s\|_\infty \leq 1) \) as
\[
Z_{mx}(s) = \hat{Z}_{mx}(s) + \delta Z_{mx}(s) = \hat{Z}_{mx} + W_m \Delta m \tag{27}
\]
\[
Z_{sx}(s) = \hat{Z}_{sx}(s) + \delta Z_{sx}(s) = \hat{Z}_{sx} + W_s \Delta s \tag{28}
\]
3.4. Robustness for perturbation of Operator and Environment

Let $\delta Z_{op}$ and $\delta Z_{env}$ define the perturbation impedances of the operator impedance $Z_{mx}(s)$ and the environment impedance $Z_{sx}(s)$ and assume they are norm bounded as

\[
|\delta Z_{op}(j\omega)| \leq |W_{op}(j\omega)|, \quad \forall \omega \in R \tag{29}
\]

\[
|\delta Z_{env}(j\omega)| \leq |W_{env}(j\omega)|, \quad \forall \omega \in R. \tag{30}
\]

Then $Z_{mx}(s)$ and $Z_{sx}(s)$ can be represented as

\[
Z_{op}(s) = \hat{Z}_{op}(s) + \delta Z_{op}(s) = \hat{Z}_{op} + W_{op}\Delta_{op} \tag{31}
\]

\[
Z_{env}(s) = \hat{Z}_{env}(s) + \delta Z_{env}(s) = \hat{Z}_{env} + W_{env}\Delta_{env} \tag{32}
\]

where $\hat{Z}_{op}(s)$ and $\hat{Z}_{env}(s)$ are nominal models, and $\Delta_{op}$ and $\Delta_{env}$ are bounded as $\|\Delta_{op}\|_{\infty} \leq 1$, $\|\Delta_{env}\|_{\infty} \leq 1$.

3.5. Robustness for Time Delay

It is well known that the time delay $e^{-j\omega L}$ is infinite-dimensional in polynomial space and cannot be represented exactly in the model. However, it can be treated as a multiplicative perturbation of the plant model in the $H_\infty/\mu$ control framework shown in Fig.4. For all $\omega$ and $0 < L < L_{max}$, the following inequality is achieved.\(^5\)

\[
|e^{-j\omega L} - 1| \leq \left| \frac{2.1j\omega}{\omega + \frac{1}{L_{max}}} \right| \leq \frac{2.1}{\omega + \frac{1}{L_{max}}} \tag{33}
\]

The time delay $e^{-Ls}(0 < L < L_{max})$ can be expressed as a multiplicative uncertainty by the weighting function $W_t$ and the normalized uncertainty $\Delta_t(\|\Delta_t\|_{\infty} \leq 1)$, where $W_t$ is given by

\[
W_t(s) = \frac{2.1s}{s + \frac{1}{L_{max}}}. \tag{34}
\]

3.6. Construction of the Generalized Plant

The above control objectives are itemized as

- Regulation Performance for Position and Velocity Reference
- Robustness for perturbation of the impedance model
- Robustness for perturbation of Operator and Environment
- Robustness for Time Delay

Figure 4. Time delay uncertainty
This multiple control objectives can be simultaneously specified in the robust $\mu$-synthesis framework. Consider the generalized plant in Fig.5 to solve this problem, where $d_m$ and $d_s$ are exogenous force disturbances to master and slave manipulators and $W_d$ is an weighting function, $W_{rel}(s)$ and $W_{abs}(s)$ are weighting functions for regulation performance, $W_{um}(s)$ and $W_{us}(s)$ are weights for control inputs and $n_v$ is an exogenous sensor disturbance.

In order to formulate the robust performance problem, the fictitious performance uncertainty block $\Delta_{perf}$ ($\|\Delta_{perf}\|_\infty \leq 1$) is introduced, and the generalized plant in Fig.5 is transformed into the LFT form in Fig.6 with the structured uncertainty.

![Figure 5. Generalized plant with uncertainties](image)

![Figure 6. Robust performance framework](image)
Let $\Delta_{mss}$ denote the block structure as
\[ \Delta_{mss} = \text{diag}[\Delta_{op}, \Delta_{env}, \Delta_{m}, \Delta_{s}, \Delta_{t}, \Delta_{perf}], \] (35)
where there six uncertainties have complex value and appropriate dimensions. The $H_\infty$ norm of these six uncertainties are normalized.

Then the robust performance condition is given by the following structure singular value $\mu$ test.
\[ \sup_{\omega \in \mathbb{R}} \mu(\Delta_{mss} [F_i(P(j\omega), K(j\omega))]) < 1 \] (36)

4. EVALUATION BY EXPERIMENTS

4.1. Control System Design
We designed a robust controller by using MATLAB $\mu$-Analysis and Synthesis Toolbox.

First, the parameters for the linearized impedance shaped model are chosen as follows.

\[
\begin{align*}
m_m &= m_s = 2.0\text{[kg]}, & d_m &= d_s = 0.2\text{[Ns/m]} \\
m_{op} &= 1.0\text{[kg]}, & d_{op} &= 2.0\text{[Ns/m]}, & k_{op} &= 10.0\text{[N/m]} \\
m_{env} &= 0\text{[kg]}, & d_{env} &= 0\text{[Ns/m]}, & k_{env} &= 100.0\text{[N/m]} \\
m_i &= 1.0\text{[kg]}, & d_i &= 0.01\text{[Ns/m]}, & c_i &= 0\text{[N/m]} \\
\lambda &= 0\text{[m/N]}
\end{align*}
\]

The design parameters for the robust control system design are selected based on experimental trial and error and the final set of design parameters is as follows.

\[
\begin{align*}
W_{rel} &= \frac{2200}{s + 10}, & W_{abs} &= \frac{80}{s + 10} \\
W_{um} &= W_{us} = \frac{0.1s + 0.01}{s + 1000}, & W_d &= 1
\end{align*}
\]

We set that generalized plant in Fig.5 includes the following parametric uncertainties which are design parameters for calculating of controllers.

- The allowable maximum time delay $L_{max} = 15\text{[msec]}$.
- 5% perturbation of impedance models of master and slave.
- 20% perturbation of impedance model of operator.
- 10% perturbation of impedance model of environment.

After the 2nd D-K iteration, the value of $\mu$ of the closed-loop system is less than 1 and the robust performance condition is achieved. The $\mu$ plot is shown in Fig.7.

4.2. Experimental Conditions
Fig.8 shows the experimental setup of the spring environment on the slave manipulator. The master manipulator is not connected with the spring. We impose the pseudo communication time delay generated in the host computer.

Two kinds of experimental environments are given as follows.

- **Case 1**: Slave Manipulator is restricted by spring I ($k_1 = 100\text{[N/m]}$) and the communication time delay is 0[msec].
- **Case 2**: Slave Manipulator is restricted by spring II ($k_2 = 110\text{[N/m]}$) and the communication time delay is 15[msec].
4.3. Experimental Results and Discussion

For the comparison, the conventional force reflecting servo type PD controllers are employed as shown in Fig.9. Force feedback is applied for the master manipulator and the following P controller $K_f(s)$ is employed.

$$K_f(s) = K_{Pf}$$

(37)

Here $K_{Pf}$ is a proportional gain. On the other side, a position feedback is applied for the slave manipulator. To avoid vibration at the high frequency range, the following saturated PD controller $K_p(s)$ is employed. By using this controller $K_p(s)$, it is expected to suppress the control gain in the high frequency.

$$K_p(s) = K_{Dp}s + K_{Pp}$$

(38)

Here, $K_{Pp}, K_{Dp}$ are proportional and derivative gains respectively. $a$ is chosen as $a = 7 \times 10^{-3}$, and $K_{Pf}, K_{Pp}, K_{Dp}$ are selected for each environment as shown in Table 1.

Experimental results on $X$-axis have the same performance on $Y$-axis in this system, then only results on $Y$-axis are shown.

Fig.10 shows the results with the Case 1 via the proposed $\mu$ controller and the conventional force reflecting servo controller. They show time responses of $Y$-axis position signals and $Y$-axis force signals of both of master and slave manipulators. This figure shows that the stability with both of two controllers is guaranteed.

Fig.11 shows the experimental results with Case 2 environment and also shows that the stability with both of two controllers is guaranteed. However, the conventional force reflecting servo controller shows a big vibration.

Note that the controller gain in FRST is tuned for each environment and two sets of controller gain are chosen in this experimental evaluation. On the other side, a $\mu$ controller is used for all experiments.

Fig.12 shows absolute value of all experimental position error data between master and slave manipulators with two controllers. The maximum position error of $\mu$ controller is about 15[mm] in the (a)Case 1 and is

Table 1. Parameters of $K_f(s)$ and $K_p(s)$

<table>
<thead>
<tr>
<th>Case</th>
<th>$K_{Pf}$</th>
<th>$K_{Pp}$</th>
<th>$K_{Dp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case1</td>
<td>3</td>
<td>350</td>
<td>50</td>
</tr>
<tr>
<td>Case2</td>
<td>1</td>
<td>300</td>
<td>20</td>
</tr>
</tbody>
</table>

Figure 7. $\mu$ Plot of the closed-loop system

Figure 8. Slave manipulator and spring environment

Figure 9. Force reflecting servo type (FRST)
about 25[mm] in the (b)Case 2. On the other side, the maximum position error of FRST is about 19[mm] in the (a)Case1 and is about 24[mm] in the (b)Case2. Compare the increase of position error of $\mu$ controller with that of FRST controller whose gain is tuned again, we can see that the increases of position error with both controllers are almost same for the environmental change from (a)Case1 to (b)Case2.

Fig.13 shows absolute value of all experimental force error data. The maximum force error of $\mu$ controller is about 1[N] in the (a)Case 1 and is about 2[N] in the (b)Case 2. We can see the force error with the (b)Case 2 via FRST controller is larger, though the controller gain is tuned again. This shows that the increase of force error with the proposed $\mu$ controller is smaller for the environmental change from (a)Case 1 to (b)Case 2.

These results means $\mu$ controller has a better robust performance for environmental uncertainties and the communication delay. Actually the robust performance is theoretically guaranteed for these uncertainties via the $\mu$ controller.

![Figure 10. Experimental results : Case 1 (Slave restricted by $k_1$, with 0[msec] time delay)](image1)

![Figure 11. Experimental results : Case 2 (Slave restricted by $k_2$, with 15[msec] time delay)](image2)
Figure 12. Absolute value of all experimental position error data between master and slave manipulators.

Figure 13. Absolute value of all experimental force error data between master and slave manipulators.
5. CONCLUSION

In this paper, a robust controller based on $\mu$-synthesis was proposed for impedance shaped 2-DOF direct-drive robot manipulators in bilateral master-slave system with environmental uncertainties and communication delay.

The dynamics of 2-DOF master-slave manipulator was linerized by using the impedance shaping, and also the dynamics of environment and operator was expressed by using the impedance model. The master slave system of two robot manipulators, environment and operator were integrated and the generalized plant was constructed. The control system that achieves the robust performance for communication delay and the perturbation of the impedance models of manipulators, operator and environment was designed by using $\mu$-synthesis.

The proposed control methodology can guarantee the robust stability and the robust performance for all the following uncertainties of the master-slave system.

- 15[msec] constant communication delay.
- 5% perturbation of impedance models of master and slave.
- 20% perturbation of impedance model of operator.
- 10% perturbation of impedance model of environment.

Experimental results showed the effectiveness of our proposed approach for various environmental uncertainties and the communication delay.

Future work is to extend this robust control method to guarantee the robust performance for a time varying communication delay.9–12

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