Proceedings of the 2004 IEEE
International Conference on Control Applications
Taipei, Taiwan, September 2-4, 2004

$\mathcal{H}_\infty$ DIA Control of Magnetic Suspension Systems

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***Abstract*— This paper deals with the $\mathcal{H}_\infty$ DIA control system design attenuating initial-state uncertainties and its application to magnetic suspension systems. Here the $\mathcal{H}_\infty$ DIA control means a mixed Disturbance and an Initial-state uncertainty Attenuation(DIA) control for linear time-invariant systems in the infinite-horizon case. The $\mathcal{H}_\infty$ DIA control problem supplies $\mathcal{H}_\infty$ controls with good transients and assures $\mathcal{H}_\infty$ controls of robustness against initial-state uncertainties. We derived a necessary and sufficient condition of the generalized $\mathcal{H}_\infty$ DIA problem.**

In this paper, we apply this $\mathcal{H}_\infty$ DIA approach to magnetic suspension systems, and evaluate a mixed attenuation property of the proposed approach via experiments. We investigate a role of the weighting matrix N for the initial state uncertainty in the control system design.

I. INTRODUCTION

Mixed **D**isturbances and **I**nitial state uncertainties **A**ttenuations are expected to supply $\mathcal{H}_\infty$ control problem with some good transient properties. The linear time-invariant $\mathcal{H}_\infty$ control attenuates the effect of disturbances on controlled outputs and is originally defined under the assumption that the initial states of the system are zero. Initial states are often uncertain where as it might be zero or non-zero. If the initial states are non-zero, the system adopting an $\mathcal{H}_\infty$ control will present some transients as the effect of the non-zero initial states, to which the $\mathcal{H}_\infty$ control is not intrinsically responsible. Such transients might be unacceptable to themselves, or might cause the performance level of disturbance attenuation of the $\mathcal{H}_\infty$ control to deteriorate intolerably. These circumstances motivated us in this paper to be concerned with $\mathcal{H}_\infty$ controls that accomplish a mixed attenuation of disturbance and initial-state uncertainty in controlled outputs. Recently, switching control for hybrid complex systems is actively studied in control theory field and this method might be one of the most reasonable and practical approach to implement it.

In the finite-horizon case, a generalized type of $\mathcal{H}_\infty$ control problem was formulated and solved by Uchida and Fujita[1] and Khargonekar et al.[2]. The problem was extended to the infinite-horizon case, and a result was derived by Kojima et al.[3] and Khargonekar et al.[2]. Here the same result was derived by the different approaches. The problem discussed by Kojima et al.[3] and Khargonekar et al.[2] is limited to the central control case. Uchida et al.[4] on the other hand, extended this result and obtained an $\mathcal{H}_\infty$ control with a free-parameter which considers a mixed attenuation of disturbance and initial-state uncertainty for linear time-invariant systems in the infinite-horizon case[4].

However, the problem discussed in [4] was limited to time-invariant systems satisfying the orthogonality assumptions [5]. This is an immensely serious problem if we apply this problem setup to the real physical control system design. The previous mixed attenuation of disturbance and initial-state uncertainty in the infinite-horizon case is not sufficient in practice[6] because time-invariant systems satisfying the orthogonality assumptions restrict the degrees of freedom of the control system design, and the previous problem setup has a difficulty in regulating control inputs[6], [7].

The authors here formulated an infinite horizon disturbance and initial state uncertainty attenuation control problem without the orthogonality assumptions[8]. The solution is given as a natural extension of the previous results in [4], [6]. A necessary and sufficient condition for a solution to exist, together with an explicit formula of the solution, was derived in [8]. Based on the given condition, a robustness property of $\mathcal{H}_\infty$ controls against initial-state uncertainty was discussed.

In this paper, we apply this approach[8] to the real magnetic suspension systems and evaluate the effectiveness of the proposed method via experiments. Magnetic suspension systems can suspend a magnetic body by magnetic force without any contact[9], which requires feedback control in order to be workable. Recently, magnetic suspension systems including active magnetic bearings and magnetic control seem to be one of the hot topics in control application field[9], [10], [11], [12]. Nonlinear control approaches are recently focused in this field[10], [11], [12], but our approach taken here is a reliable linear robust control methodology.

Comparing in the several proposed $\mathcal{H}_\infty$ DIA controllers, we show the property and effectiveness of the proposed generalized $\mathcal{H}_\infty$ DIA control attenuating initial state uncertainties. Experimental results indicate that one of the design parameter(θ_1) and the frequency responses of the $\mathcal{H}_\infty$ DIA controllers and the weight N for the initial state uncertainties $x_0 (x_0 = x(0) \neq 0)$ in the $\mathcal{H}_\infty$ DIA problem correlate closely to each other.

Finally, a role of the weighting matrix N for the initial state uncertainty is investigated in the control system design of this control problem.

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II. Problem Statement

Consider the linear time-invariant system which is defined on the time interval $[0,\infty)$ and described by

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u, \quad x(0) = x_0 \\ z &= C_1 x + D_{12} u \\ y &= C_2 x + D_{21} w \end{aligned} \tag{1}$$

where $x \in R^n$ is the state and x_0 is the initial state; $u \in R^r$ is the control input; $y \in R^m$ is the observed output; $z \in R^q$ is the controlled output and $w \in R^p$ is the disturbance. Note that this system does not have the orthogonality assumptions[5].

Without loss of generality, we regard x_0 as the initial-state uncertainty, and $x_0 = 0$ as a known initial-state case. The disturbance $w(t)$ is a square integrable function defined on $[0,\infty)$.

A, B_1, B_2, C_1, C_2, D_{12} and D_{21} are constant matrices of appropriate dimensions and satisfies that

- (A, B_1) is stabilizable and (A, C_1) is detectable.
- (A, B_2) is controllable and (A, C_2) is observable.
- $D_{12}^T D_{12} \in R^{r\times r}$ is nonsingular.
- $D_{21} D_{21}^T \in R^{m\times m}$ is nonsingular.

For system (1), every admissible output feedback control is given by a linear time-invariant system to the form

$$\begin{aligned} \dot{\zeta} &= A_K \zeta + B_K y, \quad \zeta(0) = 0 \\ u &= C_K \zeta + D_K y \end{aligned} \tag{2}$$

which makes the closed-loop system, given by (1) and (2) internally stable, where $\zeta(t)$ is the state of a controller of a finite dimension, and A_K, B_K, C_K, D_K as constant matrices of appropriate dimensions.

For the system and the class of admissible controls described above, consider a mixed-attenuation problem stated as below.

Problem 1: $\mathcal{H}_\infty$ **DIA Control Problem**
Find an admissible control attenuating disturbances and initial state uncertainties in the way that, for given $N > 0$, z satisfies

$$\|z\|_2^2 < \|w\|_2^2 + x_0^T N^{-1} x_0 \tag{3}$$

for all $w \in L^2[0,\infty)$ and all $x_0 \in R^n$, s.t., $(w, x_0) \neq 0$. Such an admissible control is called the **D**isturbance and **I**nitial state uncertainty **A**ttenuation ($\mathcal{H}_\infty$ DIA) control.

The weighting matrix N on x_0 is a measure of relative importance of the initial-state uncertainty attenuation to the disturbance attenuation. A larger choice of N in the sense of matrix inequality order means finding an admissible control which attenuates the initial-state uncertainty more.

III. $\mathcal{H}_\infty$ DIA Control

In order to solve the $\mathcal{H}_\infty$ DIA control problem, we require the Riccati equation conditions:

(A1) There exists a solution $M > 0$ to the Riccati equation

$$\begin{aligned} &M(A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1) \\ &+(A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1)^T M \\ &-M(B_2(D_{12}^T D_{12})^{-1} B_2^T - B_1 B_1^T) M \\ &+C_1^T C_1 - C_1^T D_{12}(D_{12}^T D_{12})^{-1} D_{12}^T C_1 = 0 \end{aligned} \tag{4}$$

s.t. $A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1 - B_2(D_{12}^T D_{12})^{-1} B_2^T M + B_1 B_1^T M$ is stable.

(A2) There exists a solution $P > 0$ to the Riccati equation

$$\begin{aligned} &(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2) P \\ &+P(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2)^T \\ &-P(C_2^T (D_{21} D_{21}^T)^{-1} C_2 - C_1^T C_1) P \\ &+B_1 B_1^T - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} D_{21} B_1^T = 0 \end{aligned} \tag{5}$$

s.t. $A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2 - P C_2^T (D_{21} D_{21}^T)^{-1} C_2 + P C_1^T C_1$ is stable.

(A3) $\rho(PM) < 1$,
where $\rho(X)$ denotes the spectral radius of matrix X, and $\rho(X) = \max|\lambda_i(X)|$.

Then we obtained the following results.

Theorem 1: [8] Suppose that the conditions **(A1)**, **(A2)**, and **(A3)** are satisfied. The following controller (6) is a DIA control if and only if the condition **(A4)** is satisfied.

$$\begin{aligned} \dot{x}_K &= A_K x_K + B_K y \\ u &= C_K x_K + D_K y \end{aligned} \tag{6}$$

where x_K is the state of the DIA controller and

$$\begin{aligned} A_K &= A + P C_1^T C_1 - (P C_2^T + B_1 D_{21}^T)(D_{21} D_{21}^T)^{-1} C_2 \\ &\quad -(B_2 + P C_1^T D_{12})(D_{12}^T D_{12})^{-1} \\ &\quad \times (B_2^T M + D_{12}^T C_1) L \\ B_K &= (P C_2^T + B_1 D_{21}^T)(D_{21} D_{21}^T)^{-1} \\ C_K &= -(D_{12}^T D_{12})^{-1}(B_2^T M + D_{12}^T C_1) L. \\ D_K &= 0 \end{aligned}$$

with $L := (I - PM)^{-1}$.

(A4) $Q + N^{-1} - P^{-1} > 0$,
where Q is the maximal solution of the Riccati equation

$$\begin{aligned} &Q(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2 \\ &\qquad + (B_1 B_1^T - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} D_{21} B_1^T) P^{-1}) \\ &\quad +(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2 \\ &\qquad + (B_1 B_1^T - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} D_{21} B_1^T) P^{-1})^T Q \\ &\quad -Q(B_1^T - D_{21}^T (D_{21} D_{21}^T)^{-1}(C_2 P + D_{21} B_1^T) L)^T \\ &\qquad \times (B_1^T - D_{21}^T (D_{21} D_{21}^T)^{-1}(C_2 P + D_{21} B_1^T) L) Q \\ &\quad = 0 \end{aligned} \tag{7}$$

IV. System Description and Modeling

The experimental setup of the magnetic suspension system is shown in Fig.1. An electromagnet is located at the top of the experimental system. The control problem is to levitate the iron ball stably utilizing the electromagnetic force, where the mass M of the iron ball is 238[g], and steady state gap X is 3[mm]. Note that this simple electromagnetic suspension system requires feedback control in order to be workable. As a gap sensor, a standard optical displacement sensor is placed on either side of the iron ball.

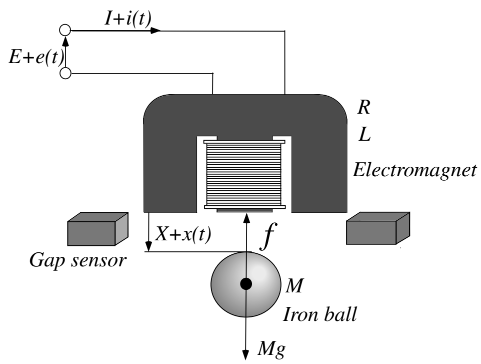

Fig. 1. Magnetic Suspension System

Under some assumptions around the steady state operation[9], we derived the 3rd-order linear state-space formulation for the system as

$$\begin{array}{rcl}\dot{x}_g &=& A_g x_g + B_g u_g + D_g v_0 \\ y_g &=& C_g x_g + w_0\end{array} \tag{8}$$

where $x_g := [x \ \ \dot{x} \ \ i]^T$, $u_g := e$, $v_0 := [v_m \ \ v_L]^T$,

$$A_g = \begin{bmatrix} 0 & 1 & 0 \\ 2670 & 0 & -23.3 \\ 0 & 0 & -31.6 \end{bmatrix}$$
$$B_g = \begin{bmatrix} 0 & 0 & 3.33 \end{bmatrix}^T, \quad C_g = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$
$$D_g = \begin{bmatrix} 0 & 0 \\ 3.50 & 0 \\ 0 & 3.33 \end{bmatrix}$$

where $x(t)$ is a gap length between the electromagnet(EM) and the iron ball, $i(t)$ is a current of the electromagnet, $e(t)$ is a control input and a voltage applied to the electromagnet and $v_m(t)$ and $v_L(t)$ are exogenous disturbance inputs.

Here (A_g, B_g) and (A_g, D_g) are controllable and (A_g, C_g) is observable.

V. CONTROL SYSTEM DESIGN

For the magnetic suspension system, our principal control objective is its stabilization. Further it should be stabilized robustly against 1) unmodeled dynamics, 2) the neglected nonlinearities, 3) the parametric uncertainties. To this end, we setup the control problem within the framework of the $\mathcal{H}_\infty$ DIA control.

A. Construction of the generalized plant and problem setup

First let us consider the system disturbance v_0. Since v_0 mainly acts on the plant in a low frequency range in practice, it is helpful to introduce a frequency weighting factor. Hence let v_0 be of the form

$$\begin{array}{rcl}v_0 &=& W_v(s)\, w_2 \\ W_v(s) &=& \Phi W(s) = \Phi C_w (sI - A_w)^{-1} B_w \\ \Phi &=& [1 \quad 1]^T\end{array} \tag{9}$$

where $W_v(s)$ is a frequency weighting whose gain is relatively large in a low frequency range, and w_2 is a $(1,2)$ element of w . These values, as yet unspecified, can be regarded as free design parameters. Note that we have not made explicit distinction in the notation between a time domain function and its Laplace transform in (9). Let us consider the system disturbance w_0 for the output. The disturbance w_0 shows an uncertain influence caused via unmodeled dynamics, and define

$$w_0 = W_w w_1 \tag{10}$$

where W_w is a weighting scalar, and w_1 is a $(1,1)$ element of w. Note that W_w is sometimes frequency dependent, but it is selected as a scalar for the sake of simplicity.

Next we consider the variables which we want to regulate. In this case, since our main concern is in the stabilization of the iron ball, the gap and the corresponding velocity are chosen; i.e.,

$$z_g = F_g x_g, \quad F_g = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{11}$$

Then, as the error vector, let us define as follows

$$z_1 = \Theta z_g, \quad \Theta = \mathrm{diag}\begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix} \tag{12}$$

where Θ is a weighting matrix on the regulated variables z_g, and z_1 is a $(1,1)$ element of z. This value Θ are also free design parameters.

Furthermore the control input u should be also regulated, and we define

$$z_2 = \rho\ u \tag{13}$$

where ρ is a weighting scalar, and z_2 is a $(1,2)$ element of z.

Finally, let $x := [\,x_g^T \quad x_w^T\,]^T$, where x_w denotes the state of the frequency weighting $W_w(s)$, and $w := [\,w_1^T \quad w_2^T\,]^T$, $z := [\,z_1^T \quad z_2^T\,]^T$, then we can construct the generalized plant as in the following;

$$\begin{array}{rcl}\dot{x} &=& Ax + B_1 w + B_2 u \\ z &=& C_1 x + D_{12} u \\ y &=& C_2 x + D_{21} w\end{array} \tag{14}$$

where A, B_1, B_2, C_1, C_2, D_{12} and D_{21} are constant matrices of appropriate dimensions.

The block diagram of the generalized plant with an unspecified controller K is shown in Fig.2. Since the disturbances w represent the various model uncertainties, the effects of these disturbances on the error vector z should be reduced. Note that this generalized plant does not have the orthogonality assumptions[5].

Next our control problem setup is: Finding an admissible controller $K(s)$ that attenuates disturbances and initial state uncertainties to achieve the $\mathcal{H}_\infty$ DIA condition in (3).

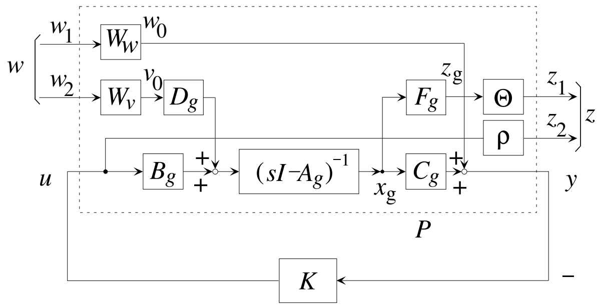

Fig. 2. Generalized Plant

B. Design Procedure of the $\mathcal{H}_\infty$ DIA Controller

We designed the $\mathcal{H}_\infty$ DIA controllers for the generalized plant derived in the previous subsection based on the following Six-Step procedure.

Iterative calculations concerning to design parameters $W_v(s)$, W_w, Θ, ρ are done to obtain appropriate numerical sets on MATLAB, then we obtain a numerical $\mathcal{H}_\infty$ DIA controller $K(s)$ directly.

[Step 1] Select a weighting function W_v: $W_v(s)$ is a frequency weighting function whose gain is relatively large in a low frequency range. This parameter is mutually related to a low gain of the controller K and the controller performance.

[Step 2] Select a weighting function W_w: $W_w(s)$ is a frequency weighting function and this is related to robustness. Bigger choice of W_w could mean allowing bigger uncertainties. Here we selected W_w as a scalar for simplicity, but it can be chosen as a frequency function.

[Step 3] Select a weighting matrix Θ: Θ is a weighting matrix on the regulated variables z_g which means that θ_1 and θ_2 regulate $x(t)$ and $\dot{x}(t)$ in $x_g(t)$ respectively.

[Step 4] Select a weighting scalar ρ: ρ is a weighting scalar on the input variable u and ρ regulates the input $u(t)$.

[Step 5] Construct a generalized plant and an $\mathcal{H}_\infty$ DIA controller: With a specified set of design parameters from [Step 1] to [Step 4], a generalized plant is constructed. The DIA controller (6) is designed for this plant.

[Step 6] Calculate the maximum matrix N: Calculating the maximum N satisfies the condition **(A4)**. For the sake of simplicity, the structure of the matrix N is limited as

$$N = nI \tag{15}$$

where n is a positive scalar number and I is a unit matrix of appropriate dimensions. This limitation on the positive definite matrix N is for easy evaluation after the $\mathcal{H}_\infty$ DIA analysis. ■

C. $\mathcal{H}_\infty$ DIA Controller

After some iteration in MATLAB environment, these parameters are chosen by the above 6-step design procedure as follows;

$$\begin{aligned} W_v(s) &= \frac{5.0\times 10^4}{s+0.010} \\ W_w &= 0.40 \\ \Theta &= \begin{bmatrix}\theta_1 & 0\\ 0 & \theta_2\end{bmatrix} = \begin{bmatrix}1.10 & 0\\ 0 & 0.00010\end{bmatrix} \\ \rho &= 1.0\times 10^{-7} \end{aligned} \tag{16}$$

Direct calculations yield the $\mathcal{H}_\infty$ DIA controller K_{DIA1} which has a four-order;

$$K_{DIA1}(s) := C_K(sI-A_K)^{-1}B_K \tag{17}$$

where

$$\begin{aligned} A_K &= \begin{bmatrix} -126 & 1.00 & 0 & 0\\ -5300 & 1.18\cdot 10^{-3} & -23.3 & 6.99\cdot 10^4\\ 4.00\cdot 10^7 & 8.09\cdot 10^4 & -1940 & 5.84\cdot 10^6\\ -2.24 & 5.62\cdot 10^{-7} & 0 & -0.01 \end{bmatrix} \\ B_K &= \begin{bmatrix} 157 & 9880 & 4320 & 2.78 \end{bmatrix}^T \\ C_K &= \begin{bmatrix} 1.20\cdot 10^7 & 2.43\cdot 10^4 & -574 & 1.73\cdot 10^6 \end{bmatrix} \end{aligned}$$

The frequency response of the controller $K_{DIA1}(s)$ is shown in Fig. 3 by a solid line. The maximum value of the weighting matrix N is given by

$$N = 5.256980\times 10^{-3}\times I_4. \tag{18}$$

Fig.3 shows that K_{DIA1} has a high gain at the low frequency and good roll-off property at high frequency range. The comprehensive frequency response looks like a modified PID controller. In the previous $\mathcal{H}_\infty$ DIA control design framework[4], [6], it was difficult to let controllers get hold an integral property.

D. Investigation for Weight N

The weighting matrix N on x_0 is a measure of relative importance of the initial-state uncertainty attenuation to the disturbance attenuation. A larger choice of N in the sense of matrix inequality order means finding an admissible control which attenuates the initial-state uncertainty more[4], [3], we treat here N as just a $n\times I$, where n is a positive scalar number and I is a unit matrix of appropriate dimensions. The mixed DIA supplies $\mathcal{H}_\infty$ control with a good transient and assures $\mathcal{H}_\infty$ control of robustness against initial-state uncertainty. Transient responses are expected to be improved via regulating of initial state uncertainties[2], [4]D

For the evaluation of a feedback performance against the weighting matrix N, we designed three other $\mathcal{H}_\infty$ DIA controllers. Here we focus on a design parameter Θ which makes a key role for a regulation of the plant state x_g. θ_1 is especially important in Θ, because it is an weight for a displacement $x(t)$ of the iron ball, hence three controllers: K_{DIA2}, K_{DIA3} and K_{DIA4} have been designed based on a variation of θ_1. Numerical values of the design parameters $W_v(s)$, W_w, θ_2 and ρ except for θ_1 are invariant throughout the control system design and experiments. A set of design results is shown in Table I.

The frequency responses of the four controllers: K_{DIA1}, K_{DIA2}, K_{DIA3} and K_{DIA4} are shown in Fig. 3 by a solid line, a dashed line, a dash-dot line and a dotted line respectively. From Fig.3 and Table I, it can be seen that a larger θ_1 supplies a controller with a higher gain at high frequency and gives a larger n.

Remark 1: A much larger choice of θ_1 $(\theta_1 > 1.1)$ supplies a controller with a much higher gain at high frequency and with a much larger n. But a time response of the resulting controller shows a vibration in experiments. $\theta_1 = 1.1$ is almost upper limit for a stable suspension.

Remark 2: A much smaller choice of θ_1 $(\theta_1 < 0.3)$ provides a controller with a lower gain not only at high frequency but at all frequency range in Fig.3 and its time response in experiments shows a different property from θ_1 is in $0.3 \le \theta_1 \le 1.1$ case.

TABLE I

$\mathcal{H}_\infty$ DIA CONTROLLERS AND THEIR PROPERTIES

Controller	θ_1	n	Line style in Fig.3
K_{DIA1}	1.10	5.256980×10^{-3}	solid line
K_{DIA2}	0.80	5.223575×10^{-3}	dashed line
K_{DIA3}	0.50	5.202185×10^{-3}	dash-dot line
K_{DIA4}	0.30	5.193773×10^{-3}	dotted line

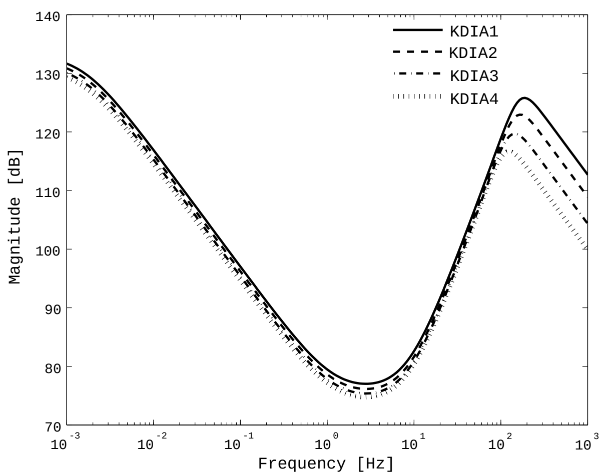

Fig. 3. Frequency Responses of $\mathcal{H}_\infty$ DIA Controllers

VI. Evaluation by Experiments

We have conducted control experiments to evaluate properties of all four controllers: K_{DIA1}, K_{DIA2}, K_{DIA3} and K_{DIA4}. The iron ball at a standstill has been suspended stably with all four controllers.

A larger choice of n means finding an admissible control which attenuates the initial-state uncertainty more. This means the controller has a better transient response[3], [4]. Table I and Fig.3 show that a larger θ_1 corresponds a higher-gain controller at high frequency which is equivalent to a larger n. This means K_{DIA1} is expected to have the best transient performance among the four controllers.

A. Transient Response

For evaluation of the above expectation for transient responses, a step reference signal is added to the system around 1.0[s], where the magnitude of the step signal is 1.0[mm] and the steady state displacement from the electromagnet to the iron ball is 3.0[mm].

Experimental results with K_{DIA1}, K_{DIA2}, K_{DIA3} and K_{DIA4} are shown respectively in Fig.4. All four setting times with these controllers are almost the same among the four responses, but overshoots are different between each other and they depend on the magnitude of n. Overshoot comparison among four $\mathcal{H}_\infty$ DIA Controllers for transient responses are summarized in Table II. K_{DIA1} shows the best transient performance among all four controllers in Table II.

B. Disturbance Response

Our concerns are not only in the attenuation of the initial state uncertainty and the transient response, but also in the basic control performance for external disturbances. Hence, a vertical step disturbance signal is added to the system downward around 1.0[s] to evaluate disturbance rejection responses, where the magnitude of the step-type disturbance force is 0.7[N], which is about 25[%] of the steady-state force.

The results with K_{DIA1}, K_{DIA2}, K_{DIA3} and K_{DIA4} are shown in Fig.5. Fig.5 has a similar feature with Fig.4.

Overshoot comparison among four $\mathcal{H}_\infty$ DIA Controllers for disturbence responses are also summarized in Table II. A larger choice of n shows a smaller and regulated overshoot.

VII. Conclusion

We had formulated and solved the $\mathcal{H}_\infty$ DIA control problem which considers a mixed attenuation of disturbance and initial-state uncertainty in the infinite-horizon case, without the orthogonality assumptions[8].

In this paper, a robustness property of $\mathcal{H}_\infty$ DIA controls against initial-state uncertainty was discussed. We evaluated the effectiveness of the proposed approach via the magnetic suspension system. The role of the weighting matrix N for the initial state x_0 was definitely shown via experiments. N is a measure of relative importance of the initial-state uncertainty attenuation to the disturbance attenuation. A larger choice of N in the sense of matrix inequality order means finding an admissible control which attenuates the initial-state uncertainty more.

Experimental results showed the design parameter θ_1 and the frequency responses of the $\mathcal{H}_\infty$ DIA controllers and the weight N of the $\mathcal{H}_\infty$ DIA problem correlate closely to each other. A larger choice of θ_1 $(\theta_1 > 1.1)$ supplies a controller with a higher gain at high frequency and with a larger n. A larger n shows a smaller and regulated overshoot. Effectiveness of the proposed $\mathcal{H}_\infty$ DIA control has been shown via these experimental results.

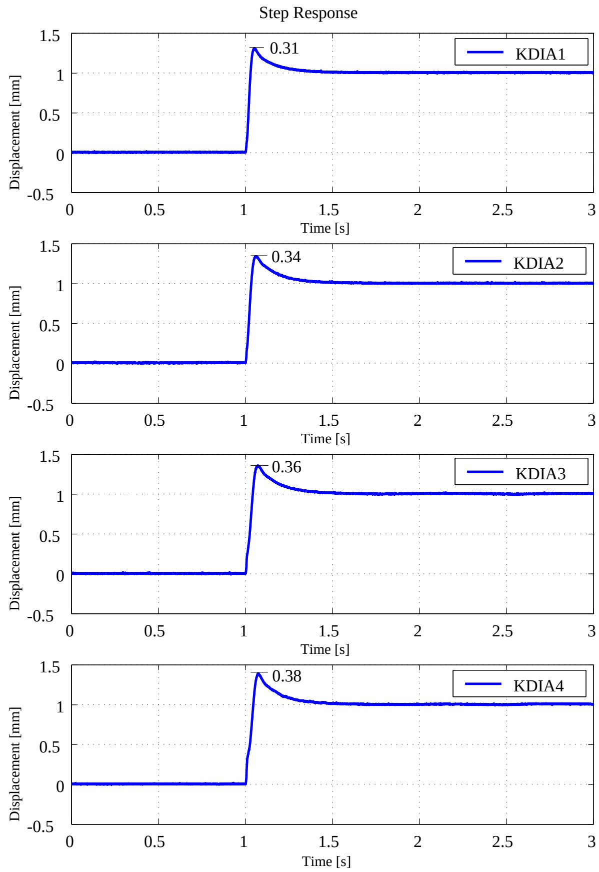

Fig. 4. Step Responses

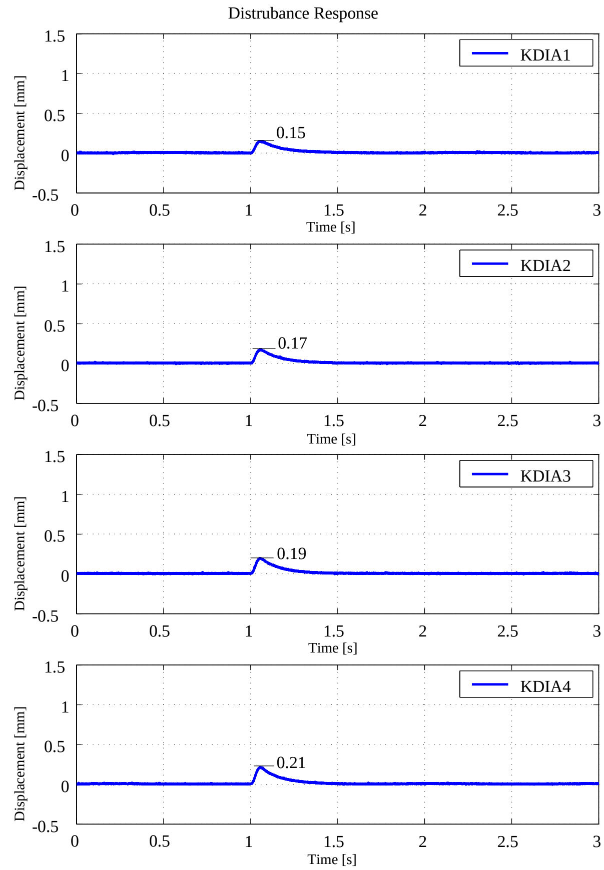

Fig. 5. Disturbance Responses

TABLE II

OVERSHOOT COMPARISON AMONG FOUR $\mathcal{H}_\infty$ DIA CONTROLLERS

Controller	n	O.S. in Fig.4 [mm]	O.S. in Fig.5 [mm]
K_{DIA1}	5.256980×10^{-3}	0.31	0.15
K_{DIA2}	5.223575×10^{-3}	0.34	0.17
K_{DIA3}	5.202185×10^{-3}	0.36	0.19
K_{DIA4}	5.193773×10^{-3}	0.38	0.21

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