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An Impedance Control Based Force-Reflection Algorithm of Bilateral Teleoperation with Communication Delay

∗Nam Duc Do and Toru Namerikawa (Kanazawa University)

Abstract– This paper addresses improved transparency of teleoperation system by using an impedance control based a new proposed force-reflection(FR) algorithm, where an exerted force by the human operator and a reflecting force from the contact environment are transferred over communication lines with time-varying delays. The input-to-output stability (IOS) small gain approach is used to show the overall FR teleoperation system to be input-to-state stable (ISS). Several experimental results show the effectiveness of our proposed algorithm.

Key Words: Teleoperation, Impedance control, Force-reflection (FR), Communication delay, Inverse dynamic

1 Introduction

Teleoperation systems allow a person to extend his/her intelligence and manipulation capability to remove and/or hazardous environments through coordinated control of two robotic arms, i.e., a master hand controller is used by human operator, and a slave robot that manipulates the environment. Therefore teleoperation system is applied in many fields such as: outer space, undersea, nuclear plants, surgical operations, vehicle steering, etc.[1].

In bilateral teleoperation, the master and slave robots are coupled via communication lines, where the position and/or force information are transferred. Communication delay is incurred in transmission of data between the master and slave. It is well known that the delay in a closed loop system may destabilize and deteriorate transparency of the system.

While accurate tracking is essential for the skillful control of tasks, it is not enough to achieve really good performance on its own since position is not only relationship that exists between both robots. In fact at the moment that the slave robot starts its interaction with the environment, reaction force appears and arises. Consequently, the feedback of the force turns out to be extremely useful and lead to so-called force reflection in master-slave system, which does not only try to achieve good tracking during unconstrained motion, but also to convey precise information of the forces that appear between the slave robot and environment, therefore the operator can actually feel them on the master robot. This system is called perfect transparency if the operator at master side can feel and exert exactly the same forces as if he/she were directly working on the remove environment with the real tool at the end-effector of slave robot [2) 3)].

In many surveys concerning the teleoperation control systems, impedance control was introduced and improved, such as in references [4)−7)]. The impedance control based computed torque approach with control objective is to make mimic a passive mechanical tool with a force-reflecting ability was used in reference [5)], in this research the slave is controlled to follow the commanded trajectory from master and to absorb interaction forces between slave and the environment. The research [6)] used the impedance control based inverse dynamic which was introduced in reference [4)] to apply to compare some controllers in a 2 DOF master-slave system, and a new force reflecting teleoperation methodology with adaptive impedance control was used in research [7)] to reduce operator energy requirements without sacrificing stability. In addition, to improve the transparency of bilateral with communication delay, a force-reflection (FR) scheme was addressed in reference [8)], and the control law was used to be PD control.

In research [8)], the force-reflection was introduced, where the environment force reflected on the master side can be altered depending on the force applied by the human operator and the alteration is not felt by the human, then this FR algorithm is not effective transparency. On the other hand, since only the force reflected from the environment, the control system of research [8)] becomes three channels communication lines teleoperation system. The problem of stable of FR teleoperation with time-varying communication delay was addressed.

In this parer, we address improved transparency of teleoperation system by using the impedance control based on researches [4)] and [6)] with a new proposal of system inputs, which relate to a new proposed FR algorithm of the bilateral teleoperation with time-varying delays in the communication lines. In the proposal, beside the reflecting force from slave side, the force that exerted by the human is also transferred to slave, then they make the system be four communication channels teleoperation. In our sense, feeling of human operator is also important, with the proposed FR algorithm, the human can feel the alteration of the force from end-effector of slave at the environment in contact tasks, therefore the transparency of teleoperation will be improved. In this paper, the input-to-output stability (IOS) small gain approach [8) 9) 10)] is used to show the overall FR teleoperation system in input-to-state stable (ISS), and several experimental results show the effectiveness of our proposed algorithm.

2 Problem Formulation

2.1 Dynamics of Teleoperation System

In this paper, we consider a pair of robotic system couple via communication lines with time-varying delays. Assuming absence of fiction, other disturbances and gravity term, the master and slave dynamics with $n-$ DOF are described as:

$$\begin{cases} M_m(q_m)\ddot{q}_m + C_m(q_m,\dot{q}_m)\dot{q}_m = \tau_m + J_m^T F_{op} \\ M_s(q_s)\ddot{q}_s + C_s(q_s,\dot{q}_s)\dot{q}_s = \tau_s - J_s^T F_e \end{cases} \quad (1)$$

Where the subscript "m" and "s" denote the master and slave indexes, respectively, $q_m, q_s \in R^{n\times 1}$ are the joint angle vectors, $\dot{q}_m, \dot{q}_s \in R^{n\times 1}$ are the joint velocity vectors, $\ddot{q}_m, \ddot{q}_s \in R^{n\times 1}$ are the joint acceleration vectors, $\tau_m, \tau_s \in R^{n\times 1}$ are the input torque vectors, $F_{op} \in R^{n\times 1}$ is the operational force vector, $F_e \in R^{n\times 1}$ is the environmental force vector, $M_m, M_s \in R^{n\times n}$ are the symmetric and positive definite inertial matrices, $C_m\dot{q}_m, C_s\dot{q}_s \in R^{n\times 1}$ are the centripetal and Coriolis torque vectors, and $J_m, J_s \in R^{n\times n}$ are Jacobian matrices.

Consider that position encoders measure manipulator coordinate q_i Cartesian coordinate must be relate to the former, and also their derivatives through the above Jacobian matrix $J_i(q_i)$ with $i = m, s$ as:

$$z_i = h_i(q_i(t)) \Rightarrow \dot{z}_i = J_i(q_i)\dot{q}_i \tag{2}$$

Following the motion of the master, the slave manipulator interacts with the environment. Here the environment is assumed to be a dynamical system described by the equations below:

$$\begin{cases} \dot{x}_e = F_{env}(x_e, z_s, \dot{z}_s, t) \\ F_e = G_{env}(x_e, z_s, \dot{z}_s, t) \end{cases} \tag{3}$$

where $x_e \in R^z$ is a state of environment, we assume that $F_{env}(x_e, z_s, \dot{z}_s, t)$ is measurable in t, locally Lipschitz in the $x_e, z_s, \dot{z}_s$ and essentially bounded on any compact set of $x_e, z_s, \dot{z}_s$ uniformly in $t \geq 0$. Additionally suppose:

$$|G_{env}(x_e, z_s, \dot{z}_s, t)| \leq a(|x_e| + |z_s| + |\dot{z}_s| + |b|) \tag{4}$$

hold for all $t \geq 0$, where $a, b \geq 0$. The following assumption is imposed on the environmental dynamics:

Assumption 1. *There exists a locally Lipschitz storage function $V_e : R^z \to R, \alpha_{1e}, \alpha_{2e} \in \mathcal{K}_\infty$, and $\alpha_{3e} > 0$ such that: $\alpha_{1e}(|x_e|) \leq V_e(x_e) \leq \alpha_{2e}(|x_e|)$ holds for all $x_e \in R^z$, and the time derivative of V_e along trajectories of (3) satisfies: $\dot{V}_e(t) \leq -\alpha_{3e}|x_e|^2 + F_e^T s_e(t)$ for almost all $t \geq 0$, where:*

$$s_e(t) = \dot{z}_s(t) - z_2^*(t) + \Lambda_{env}(z_s(t) - z_1^*(t)) \tag{5}$$

where $\Lambda_{env} = \Lambda_{env}^T \in R^{n\times n}$, and $z_1^*, z_2^* : R \to R^n$ are some continuous uniformly bounded functions.

2.2 Control Objectives

We would like to design the control input τ_m and τ_s to achieve a task-space synchronization and the transparency improvement with proposed force reflection algorithm of teleoperation. Let us define the position tracking errors of the end-effector as:

$$\begin{cases} e_m(t) = z_m(t) - z_s(t - T(t)) \\ e_s(t) = z_s(t) - z_m(t - T(t)) \end{cases}$$

where $z_m,\ z_s \in R^{n\times 1}$ are the end-effector position vectors. Then the control objectives in this paper as:

1. The synchronization is achieved as:

$$e_i(t),\ \dot{e}_i \to 0 \ as \ t \to \infty,\ i = m, s$$

2. The transparency is achieved with $\ddot{z}_i(t) = \dot{z}_i(t) = 0$, $i = m, s$ as:

$$F_{op} = F_e$$

2.3 Impedance Controller

A precise knowledge of values of the dynamic parameters of the system allows the implementation of an inverse dynamics algorithm as impedance controller. Here, following the proposal in research [6], the torque given by the motors can be split into two terms, the first arising from the teleoperation τ_{tel}, and the second from the impedance control τ_{inv}, then the torque inputs of the system as:

$$\tau_i = \tau_{inv_i} + \tau_{tel_i} \tag{6}$$

where the second term is defined as: $\tau_{tel\,i} = J_i^T F_{tel\,i}$ $(i = m, s)$. If we call H_i and B_i to be the mass and damping and they are assumed positive definite diagonal matrices, and z_i is a vector containing the Cartesian coordinates as following (2). $F_{ext\,m/s}$ are the forces exert on each robot which include reflection force information in, and $F_{tel\,m/s}$ are the forces via teleoperation. Applying approach in research [4], the target relationship between the movement of each robot and the force that act on it is expressed as:

$$\begin{cases} H_m\ddot{z}_m + B_m\dot{z}_m = F_{ext\,m} + F_{tel\,m} \\ H_s\ddot{z}_s + B_s\dot{z}_s = F_{tel\,s} + F_{ext\,s} \end{cases} \tag{7}$$

Concerning (2) we get the further differentiation:

$$\ddot{z}_i = J_i(q_i)\ddot{q}_i(t) + \dot{J}(q_i)\dot{q}_i \tag{8}$$

Substituting (8) and (2) in to (7) and operating, we can calculate the acceleration of system as follows:

$$\begin{cases} \ddot{q}_m = H_m^{-1}J_m^{-1}[F_{ext\,m} + F_{tel\,m} - B_mJ_m\dot{q}_m] \\ \qquad\qquad - J_m^{-1}\dot{J}_m\dot{q}_m \\ \ddot{q}_s = H_s^{-1}J_s^{-1}[F_{ext\,s} + F_{tel\,s} - B_sJ_s\dot{q}_s] - J_s^{-1}\dot{J}_s\dot{q}_s \end{cases} \tag{9}$$

Here for simplicity, we assume that:

Assumption 2. *The Jacobian (J_m, J_s) are invertible, i.e. they are nonsingular matrices at all the time in operation. They are also called pseudoinverse matrices.*

Substituting (9) and (6) in to (1) and enclosing above assumption, we receive:

$$\begin{cases} \tau_{inv\,m} = \\ \quad M_mH_m^{-1}J_m^{-1}[F_{ext\,m} + F_{tel\,m}] - M_mH_m^{-1}B_m\dot{q}_m \\ \quad -M_mJ_m^{-1}\dot{J}_m\dot{q}_m + C_m\dot{q}_m - (J_m^TF_{tel\,m} + J_m^TF_{op}) \\ \tau_{inv\,s} = \\ \quad M_sH_s^{-1}J_s^{-1}[F_{ext\,s} + F_{tel\,s}] - M_sH_s^{-1}B_s\dot{q}_s \\ \quad -M_sJ_s^{-1}\dot{J}_s\dot{q}_s + C_s\dot{q}_s - (J_s^TF_{tel\,s} - J_m^TF_e) \end{cases} \tag{10}$$

We receive the master slave robot dynamics with impedance controller by substituting (10) into (1) as:

$$\begin{cases} M_m\ddot{q}_m + C_m\dot{q}_m = M_mH_m^{-1}J_m^{-1}[F_{ext\,m} + F_{tel\,m}] \\ \quad -M_mH_m^{-1}B_m\dot{q}_m - M_mJ_m^{-1}\dot{J}_m\dot{q}_m + C_m\dot{q}_m \\ M_s\ddot{q}_s + C_s\dot{q}_s = M_sH_s^{-1}J_s^{-1}[F_{ext\,s} + F_{tel\,s}] \\ \quad -M_sH_s^{-1}B_s\dot{q}_s - M_sJ_s^{-1}\dot{J}_s\dot{q}_s + C_s\dot{q}_s \end{cases} \tag{11}$$

From (11) with noticing (2) and (8) we receive the task space dynamics of the teleoperation system as:

$$\begin{cases} H_m\ddot{z}_m + B_m\dot{z}_m = F_{ext\,m} + F_{tel\,m} \\ H_s\ddot{z}_s + B_s\dot{z}_s = F_{tel\,s} + F_{ext\,s} \end{cases} \tag{12}$$

In the impedance controller, we propose the exerted forces of each robot at both sides of the system, in which, the reflecting forces are also addressed:

$$\begin{cases} F_{ext\,m}(t) = F_{op}(t) - \hat{F}_e(t - T_s(t)) \\ F_{ext\,s}(t) = \hat{F}_{op}(t - T_m(t)) - F_e(t) \end{cases} \tag{13}$$

where $\hat{F}_{op}(t - T_m(t))/\hat{F}_e(t - T_s(t))$ are reflection forces from master/slave sides of teleoperation.

We assume $K_m, K_s \in R^{n\times n}$ to be positive definite diagonal gain matrices. The controller of the torque arises from teleoperation is proposed as:

$$\begin{cases} F_{tel\,m}(t) = K_m[z_s(t - T_s(t)) - z_m(t)] \\ F_{tel\,s}(t) = K_s[z_m(t - T_m(t)) - z_s(t)] \end{cases} \tag{14}$$

where $T_m(t)$ and $T_s(t)$ are time varying delays in the communication lines. Fig.1 shows a block diagram of the control system with impedance based force-reflection teleoperation, Fig.2 is a block of master/slave robot dynamics with impedance controller.

2.4 Communication Delay

Let $T_i : R \to R^+, i \in m, s$ be time-dependent time-delay in the forward ($i = m$) and backward($i = s$) communication channels, respectively. The positions and velocities of the master and slave are transmitted to the each side with communication delays $T_{m/s}(\cdot)$, then the following signals

$$\hat{z}_m(t) = z_m(t - T_m(t)); \; \hat{\dot{z}}_m(t) = \dot{z}_m(t - T_m(t))$$
$$\hat{z}_s(t) = z_s(t - T_s(t)); \; \hat{\dot{z}}_s(t) = \dot{z}_s(t - T_s(t)) \qquad (15)$$

are available for the controller on both sides of teleoperation.

On the other hand, a contact force due to the environment is measured on the slave side and transmitted back to the master, and similarly, the force exerted on the master manipulator also is measured and transmitted forward to the slave side, with communication delay $T_{s/m}(\cdot)$, i.e.

$$\tilde{F}_e(t) = \hat{F}_e(t - T_s(t)); \; \tilde{F}_{op}(t) = \hat{F}_{op}(t - T_m(t)) \qquad (16)$$

Both $T_{m/s}(t)$ are assumed to be time-varying and possibly unbounded. More precisely, the assumption imposed on $T_{m/s}(\cdot)$ is given below:

Assumption 3. *The communication delay $T_i(\cdot)$: $R \to R^+, (i = m, s)$ satisfy the following properties:*

1. $T_i(t_2) - T_i(t_1) \leq t_2 - t_1$ *for any* $t_2 \geq t_1$;
2. $-\Upsilon(t_2 - t_1) \leq T_i(t_2) - T_i(t_1)$ *for some any* $\Upsilon \geq 0$ *and for any* $t_2 \geq t_1$
3. $t - T_i(t) \to +\infty$ *as* $t \to +\infty$

3 Force-Reflection Scheme

In this section, we will consider the FR teleoperation system with communication delay as a system of functional-differential equations (FDE). A state of overall teleoperation system at time $t \in R$ can be choosen as follows:

$$x_t = (z_m^T, \dot{z}_m^T, z_s^T, \dot{z}_s^T, e_m^T, \dot{e}_m^T, e_s^T, \dot{e}_s^T)^T \qquad (17)$$

The research[8)] introduced a new FR algorithm that used reflecting force which transfers from slave side to master or operator side. In this strategy, to avoid the excessively force pushing against the human operator, the saturation function of FR was used. Note that, since the algorithm proposed changes the FR only when the human operator does not push against the environmental force, this alteration is not felt by the human, then the transparency of the teleoperation system is deteriorated. However, it prevents the teleoperator system from going into unstable mode. But in our sense, feeling of human operator is important, it makes the human can feel the alteration of the force exert on the environment by the FR from slave side. It supports to the human to apply an appropriate force in the real task at teleoperation. Therefore, we propose using one more communication channel to transfer the force of operator to slave side, then some better results can be obtained in comparison with the early research [8)]. The output of Master+operator and Slave+environment interconnection are defined as:

$$y_m = \bar{P}_1 F_{op} + \bar{P}_2 e_m + \bar{P}_3 \dot{e}_m$$
$$y_s = \bar{K}_1 F_e + \bar{K}_2 e_s + \bar{K}_3 \dot{e}_s \qquad (18)$$

where $e_m = z_m - \hat{z}_s$ and $e_s = z_s - \hat{z}_m$ are the position errors, $\dot{e}_m = \dot{z}_m - \hat{\dot{z}}_s$ and $\dot{e}_s = \dot{z}_s - \hat{\dot{z}}_m$ are the velocity errors in the slave and master sides; $\bar{K}_1 = \bar{K}_1^T \geq 0, \; \bar{K}_2 = \bar{K}_2^T \geq 0, \; \bar{K}_3 = \bar{K}_3^T \geq 0$ and $\bar{P}_1 = \bar{P}_1^T \geq 0, \; \bar{P}_2 = \bar{P}_2^T \geq 0, \; \bar{P}_3 = \bar{P}_3^T \geq 0$ are the gain matrices.

We define the FR signal of both sides as follows:

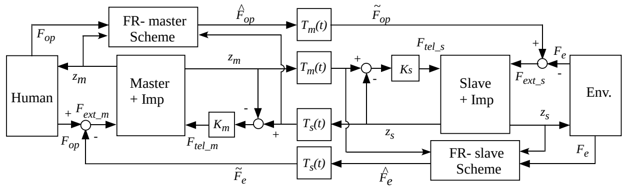

Fig. 1: Teleoperation system

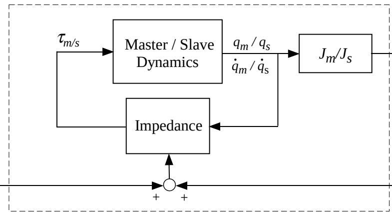

Fig. 2: The master and slave robot dynamics with impedance controller

$$\hat{F}_e(t) = K_{fb} y_s(t); \; \hat{F}_{op}(t) = P_{fw} y_m(t) \qquad (19)$$

where $K_{fb} \geq 0$ and $P_{fb} \geq 0$ are FR gains, and the signal $\hat{F}_e(t)$ is then transmitted to the master side, and $\hat{F}_{op}(t)$ is then transmitted to slave side with communication delays $T_s(t)$ and $T_m(t)$.

Concerning (18) and (19), the general formulas of FR signals are given as:

$$\hat{F}_e(t) = K_{fb}(\bar{K}_1 F_e + \bar{K}_2 e_s + \bar{K}_3 \dot{e}_s),$$
$$\hat{F}_{op}(t) = P_{fw}(\bar{P}_1 F_{op} + \bar{P}_2 e_m + \bar{P}_3 \dot{e}_m) \qquad (20)$$

The block of force reflection scheme is in Fig.1.

4 Stability Analysis

This section deals with stability of the overall teleoperation system that includes master and slave subsystems, we defined a state of the system similarly to (17). First, see approach in research [8)], we consider the master subsystem in (1) following the below Lemma

Lemma 1. *The close-loop master subsystem with state $x_M = (z_m^T, \dot{z}_m^T)^T$, input $u_M = (F_{op} - \tilde{F}_e)^T$ and output $y_M = (z_m^T, \dot{z}_m^T)^T$ is input-to-state stable (ISS), and also is input-to-output stable (IOS)*

Proof. Consider ISS-Lyapunov function candidate: $V_m = \frac{1}{2}\xi_m^T H_m \xi_m$, where ξ_s is defined as: $\xi_m = \dot{z}_m - \sigma_2 + \Lambda_m(z_m - \sigma_1)$. We can easily check that $\underline{\alpha}_m(|x_M|) \leq V_m \leq \bar{\alpha}_m(|x_M|)$ for some $\underline{\alpha}_m, \bar{\alpha}_m \in \mathcal{K}_\infty$. The time derivative of V_m along trajectories of the system is:

$$\dot{V}_m = \xi_m^T H_m \dot{\xi}_m \qquad (21)$$

We consider the FR stabilization algorithm where the velocity measurements are replaced by the estimates obtained using the so-called "dirty derivative" filters, which was also introduced in reference [8)] for the system transparency improvement as follows:

$$\begin{cases} \dot{\sigma}_1 = \sigma_2 - \Lambda_m(z_m - \sigma_1) \\ \dot{\sigma}_2 = H_m^{-1}[-B_m \dot{\sigma}_1 - K_m(z_m - \hat{z}_s)] \end{cases} \qquad (22)$$

We have: $\dot{\xi}_s = \ddot{z}_m - \dot{\sigma}_2 + \Lambda_m(\dot{z}_m - \dot{\sigma}_1)$

Substituting $\ddot{z}_m$ from (12) into $\dot{\xi}$ and then put them in (21) with noticing the formulas of $F_{tel m}$ and $F_{ext m}$, we get:

$$\dot{V}_m = -\xi_m^T H_m[H_m^{-1} - B_m(\dot{z}_m - \sigma_2 + \Lambda_m(z_m - \sigma_1))$$
$$+ F_{ext m} + \Lambda_m \xi_m]$$
$$= -\xi_m^T(B_m + H_m\Lambda_m)\xi_m + \xi_m^T(F_{op} - \tilde{F}_e) \qquad (23)$$

We use the Young's quadratic inequality with $|a^Tb| \leq (\varepsilon/2)|a|^2 + (1/2\varepsilon)|b|^2$ holds for all $\varepsilon > 0$, therefore we can obtain the following estimate:

$$\xi_m^T(F_{op} - \tilde{F}_e) \leq \frac{\lambda_{min}(\Lambda_m)}{4}|\xi_m|^2 + \frac{1}{\lambda_{min}(\Lambda_m)}|F_{op} - \tilde{F}_e|^2 \quad (24)$$

We get:

$$\dot{V}_m \leq -\xi_m^T(B_m + H_m\Lambda_m)\xi_m + \frac{\lambda_{min}(\Lambda_m)}{4}|\xi_m|^2 + \frac{1}{\lambda_{min}(\Lambda_m)}|F_{op} - \tilde{F}_e|^2 \quad (25)$$

Now, let $\gamma_\Lambda \in \mathcal{K}$ be defined as: $\gamma_\Lambda = \lambda_{min}(\Lambda_m)/4$ then we can choose $K_m = K_m^T > 0$ satisfying $\gamma_\Lambda(\Lambda_m) \leq 4[B_m + H_m\mathcal{K}_m]$.

Applying the results of Sontag and Wang (1995) [11] (see Appendix) and in reference [10], the subsystem is input-to-state stable with the state $(z_m^T, \dot{z}_m^T)^T$.

□

Now, we consider the slave-environment interconnection with slave subsystem.

Lemma 2. *State of the closed-loop slave subsystem is assumed as: $x_S = (\tilde{z}_s^T, \xi_s^T, x_e^T)^T$, and input: $u_S = ((z_1^*)^T, (z_2^*)^T, \varsigma_1^T, \varsigma_2^T, z_s^T, \tilde{F}_{op}^T)^T$. We suppose the environment dynamics (3) satisfy Assumption 1. Then there exits $\gamma_\Lambda \in \mathcal{K}$ such that if $\lambda_{min}(K_s) \geq \gamma_\Lambda(\|\Lambda_s - \Lambda_{env}\|)$, then the slave-environment interconnection is input-to-state stable.*

Proof. To prove the Lemma, following the proposal in research [8], we consider the ISS-Lyapunov function candidate:

$$V_s = \frac{1}{2}\xi_s^T H_s\xi_s + \frac{1}{2}k_z\tilde{z}_s^T\tilde{z}_s + V_e \quad (26)$$

where V_e is introduced in Assumption 1, $k_z > 0$ is a constant to be determined, $\tilde{z}_s = z_s - \varsigma_1$. And one can easily check that V_s satisfies the inequality $\underline{\alpha}_s(|x_S|) \leq V_s \leq \bar{\alpha}_s(|x_S|)$ for some $\underline{\alpha}_s, \bar{\alpha}_s \in \mathcal{K}_\infty$. Calculating the time derivative of V_s along the trajectories of the system as:

$$\dot{V}_s = \xi_s^T H_s\dot{\xi}_s + k_z\tilde{z}_s^T\dot{\tilde{z}}_s + \dot{V}_e \quad (27)$$

Similar to the master subsystem, we set: $\xi_s = (\dot{z}_s - \varsigma_2) + \Lambda_s(z_s - \varsigma_1)$; $\dot{\xi}_s = (\ddot{z}_s - \dot{\varsigma}_2) + \Lambda_s(\dot{z}_s - \dot{\varsigma}_1)$; and use the "dirty-derivative" filter as follows:

$$\begin{cases} \dot{\varsigma}_1 = \varsigma_2 - \Lambda_s(z_s - \varsigma_1) \\ \dot{\varsigma}_2 = H_s^{-1}[-B_s\dot{\varsigma}_1 - K_s(z_s - \hat{z}_m)] \end{cases} \quad (28)$$

Considering (12) to get $\ddot{z}_s$ and substituting (28) into $\dot{\xi}_s$ with noticing the formula of $F_{tel\,s}$ in (14), we receive:

$$\dot{\xi}_s = H_s^{-1}[-B_s(\dot{z}_s - \dot{\varsigma}_1) + F_{ext\,s}] - \Lambda_s\xi_s \quad (29)$$

The fact that:

$$\dot{\tilde{z}}_s = -\Lambda_s\tilde{z}_s + \xi_s + \varsigma_2 - \dot{\varsigma}_1 \quad (30)$$

and substituting (29) into (27) and concerning $\dot{V}_e$ in Assumption 1, the formula of $F_{ext\,s}$ in (13), we get:

$$\dot{V}_s \leq -\xi_s^T(B_s + H_s\Lambda_s)\xi_s - k_z\tilde{z}_s^T\Lambda_s\tilde{z}_s - \alpha_{3e}|x_e|^2 + \tilde{F}_{op}^T\xi_s + F_e^T(s_e - \xi_s) + k_z\tilde{z}_s^T\xi_s + k_z\tilde{z}_s^T(\varsigma_2 - \dot{\varsigma}_1) \quad (31)$$

Modifying the formula of ξ_s as follows:

$$\xi_s = (\dot{z}_s - \varsigma_2) + \Lambda_{env}(z_s - \varsigma_1) - (\Lambda_{env} - \Lambda_s)(z_s - \varsigma_1)$$

Seeing (5), we receive:

$$(s_e - \xi_s) = \underbrace{\varsigma_2 - z_2^* + \Lambda_{env}(\varsigma_1 - z_1^*)}_{\Omega_1} + \underbrace{(\Lambda_{env} - \Lambda_s)}_{\Omega_2}\tilde{z}_s \quad (32)$$

Using the formulas: $z_s = \tilde{z}_s + \varsigma_1$, $\dot{z}_s = \xi_s - \Lambda_s\tilde{z}_s + \varsigma_2$; and noticing the inequality (4), we get:

$$|F_e| \leq a(|x_e| + \| \Lambda_s + I \| |\tilde{z}_s| + |\xi_s| + |\varsigma_1| + |\varsigma_2|) + b \quad (33)$$

Combining (32) and (33), we get the estimate:

$$\begin{aligned} |(s_e - \xi_s)^T F_e| \leq\; & a|x_e||\Omega_1| + a \| \Lambda_s + I \| |\tilde{z}_s||\Omega_1| \\ & + a|\xi_s||\Omega_1| + a(|\varsigma_1| + |\varsigma_2|)|\Omega_1| + b|\Omega_1| \\ & + a|x_e| \| \Omega_2 \| |\tilde{z}_s| + a \| \Lambda_s + I \| \| \Omega_2 \| |\tilde{z}_s|^2 \\ & + a \| \Omega_2 \| |\xi_s||\tilde{z}_s| + a \| \Omega_2 \| (|\varsigma_1| + |\varsigma_2|)|\tilde{z}_s| \\ & + b \| \Omega_2 \| |\tilde{z}_s| \end{aligned} \quad (34)$$

We use the fact that $(\varsigma_2 - \dot{\varsigma}_1) = \Lambda_s(z_s - \varsigma_1)$, and applying the Young's quadratic inequality form, we can obtain the following set of estimates:

$$\tilde{F}_{op}^T\xi_s \leq \frac{\lambda_{min}(\Lambda_s)}{4}|\tilde{F}_{op}|^2 + \frac{1}{\lambda_{min}(\Lambda_s)}|\xi_s|^2 \quad (35)$$

$$k_z\tilde{z}_s^T\xi_s \leq k_z\left(\frac{\lambda_{min}(\Lambda_s)}{4}|\tilde{z}_s|^2 + \frac{1}{\lambda_{min}(\Lambda_s)}|\xi_s|^2\right) \quad (36)$$

$$k_z\tilde{z}_s^T\Lambda_s(z_s - \varsigma_1) \leq \frac{k_z\lambda_{min}(\Lambda_s)}{4}|\tilde{z}_s|^2 + \frac{k_z\Lambda_s^2}{\lambda_{min}(\Lambda_s)}|z_s - \varsigma_1|^2 \quad (37)$$

$$a|x_e||\Omega_1| \leq \frac{\alpha_{3e}}{4}|x_e|^2 + \frac{a^2}{\alpha_{3e}}|\Omega_1|^2 \quad (38)$$

$$a \| \Lambda_s + I \| |\tilde{z}_s||\Omega_1| \leq \frac{k_z\lambda_{min}(\Lambda_s)}{4}|\tilde{z}_s|^2 + \frac{a^2\|\Lambda_s + I\|^2}{k_z\lambda_{min}(\Lambda_s)}|\Omega_1|^2 \quad (39)$$

$$a|\xi_s||\Omega_1| \leq \frac{\lambda_{min}(K_s)}{4}|\xi_s|^2 + \frac{a^2}{\lambda_{min}(K_s)}|\Omega_1|^2 \quad (40)$$

$$a|x_e| \| \Omega_2 \| |\tilde{z}_s| \leq \frac{\alpha_{3e}}{4}|x_e|^2 + \frac{a^2}{\alpha_{3e}} \| \Omega_2 \|^2 |\tilde{z}_s|^2 \quad (41)$$

$$a \| \Omega_2 \| |\xi_s||\tilde{z}_s| \leq \| \Omega_2 \| (a^2|\xi_s|^2 + \frac{1}{4}|\tilde{z}_s|^2) \quad (42)$$

$$a \| \Omega_2 \| (|\varsigma_1| + |\varsigma_2|)|\tilde{z}_s| \leq \| \Omega_2 \| (a^2(|\varsigma_1| + |\varsigma_2|)^2 + \frac{1}{4}|\tilde{z}_s|^2) \quad (43)$$

$$b \| \Omega_2 \| |\tilde{z}_s| \leq \| \Omega_2 \| (b^2 + \frac{1}{4}|\tilde{z}_s|^2) \quad (44)$$

Combining (35)-(44) and (31), (34), we get:

$$\begin{aligned} \dot{V}_s \leq\; & -\xi_s^T(B_s + H_s\Lambda_s)\xi_s - k_z\tilde{z}_s^T\Lambda_s\tilde{z}_s - \frac{\alpha_{3e}}{2}|x_e|^2 \\ & + \Big[\frac{1 + k_z}{\lambda_{min}(\Lambda_s)} + a^2 \| \Omega_2 \| \Big]|\xi_s|^2 \\ & + \Big[\Big(a \| \Lambda_s \| + a + \frac{3}{4}\Big) \| \Omega_2 \| + \frac{a^2}{\alpha_{3e}} \| \Omega_2 \|^2 \Big]|\tilde{z}_s|^2 \\ & + \Big[\frac{a^2}{\alpha_{3e}} + \frac{a^2 \| \Lambda_s + I \|^2}{k_z\lambda_{min}(\Lambda_s)} + \frac{a^2}{\lambda_{min}(K_s)}\Big]|\Omega_1|^2 \\ & + \big[a(|\varsigma_1| + |\varsigma_2|) + b\big]|\Omega_1| + \| \Omega_2 \| \big[(a^2(|\varsigma_1| + |\varsigma_2|)^2 \\ & + b^2\big] + \frac{\lambda_{min}(\Lambda_s)}{4}|\tilde{F}_{op}|^2 + \frac{k_z\Lambda_s^2}{\lambda_{min}(\Lambda_s)}|z_s - \varsigma_1|^2 \end{aligned} \quad (45)$$

Now, let $\gamma_{1\Lambda}, \gamma_{2\Lambda} \in \mathcal{K}_\infty$ be defined for each $s \geq 0$ as follows:

$$\gamma_{1\Lambda}(s) = \frac{\big(a \| \Lambda_s \| + a + (3/4)\big)s + \big(a^2/\alpha_{3e}\big)s^2}{2\lambda_{min}(\Lambda_s)}$$

$$\gamma_{2\Lambda}(s) = \frac{s}{\lambda_{min}(\Lambda_s)} + a^2\gamma_{1\Lambda}^{-1}(s)$$

and let consider the small gain $\gamma_\Lambda = \gamma_{1\Lambda}(s) \circ \gamma_{2\Lambda}(s) \in \mathcal{K}_\infty$. Choosing $K_s = K_s^T > 0, k_z > 0$ satisfying:

$$\lambda_{min}(K_s) \geq \gamma_{2\Lambda}(k_z) \geq \gamma_\Lambda(\| \Lambda_{env} - \Lambda_s \|) \quad (46)$$

implies that:

$$\dot{V}_s \leq -\xi_s^T(B_s + H_s\Lambda_s)\xi_s - k_z\tilde{z}_s^T\Lambda_s\tilde{z}_s - \frac{\alpha_{3e}}{2}|x_e|^2$$
$$+ \Big[\frac{a^2}{\alpha_{3e}} + \frac{a^2 \parallel \Lambda_s + I \parallel^2}{k_z\lambda_{min}(\Lambda_s)} + \frac{a^2}{\lambda_{min}(K_s)}\Big]|\Omega_1|^2$$
$$+ \big[a(|\varsigma_1| + |\varsigma_2|) + b\big]|\Omega_1| + \parallel \Omega_2 \parallel \big[(a^2(|\varsigma_1| + |\varsigma_2|)^2$$
$$+ b^2\big] + \frac{\lambda_{min}(\Lambda_s)}{4}|\tilde{F}_{op}|^2 + \frac{k_z\Lambda_s^2}{\lambda_{min}(\Lambda_s)}|z_s - \varsigma_1|^2 \qquad (47)$$

Similar to the master subsystem, the results in research [11] of Sontag and Wang were used (see Appendix), then the slave subsystem is input-to-state stable with state $(\tilde{z}_s^T, \xi_s^T, x_e^T)^T$.

□

Remark 1. *We consider the output of master and the input of slave subsystem with the formula of s_e, which is the tracking set in the environment dynamics. When s_e set to zero, then the function $\dot{V}_e$ satisfies the ISS property [11], the control objectives (tracking performance and transparency) are achieved. In this case, the inputs of slave subsystem $(z_1^*, z_2^*)^T$, $(\varsigma_1, \varsigma_2)^T$ are both converged to $(\hat{z}_m, \dot{\hat{z}}_m)^T$, respectively, ones also relate to the output of the master subsystem over the communication delays. Therefore in the close-loop teleoperation system, some outputs of master relate to the inputs of slave subsystem.*

Concerning the Lemma 1 and Lemma 2, the following theorem describes stability properties of the close-loop system:

Theorem 1. *Consider the force-reflecting teleoperator system (1), (3), (13)-(14) with FR algorithm (20). Suppose the environment dynamics satisfy the Assumption 1, and the communication delays $T_{m/s}(\cdot)$ satisfy Assumption 3, there exists $\gamma_\Lambda(\cdot) \in \mathcal{K}$ such that $\lambda_{min}(K_s) \geq \gamma_\Lambda(\parallel \Lambda_s - \Lambda_{env} \parallel)$ implies that: for the FR algorithm (20), the overall teleoperation system is input-to-state stable (ISS).*

Proof. Now we can combine the above presented results and the consecutive application of the IOS small gain theorem in research [8]. Indeed, denote by $\gamma_{[u_M \to y_M]}(\cdot) \in \mathcal{K}$ the ISS gain of the closed-loop master subsystem whose existence is guaranteed by Lemma 1. And also, we let $\gamma_{[u_S \to x_S]}(\cdot) \in \mathcal{K}$ be the IOS gain of the closed-loop slave+environment subsystem (3), (29), (30). Choose $\alpha^*(\cdot) \in \mathcal{K}_\infty$ such that the inequality:

$$\alpha^* \circ \gamma_{[u_S \to x_S]}(\cdot) \circ \gamma_{[u_M \to y_M]}(\cdot)(s) < s \qquad (48)$$

holds for all $s > 0$. Applying the IOS small gain theorem, the overall teleoperation system is input-to-state stable for any $\alpha \in \mathcal{N}$ satisfying $\alpha(s) \leq \alpha^*$ for all $s \geq 0$. □

5 Evaluation by Control Experiment

In this section, we verify efficacy of the proposed FR teleoperation. The experiments were carried out on a pair of identical direct-drive planar 2 links revolute-joint robots. The inertia matrices, the Coriolis matrices are identified as:

$$M_i = \begin{bmatrix} M_{i1} + 2R_i cos(q_{i2}) & M_{i2} + R_i cos(q_{i2}) \\ M_{i2} + R_i cos(q_{i2}) & M_{i2} \end{bmatrix}$$

$$C_i = \begin{bmatrix} -R_i sin(q_{i2})\dot{q}_{i2} & -R_i sin(q_{i2})(\dot{q}_{i1} + \dot{q}_{i2}) \\ R_i sin(q_{i2})\dot{q}_{i1} & 0 \end{bmatrix}$$

where $M_{i1} = 0.366\ kgm^2, M_{i2} = 0.0291\ kgm^2, R_i = 0.0227\ kgm^2; l_{i1} = l_{i2} = 0.2\ m$; with $i = m, s$. A

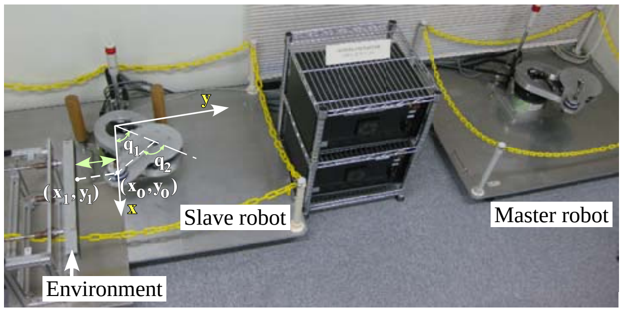

Fig. 3: Experimental setup

remove environment on the slave side is a hard aluminum wall and its surface is covered by rubber as shown in Fig.3. We also receive the joint angle values from encoders in each joint of robots and measure the operational and the environment reflecting forces by using the force sensors at the end-effectors of robots. For implementation of the controllers and communication line, we use a dSPACE system (dSPACE Inc.) All experiments have been done with the artificial time varying communication delays as:

$$T_m(t) = 0.2sin(0.3t + 0.3) + 0.3\ [s]$$
$$T_s(t) = 0.2sin(0.3t + 0.3) + 0.3\ [s]$$

Fig.3 shows the experiment task, the slave here is controlled to contact the surface of environment at $(x_1,\ y_1)$ in a line from initial position $(x_0,\ y_0)$. The initial joint angles of robots are chosen to satisfy the Assumption 1, then we set $q_1 = 45^0; q_2 = -90^0$ and they are equivalent in task space with $x_0 = 0.2828\ m;\ y_0 = 0.0\ m$. The contact position is set as: $x_1 = 0.2828\ m,\ y_1 = -0.15\ m$. The controller parameters are selected as follows:

$$H_m = H_s = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},\ B_m = B_s = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix};$$

$$K_m = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix},\ K_s = \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix}$$

and some gains of force reflection scheme are chosen: $\bar{P}_1 = 1.6,\ \bar{P}_2 = \bar{P}_3 = 0.3;\ \bar{K}_1 = 0.5,\ \bar{K}_2 = \bar{K}_3 = 0.3;\ K_{fb} = \bar{K}_{fb} = 1.25$.

Two kinds of experiment conditions are given as follows:

1. The slave moves without any contact.
2. The slave moves in contact with environment.

Figs.4, 5 show the results of case 1, the end-effector of slave robot is controlled by the master from position $(x_0,\ y_0)$, we see that the free movement of the slave accurately track those of the master robot. Fig.6 of this case shows the force of the human operator that exerted on the master, and obviously, there was not the force reflected from the environment. Figs.6, 7 show the results of case 2. In the Fig.7, while the slave robot is pushing the surface of environment (10-37[sec]), the human exerts an increasing force and the reflecting force from environment is also increasing. This force is transferred back to the master side to track the master position (see (20)) and the human also can feel the alteration of the force. The second case also proved that: with the proposed FR scheme, the teleoperation system can get better position tracking performance (see Fig.6), specially in the contact time.

6 Conclusion

In this paper, we proposed an impedance control based a new force reflection (FR) algorithm of the bilateral teleoperation with time varying delays. This proposal has improved the transparency of teleoperation with the effectively tracking performance. Using

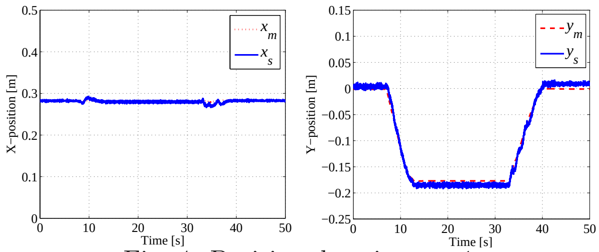

Fig. 4: Position data in case 1

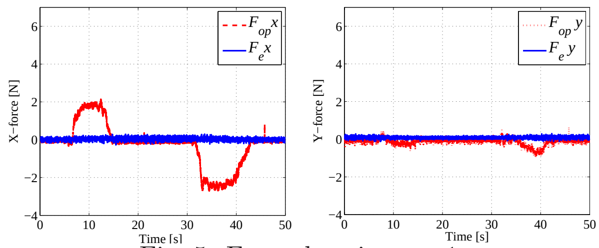

Fig. 5: Force data in case 1

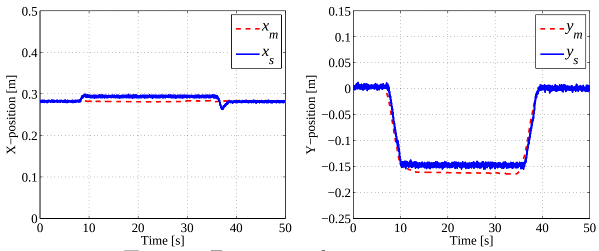

Fig. 6: Position data in case 2

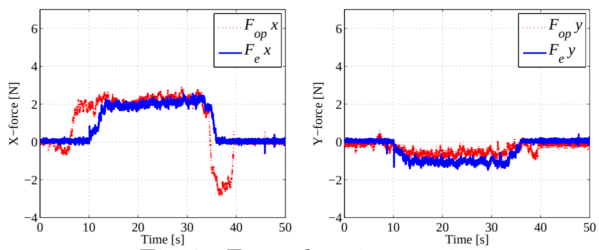

Fig. 7: Force data in case 2

the input-to-output stability (IOS) small gain theorem to show the overall FR teleoperation system in input-to-state stable (ISS). Several experimental results showed the effectiveness of our proposed methods. In future works, we will improve this algorithm with lower or varying damping without the deteriorate transparency of the teleoperation.

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Appendix

Consider the following general system:

$$\dot{x} = f(x, y) \qquad (49)$$

here $f \in R^n \times R^m \to R^n$ is continuously differentiable and satisfies $f(0,0) = 0$.

Definition 1. *The system (49) is (globally) input-to-state stable (ISS) if there exist a $\mathcal{KL}$-function $\beta : R_{\geq 0} \times R_{\geq 0} \to R$ and a $\mathcal{K}$-function γ such that, for each input $u \in L_\infty^m$ and each $\xi \in R^n$, it hold that:*

$$|x(t, \xi, u)| \leq \beta(|\xi|, t) + \gamma(\|u\|) \qquad (50)$$

for each $t \geq 0$.

Definition 2. *A smooth function $V : R^n \to R_{>0}$ is called ISS-Lyapunov function for system (49) if there exits $\mathcal{K}_\infty$-function α_1, α_2 and $\mathcal{K}$-function α_3 and χ, such that:*

$$\alpha_1(|\xi|) \leq V(\xi) \leq \alpha_2(|\xi|) \qquad (51)$$

for any $\xi \in R^n$ and

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, \xi, u) \leq -\alpha_3(|\xi|) \qquad (52)$$

for any $\xi \in R^n$ and any $u \in R^m$ so that $|\xi| \geq \chi(|u|)$

Remark 2. *A smooth function V is an ISS-Lyapunov function for (49) if and only if there exits $\alpha_i \in \mathcal{K}_\infty$ $(1 \leq i \leq 4)$ such that (51) holds, and*

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, \xi, u) \leq -\alpha_3(|\xi|) + \alpha_4(|u|) \qquad (53)$$

This provides a "dissipation" type of characterization for the ISS property. Clearly (53) implies (52). Assume now that (52) holds with some $\alpha_3 \in \mathcal{K}$ and $\chi \in \mathcal{K}$. Let $\alpha_4(r) = max\{0, \hat{\alpha}_4(r)\}$ where $\hat{\alpha}_4(r) = max\{(\partial V/\partial t + \partial V/\partial x) f(t, \xi, u) + \alpha_3(\chi(|u|)) : |u| \leq r, \xi \leq \chi(r)\}$. Then α_4 is continuous and $\alpha_4(0) = 0$, and can assume that α_4 is a $\mathcal{K}$-function. Note then that (53) is holds because $\alpha_4(r) \geq sup_{|u|=r}(\partial V/\partial t + \partial V/\partial x) f(t, \xi, u) + \alpha_3(|\xi|)$.