

On Predictive Control for Systems with Information Structured Constraints

Toru NAMERIKAWA *, Takeshi HATANAKA **, and Masayuki FUJITA **

Abstract: In this paper, we investigate a predictive control problem with information structured constraints motivated by control of micro grid. A system with information structures is defined as a system in which each subsystem collects spatio-temporally different information. For the system, we consider a predictive control law and the finite time constrained optimization problem to be solved online is reduced to a deterministic convex programming problem. Then, we reduce a control problem for a simple micro grid system to the framework and the effectiveness of the proposed control and estimation law is demonstrated through a numerical simulation.

Key Words: information structure, predictive control, micro grid, constrained system.

1. Introduction

Systems with information structures mean the systems which consist of multiple subsystems collecting spatio-temporally different information. Such systems were extensively studied in the 1970s, and a variety of control methodologies were presented and proposed (see [1],[2]).

In recent years, systems with information structures receive a lot of attention again (see [3]–[6]) due to the growing interests in new environmental and energy technologies such as smart grid, micro grid, sensor networks, and so on. For example, it is required in the micro grid (Fig. 1) that different power generators (photo-voltaic generator, wind farm, fuel cell, micro gas turbine, and so on), and power storages cooperate in the energetically and environmentally optimal fashion.

Note that this research subject is also closely related to cooperative control theory which has attracted attention over the years in the systems and control society (see [7],[8]). Among numerous research works on systems with information structures, our focus is on the distributed optimal control and estimation approach ([9]–[20]). Especially, we focus on the work of [5], where the information structures are modelled by covariance constraints and a stationary linear quadratic Gaussian (LQG) control law is presented based on the model. A finite and infinite time LQG control with covariance constraints is also proposed in [6].

Though the present control method might be useful for other objectives, this work is basically motivated by the aforementioned network-connected micro grid/smart grid, where it is required to satisfy input and state constraints with the system inherently possesses and to use information available online such as weather, wind, and demand forecast [21].

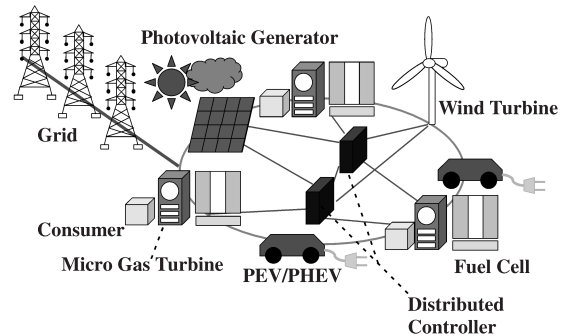


Fig. 1 Micro grid.

The main objective of this paper is thus to present a predictive control scheme, which is known to be a useful control methodology [22],[23] in order to meet the above requirements for systems with information structures. We first propose a predictive control scheme for a finite-time optimal control problem. Then, we prove that the constrained finite-time optimal control problem is reduced to a deterministic convex programming problem. Finally, a control problem for a simple micro grid system is shown to be reduced to the present framework and the effectiveness of the proposed control scheme is shown via a numerical simulation.

The following notation is used in this paper. \mathbb{Z}_+ is the non-negative integer set, \mathbb{R}^n is the n -dimensional real space, \mathbb{R}_+^n is the n -dimensional non-negative real space, $\mathbb{R}^{m \times n}$ is the $m \times n$ -dimensional real space, \mathbf{E} is an expectation operator, $Q > 0$ ($Q \geq 0$) means that Q is a positive (non-negative) definite matrix, and Tr is a trace operator.

2. System Description

2.1 Motivating Scenario

This paper is mainly motivated by control of micro grid. Its main objective is to meet the demand of each grid by operating the controllable power generators and uncontrollable generators such as renewable energy resources in a coordinated manner. For example, let us consider a micro grid system consisting of three grids in Fig. 2. Here, each grid contains as power generators a micro gas turbine and a photo-voltaic generator. The

* Department of System Design Engineering, Keio University, 3-14-1 Hiyoshi, Kohoku-ku, Yokohama 223-8522, Japan

** Department of Mechanical and Control Engineering, Tokyo Institute of Technology, 2-12-1 O-okayama Meguro-ku, Tokyo 152-8550, Japan

E-mail: namerikawa@sd.keio.ac.jp, hatanaka@ctrl.titech.ac.jp, fujita@ctrl.titech.ac.jp

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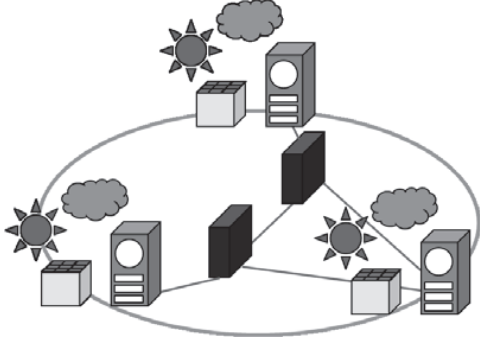


Fig. 2 An example of micro grid.

power from the gas turbine is controllable, while that from the photo-voltaic generator is uncontrollable.

Let us now use the following notations: $P_i(t)$ is the total power of the i -th subsystem, $u_i(t)$ is the power generated by the i -th micro gas turbine, $\Delta P_i(t)$ is by the i -th photo-voltaic generator, $P_i^{ref}(t)$ is the desirable power of i -th grid. Then, we formulate the time evolution of P_i . The power $P_i(t)$ at each time is in general determined by $P_i(t-1)$ at the previous time, the power of the other grid $P_j(t-1)$ connected to grid i and the generated power. For example in Fig. 2, since grid 2 is connected to grid 1, the evolution of P_1 can be described as

$$P_1(t+1) = a_1 P_1(t) + a_{12} P_2(t) + u_1(t) + \Delta P_1(t) + f_1 w_1(t)$$

with some coefficients a_1 , a_{12} , b_1 and f_1 , where $w(t)$ is the zero mean white noise which may describe the uncertainties of the power generated by a photo-voltaic generator. Let the sensor measurement $y_i(t)$ of grid i be $P_i(t)$ with zero mean white noise $v_i(t)$, i.e. $y_i(t) = P_i(t) + v_i(t)$.

Recall that the objective here is to make $P_i(t)$ close to the demand $P_i^{ref}(t)$ for all grids and for all time t . Since ΔP_i is uncontrollable, the problem is also reformulated as minimization of $\|P_i(t) - (P_i^{ref}(t) - \Delta P_i)\|$ for the modified system

$$P_1(t+1) = a_1 P_1(t) + a_{12} P_2(t) + u_1(t) + f_1 w_1(t).$$

We next draw some characteristics of micro grid systems. Though the power $\Delta P_i(t)$ is uncontrollable, its future values at least over a certain finite future time interval at each time instant are obtained online by weather forecasts. Similarly, the finite future demands P_i^{ref} are gained online by some existing energy demands forecasting technique [24]–[26]. These information should be utilized efficiently in order to achieve successful cooperation between artificial and renewable energy generators. The total control system also should be inherently multi-variable. Moreover, the physical constraints such as the maximal values of power generation ubiquitously exist over the network. The most suitable control methodology to deal with such unique characteristics current control theory provides is predictive control.

In addition to the above issues, the micro grid subsystem has information structures, namely each grid collects spatio-temporally different information. Consequently, the information available for control and estimation differs from grid to grid. In this paper, we thus present a predictive control scheme for systems with information structures.

2.2 Plant Model with Information Structures

In this subsection, we present a general plant model with information structured including the above micro grid system as a special case. Let us consider the following n linear time invariant systems.

$$z_i(t+1) = A_i z_i(t) + A_{ij} \sum_{j \in \mathcal{N}_i} z_j(t) + B_i u_i(t) + F_i w_i(t) \quad (1a)$$

$$y_i(t) = C_i z_i(t) + v_i(t), \quad i \in \mathcal{V} := \{1, \dots, n\} \quad (1b)$$

where $t \in \mathbb{Z}_+$, $z_i(t) \in \mathbb{R}^{n_x}$ is the state, $u_i(t) \in \mathbb{R}^{n_u}$ is the control input, $y_i(t) \in \mathbb{R}^{n_y}$ is the measurement, $w_i(t) \in \mathbb{R}^{n_w}$ and $v_i(t) \in \mathbb{R}^{n_v}$ are respectively zero mean white process and sensor noises. The set \mathcal{N}_i is a set of subsystems which have direct effects on the state evolution of the i -th subsystem. Then, we can define a graph $G = (\mathcal{V}, \mathcal{E})$ representing the network of interactions, where the edge set \mathcal{E} is defined by $(j, i) \in \mathcal{E} \Leftrightarrow j \in \mathcal{N}_i$.

Collecting all the subsystems (1), the total system is represented by

$$z(t+1) = A_z z(t) + B_z u(t) + F_z w(t), \quad (2a)$$

$$y(t) = C_z z(t) + v(t), \quad (2b)$$

where

$$\begin{aligned} z &:= [z_1^T \quad \dots \quad z_n^T]^T, \quad u := [u_1^T \quad \dots \quad u_n^T]^T, \\ y &:= [y_1^T \quad \dots \quad y_n^T]^T, \quad w := [w_1^T \quad \dots \quad w_n^T]^T, \\ v &:= [v_1^T \quad \dots \quad v_n^T]^T. \end{aligned}$$

The (i, j) -block of the matrix A_z is given by

$$(A_z)_{ij} = \begin{cases} A_i, & \text{if } i = j \\ A_{ij}, & \text{if } j \in \mathcal{N}_i \\ 0, & \text{otherwise} \end{cases}$$

and B_z , F_z , C_z are also defined by

$$\begin{aligned} B_z &:= [B_1^T \quad \dots \quad B_n^T]^T, \quad F_z := [F_1^T \quad \dots \quad F_n^T]^T, \\ C_z &:= \begin{bmatrix} C_1 & & 0 \\ & \ddots & \\ 0 & & C_n \end{bmatrix}^T. \end{aligned}$$

Throughout this paper, we assume that the system (2) satisfies the following equations.

$$\mathbf{E} \begin{bmatrix} w(t) \\ v(t) \end{bmatrix} \begin{bmatrix} w^T(s) & v^T(s) \end{bmatrix} = \begin{bmatrix} R_{ww} & 0 \\ 0 & R_{vv} \end{bmatrix} \delta_{ts}, \quad (3)$$

$$\delta_{ts} = 1 \quad \text{if } t = s, \quad \delta_{ts} = 0 \quad \text{if } t \neq s,$$

$$\mathbf{E} w(t)x^T(s) = 0, \quad \mathbf{E} v(t)x^T(s) = 0 \quad \text{if } t \geq s. \quad (4)$$

In this paper, we suppose that the total system (2) has information structures, namely each subsystem collects spatio-temporally different information. Thus, the information available for control and estimation differs from subsystem to subsystem. We also assume that information propagates through the communication channels at least as fast as it propagates through the plant itself (funnel causality (see [4]) or partially nested information structure (see [1],[2])). Then, it is known that the difference can be represented by communication delays between subsystems. In particular, the paper [5] shows

that information structures can be modeled by the covariance constraints

$$\mathbf{E} u_i(t)w_j^T(t-\tau) = 0 \quad \text{if } \tau \leq \text{dist}(i, j), \quad (5)$$

where $\tau \in \mathbb{Z}_+$, $\text{dist}(i, j)$ is the size of the shortest path from node j to node i along with the interaction graph G . This means there are no correlation between the control input and the disturbance beyond τ steps. Similarly to (5), the covariance constraints are also imposed on sensing as

$$\mathbf{E} y_i(t)v_j^T(t-\tau) = 0 \quad \text{if } \tau \leq \text{dist}(i, j) - 1. \quad (6)$$

In summary, the plant model under consideration is formulated as (2) with covariance constraints (5) and (6). Since the formulation is not suitable for reduction of control problem to an optimization framework, in the next subsection, we provide another description of the plant model.

2.3 Reformulation of Plant Model

In this section, we reformulate the plant model (2) with covariance constraints (5) and (6). Let us first define the augmented state $x(t)$ as

$$x(t) = \left[z^T(t) \mid w^T(t-1) \mid w^T(t-2) \quad \dots \quad w^T(t-D) \right]^T$$

with $D = \max_{i,j} \text{dist}(i, j)$. Then, the plant model (2b) is described by

$$x(t+1) = Ax(t) + Bu(t) + Fw(t), \quad (7a)$$

$$y(t) = Cx(t) + v(t) \quad (7b)$$

with

$$A := \begin{bmatrix} A_z & 0 & 0 \\ 0 & 0 & 0 \\ 0 & D_z & E_z \end{bmatrix}, B := \begin{bmatrix} B_z \\ 0 \\ 0 \end{bmatrix}, F := \begin{bmatrix} F_z \\ I \\ 0 \end{bmatrix},$$

$$C := \left[C_z \mid 0 \mid 0 \right], D_z := \begin{bmatrix} I \\ 0 \\ \vdots \\ 0 \end{bmatrix}, E_z := \begin{bmatrix} 0 & \dots & 0 \\ \vdots & & \vdots \\ I & \dots & 0 \end{bmatrix}.$$

This reformulation reduces the covariance constraints (5) to the constraints in the form of

$$\mathbf{E} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}^T Q_r \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} = 0, \quad r = 1, \dots, m \quad (8)$$

with an appropriately chosen Q_r . A structure of the covariance constraints Q_r depends on each control problem and might not be invertible. An example of Q_r is shown in Section 4.1.

In addition to the constraint (8), we deal with other two kinds of constraints into the system description: The first one is the power constraints represented by the following covariance constraints.

$$\mathbf{E} x^T(t)Q_x x(t) + u^T(t)Q_u u(t) \leq \gamma, \quad Q_x, Q_u > 0 \quad (9)$$

The second one is the mean polytopic constraints represented by

$$\mathbf{E} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \in \mathcal{D}, \quad (10)$$

where \mathcal{D} is a convex polytope including the origin as an interior and it is given by

$$\mathcal{D} := \left\{ \begin{bmatrix} x \\ u \end{bmatrix} \mid M_D \begin{bmatrix} x \\ u \end{bmatrix} \leq \mathbf{1} \right\}, \quad \mathbf{1} = [1 \quad 1 \quad \dots \quad 1]^T \quad (11)$$

In summary, the system model under consideration in this paper is given by (7) with constraints (6), (8), (9) and (10).

3. Predictive Control Law

3.1 Constrained Finite-Time Optimal Control Problem

In this section, we propose a state feedback predictive control law i.e. $C = I$ and $v \equiv 0$ for systems with information structures. For this purpose, we first consider the following constrained finite-time optimal control (CFTOC) problem.

Problem 1

$$\min_{u(0), \dots, u(N_c-1)} \mathbf{E} \left\{ x^T(N_c)P_{N_c}x(N_c) + \sum_{j=0}^{N_c-1} \begin{bmatrix} x(j) \\ u(j) \end{bmatrix}^T P \begin{bmatrix} x(j) \\ u(j) \end{bmatrix} \right\} \quad (12a)$$

subject to

$$x(k+1) = Ax(k) + Bu(k) + Fw(k), \quad x(0) = y_0$$

$$\mathbf{E} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix}^T Q_j \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \leq \epsilon_j, \quad j = 1, 2, \dots, m \quad (12b)$$

$$\mathbf{E} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix}^T Q \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} < \gamma, \quad Q = \begin{bmatrix} Q_x & 0 \\ 0 & Q_u \end{bmatrix} \quad (12c)$$

$$\mathbf{E} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \in \mathcal{D} \subset \mathbb{R}^{n_x+n_u} \quad (12d)$$

$$\mathbf{E} x(N_c) \in \mathcal{F}. \quad (12e)$$

$Q = H^T H > 0$ and Q_r are symmetric matrices. Note that $x(0)$ is a probability variable with mean y_0 and variance R_{y_0} . (12b) represents the covariance constraint (8) introduced by the information structures, where we relax the constraint by inserting a sufficiently small parameters ϵ_j . The constraint (12c) describes the power constraints (9) and (12d) describes mean constraints (10) on states and inputs, \mathcal{F} is a terminal constraint set, where

$$\mathcal{F} := \left\{ \begin{bmatrix} x \\ u \end{bmatrix} \mid M_F \begin{bmatrix} x \\ u \end{bmatrix} \leq \mathbf{1} \right\}. \quad (13)$$

The present control law determines the control input according to the receding horizon control policy, i.e. the above problem with the initial state $y_0 = x(t)$ is solved, the first one of the computed control moves is implemented and then the optimal control problem is newly solved at the next step with the horizon shifted forward by one time instant. In the following, the numbers of linear constraints in (12d) and (12e) are denoted by N_D and N_F respectively.

3.2 Solution to CFTOC

In this subsection, we present a solution to Problem 1. In terms of this issue, we have the following theorem.

Theorem 1 Problem 1 is reduced to the following deterministic optimization problem.

Problem 2

$$\min_{s,\lambda} s \text{ subject to } \begin{bmatrix} \bar{\Phi}(\lambda) & \bar{\Psi}(\lambda) \\ * & \bar{\eta}(\lambda) + s \end{bmatrix} > 0. \quad (14)$$

The definitions of $\bar{\Phi}$, $\bar{\Psi}$ and $\bar{\eta}$ are given in the proof.

Proof 1 First, we define

$$U(0) := \begin{bmatrix} u^T(0) & u^T(1) & \cdots & u^T(N_c - 1) \end{bmatrix}^T$$

$$W(0) := \begin{bmatrix} w^T(0) & w^T(1) & \cdots & w^T(N_c - 1) \end{bmatrix}^T$$

Then, as well known, $x(k)$, $k \in [0, N_c]$ is described by the form of

$$x(k) = \Phi_k U(0) + \Psi_k \begin{bmatrix} x(0) \\ W(0) \end{bmatrix} \quad (15)$$

where

$$\Phi_k := \begin{bmatrix} A^{k-1}B & \cdots & B & 0 & \cdots & 0 \end{bmatrix},$$

$$\Psi_k := \begin{bmatrix} A^k & A^{k-1}F & \cdots & F & 0 & \cdots & 0 \end{bmatrix}.$$

Substituting (15) to Problem 1 yields the following optimization problem

Problem 3

$$\min_{U(0)} \mathbf{E} f(U(0), W(0), x(0)) \quad (16a)$$

subject to

$$\mathbf{E} h_{i,k}(U(0), W(0), x(0)) \leq 0, \quad i = 1, \dots, m, \quad k = 0, \dots, N_c \quad (16b)$$

$$\mathbf{E} g_l(U(0), W(0), x(0)) \leq 0, \quad l = 0, \dots, N_M, \quad (16c)$$

$$N_M := N_c(N_D + 1) + N_F.$$

where

$$f(U(0), W(0), x(0)) = U^T(0)\Phi U(0)$$

$$+ U^T(0)\Psi \begin{bmatrix} x(0) \\ W(0) \end{bmatrix} + \begin{bmatrix} x(0) \\ W(0) \end{bmatrix}^T \Gamma \begin{bmatrix} x(0) \\ W(0) \end{bmatrix}$$

$$h_{i,k}(U(0), W(0), x(0)) = U^T(0)H_{i,k}^u U(0)$$

$$+ U^T(0)H_{i,k}^{ux} \begin{bmatrix} x(0) \\ W(0) \end{bmatrix} + \begin{bmatrix} x(0) \\ W(0) \end{bmatrix}^T H_{i,k}^x \begin{bmatrix} x(0) \\ W(0) \end{bmatrix}$$

$$- \epsilon_i \quad (17)$$

$$g_l(U(0), W(0), x(0)) = U^T(0)G_l^u U(0)$$

$$+ U^T(0)G_l^{ux} \begin{bmatrix} x(0) \\ W(0) \end{bmatrix} + \begin{bmatrix} x(0) \\ W(0) \end{bmatrix}^T G_l^x \begin{bmatrix} x(0) \\ W(0) \end{bmatrix} - \gamma, \quad (18)$$

$$l = 0, \dots, N_c$$

$$g_l(U(0), W(0), x(0)) =$$

$$A_{l-N_c}^x \begin{bmatrix} x(0) \\ W(0) \end{bmatrix} + A_{l-N_c}^u U(0) - 1, \quad l = N_c + 1, \dots, N_M$$

The constraints (17), (18) and (19) respectively correspond to the constraints (12b), (12c), and (12d), (12e) in Problem 1.

The matrices in the above equations are defined by

$$\Phi := \sum_{k=0}^{N_c-1} \Phi_k^T P \Phi_k + \Phi_{N_c}^T P_N \Phi_{N_c}$$

$$\Psi := \sum_{k=0}^{N_c-1} \Psi_k^T P \Psi_k + \Psi_{N_c}^T P_N \Psi_{N_c}$$

$$\Gamma := 2 \sum_{k=0}^{N_c-1} \Phi_k^T P \Psi_k + 2 \Phi_{N_c}^T P_N \Psi_{N_c}$$

$$H_{i,k}^u = \Phi_k^T Q_i \Phi_k, \quad H_{i,k}^{ux} = 2 \Psi_k^T Q_i \Phi_k, \quad H_{i,k}^x = \Psi_k^T Q_i \Psi_k,$$

$$i = 1, \dots, m, \quad k = 0, \dots, N_c,$$

$$G_l^u = \Phi_l^T Q \Phi_l, \quad G_{l,ux}^u = 2 \Psi_l^T Q \Phi_l, \quad G_l^x = \Psi_l^T Q \Psi_l,$$

$$l = 0, \dots, N_c,$$

$$A^u = \begin{bmatrix} M_D \Phi_0 \\ \vdots \\ M_D \Phi_{N_c-1} \\ M_F \Phi_{N_c} \end{bmatrix}, \quad A^x = \begin{bmatrix} M_D \Psi_0 \\ \vdots \\ M_D \Psi_{N_c-1} \\ M_F \Psi_{N_c} \end{bmatrix},$$

and A_l^x and A_l^u denotes l -th lows of A_x and A_u respectively.

Let us now define the Lagrange function

$$L(U(0), \lambda) = \mathbf{E} f + \sum_{i=1}^m \sum_{k=0}^{N_c} \lambda_{(N_c+1)(i-1)+k+1} \mathbf{E} h_{i,k}$$

$$+ \sum_{l=0}^{N_M} \lambda_{l+1+m(N_c+1)} \mathbf{E} g_l,$$

with the Lagrange multipliers $\lambda = (\lambda_1, \dots, \lambda_{m(N_c+1)+N_M+1})$. We assume that all the elements of λ are nonnegative. More specifically, the function is equivalent to

$$L(U(0), \lambda) = \mathbf{E} U^T(0) \bar{\Phi}(\lambda) U(0) + \mathbf{E} \bar{\Psi}(\lambda) U(0) + \mathbf{E} \eta(\lambda), \quad (19)$$

where

$$\bar{\Phi}(\lambda) = \Phi + \sum_{i=1}^m \sum_{k=0}^{N_c} \lambda_{(N_c+1)(i-1)+k+1} H_{i,k}^u + \sum_{l=0}^{N_c} \lambda_{l+1+m(N_c+1)} G_l^u,$$

$$\bar{\Psi}(\lambda) = \Psi + \sum_{i=1}^m \sum_{k=0}^{N_c} \lambda_{(N_c+1)(i-1)+k+1} \begin{bmatrix} x(0) \\ W(0) \end{bmatrix}^T H_{i,k}^{ux}$$

$$+ \sum_{l=0}^{N_c} \lambda_{l+1+m(N_c+1)} \begin{bmatrix} x(0) \\ W(0) \end{bmatrix}^T G_l^{ux} + \sum_{l=N_c+1}^{N_M} \lambda_{l+1+m(N_c+1)} A_{l-N_c}^u$$

$$\eta(\lambda) = \begin{bmatrix} x(0) \\ W(0) \end{bmatrix}^T \tilde{\Gamma}(\lambda) \begin{bmatrix} x(0) \\ W(0) \end{bmatrix} - \sum_{i=1}^m \sum_{k=0}^{N_c} \lambda_{(N_c+1)(i-1)+k+1} \epsilon_i$$

$$- \gamma \sum_{l=0}^{N_c} \lambda_{l+1+m(N_c+1)} + \sum_{l=N_c+1}^{N_M} \lambda_{l+1+m(N_c+1)} \left(A_{l-N_c}^x \begin{bmatrix} x(0) \\ W(0) \end{bmatrix} - 1 \right)$$

$$\tilde{\Gamma}(\lambda) = \Gamma + \sum_{i=1}^m \sum_{k=0}^{N_c} \lambda_{(N_c+1)(i-1)+k+1} H_{i,k}^x + \sum_{l=0}^{N_c} \lambda_{l+1+m(N_c+1)} G_l^x$$

Then, we also define $\theta(U(0)) = \sup_{\lambda \geq 0} L(U(0), \lambda)$ and $\omega(\lambda) = \inf_{U(0)} L(U(0), \lambda)$. Note that the problem of $\inf_{U(0)} \theta(U(0))$ is the same as Problem 3. The dual problem of the problem is represented by $\sup_{\lambda \geq 0} \omega(\lambda)$.

In the following, we attempt to solve the dual problem $\sup_{\lambda \geq 0} \omega(\lambda)$. For this purpose, we first consider $\omega(\lambda) = \inf_{U(0)} L(U(0), \lambda)$. From optimality conditions of $\inf_{U(0)} L(U(0), \lambda)$, for any optimal solution \tilde{U} , we get

$$2\bar{\Phi}(\lambda)\tilde{U} + \mathbf{E}\bar{\Psi}(\lambda) = 0,$$

namely $\tilde{U} = -\bar{\Phi}^{-1}(\lambda)\mathbf{E}\bar{\Psi}(\lambda)/2$ holds true. Notice that $\mathbf{E}\bar{\Psi}(\lambda)$ is given by

$$\begin{aligned} \mathbf{E}\bar{\Psi}(\lambda) &= \Psi \\ &+ \left(\sum_{i=1}^m \sum_{k=0}^{N_c} \lambda_{(N_c+1)(i-1)+k+1} H_{i,k}^{ux} + \sum_{l=0}^{N_c} \lambda_{l+1+m(N_c+1)} G_l^{ux} \right) \begin{bmatrix} y_0 \\ 0 \end{bmatrix}^T \\ &+ \sum_{l=N_c+1}^{N_M} \lambda_{l+1+m(N_c+1)} A_{l-N_c}^u \end{aligned} \quad (20)$$

For notational simplicity, we define $\bar{\Psi}(\lambda) := \mathbf{E}\bar{\Psi}(\lambda)/2$. Then, \tilde{U} is represented by $\tilde{U} = -\bar{\Phi}^{-1}(\lambda)\bar{\Psi}(\lambda)$. Hence, we have

$$\begin{aligned} \omega(\lambda) &= \inf_{U(0)} L(U(0), \lambda) = L(\tilde{U}, \lambda) \\ &= (\bar{\Phi}^{-1}(\lambda)\bar{\Psi}(\lambda))^T \bar{\Psi}(\lambda) - 2\bar{\Psi}(\lambda)\bar{\Phi}^{-1}(\lambda)\bar{\Psi}(\lambda) + \mathbf{E}\eta(\lambda) \\ &= -\bar{\Psi}(\lambda)\bar{\Phi}^{-1}(\lambda)\bar{\Psi}(\lambda) + \mathbf{E}\eta(\lambda). \end{aligned}$$

Notice that $\mathbf{E}\eta(\lambda)$ (denoted by $\bar{\eta}(\lambda)$) is given by

$$\begin{aligned} \bar{\eta}(\lambda) &= \mathbf{E} \begin{bmatrix} x(0) \\ W(0) \end{bmatrix}^T \bar{\Gamma}(\lambda) \begin{bmatrix} x(0) \\ W(0) \end{bmatrix} - \sum_{i=1}^m \sum_{k=0}^{N_c} \lambda_{(N_c+1)(i-1)+k+1} \epsilon_i \\ &- \gamma \sum_{l=0}^{N_c} \lambda_{l+1+m(N_c+1)} + \sum_{l=N_c+1}^{N_M} \lambda_{l+1+m(N_c+1)} \left(A_{l-N_c}^x \mathbf{E} \begin{bmatrix} x(0) \\ W(0) \end{bmatrix} - 1 \right) \\ &= \mathbf{Tr} \bar{\Gamma}(\lambda) \begin{bmatrix} y_0^T y_0 + 2R_{vv} & 0 \\ 0 & R_{ww} \end{bmatrix} - \sum_{i=1}^m \sum_{k=0}^{N_c} \lambda_{(N_c+1)(i-1)+k+1} \epsilon_i \\ &- \gamma \sum_{l=0}^{N_c} \lambda_{l+1+m(N_c+1)} + \sum_{l=N_c+1}^{N_M} \lambda_{l+1+m(N_c+1)} \left(A_{l-N_c}^x \mathbf{E} \begin{bmatrix} y_0 \\ 0 \end{bmatrix} - 1 \right) \end{aligned}$$

In summary, $\omega(\lambda)$ is given by

$$\omega(\lambda) = -\bar{\Psi}(\lambda)\bar{\Phi}^{-1}(\lambda)\bar{\Psi}(\lambda) + \bar{\eta}(\lambda),$$

which is deterministic. By letting s be an upper bound of $-\omega(\lambda)$ and using Schur Complement [27], the problem $\sup_{\lambda \geq 0} \omega(\lambda)$ is reduced to the problem (14). This completes the proof. \square

Note that since $\bar{\Phi}$ and $\bar{\Psi}$ are linear in terms of λ , Problem 2 is a linear matrix inequality (LMI) optimization problem, which is solvable by some existing solvers.

3.3 Additional Issue

In the previous subsection, we presented a state feedback predictive control law for systems with information structures. However, it is difficult to apply it to practical systems because the augmented systems usually include the disturbances as state variables. We thus need to present a state estimation scheme for systems with information structure. Here we employ a moving horizon estimator [28] with variance minimization for state estimation in order to get a state estimate $\hat{x}^i(k)$. In terms of this issue, if we employ the variance minimization for state estimation the objective function to be minimized is given by

$$\mathbf{E} \sum_{i=0}^{N_c} (y(t-i) - C\hat{x}(t-i))^T Q_e (y(t-i) - C\hat{x}(t-i)). \quad (21)$$

By using the same procedure as the previous subsection, the minimization problem of (21) under the communication delay

constraint (6) is also reduced to an LMI problem and it can be solved in a distributed fashion via dual decomposition techniques. Hence, we can implement an output feedback predictive control scheme at least according to the certainly equivalence principle. Though we guess that separation principle holds for the control and estimation in the absence of power and mean constraints, its theoretical investigations are one of future works of this paper. The separation principal in the presence of constraints does not hold and analysis on the integrated system is also left as a future work.

4. Application to Micro Grid and Numerical Verification

In this section, we reduce the example in Fig. 2 to the present framework and verify the effectiveness of the present scheme through simulation.

4.1 Dynamics and Information Structures

Let us define $z_i = P_i$ and $z_i^{ref} = P_i^{ref} - \Delta P_i$. Then, the dynamics of the total system is represented as (2).

For the network of Fig. 2, the available information $\mathcal{Z}_i(t)$ of i -th subsystem at time t for determining $u_i(t)$ is respectively given by

$$\begin{aligned} \mathcal{Z}_1(t) &= (\bar{z}_1(t), \bar{z}_2(t-1), \bar{z}_3(t-2)), \\ \mathcal{Z}_2(t) &= (\bar{z}_1(t-1), \bar{z}_2(t), \bar{z}_3(t-1)), \\ \mathcal{Z}_3(t) &= (\bar{z}_1(t-2), \bar{z}_2(t-1), \bar{z}_3(t)), \end{aligned} \quad (22)$$

where $\bar{z}_i(t) := (z_i(t), z_i(t-1), \dots, z_i(0))$, $i = 1, 2, 3$. Substituting (2a) into the variables in (22), equations in (22) are rewritten as

$$\begin{aligned} \mathcal{Z}_1(t) &= (\bar{z}(t-2), w_1(t-1), w_1(t-2), w_2(t-2)), \\ \mathcal{Z}_2(t) &= (\bar{z}(t-2), w_1(t-2), w_2(t-1), w_2(t-2), \\ &\quad , w_3(t-2)) \\ \mathcal{Z}_3(t) &= (\bar{z}(t-2), w_2(t-2), w_3(t-1), w_3(t-2)), \end{aligned} \quad (23)$$

where $\bar{z}(t) := (z(t), z(t-1), \dots, z(0))$ and $z(t) := [z_1(t) z_2(t) z_3(t)]^T$. The above equations mean that the covariance constraints (5) are formulated as

$$\begin{aligned} \mathbf{E} u_1(t)w_2(t-1) &= 0, & \mathbf{E} u_2(t)w_3(t-1) &= 0, \\ \mathbf{E} u_1(t)w_3(t-1) &= 0, & \mathbf{E} u_3(t)w_1(t-1) &= 0, \\ \mathbf{E} u_1(t)w_3(t-2) &= 0, & \mathbf{E} u_3(t)w_1(t-2) &= 0, \\ \mathbf{E} u_2(t)w_1(t-1) &= 0, & \mathbf{E} u_3(t)w_2(t-1) &= 0. \end{aligned}$$

Accordingly, the augmented state vector x is given by

$$x(t) := [z^T(t) w^T(t-1) w^T(t-2)]^T,$$

we have the augmented state equation in the same form as (7). Then, we obtain the covariance constraint (8) with

$$Q_r = \begin{bmatrix} \mathbf{0} & & 0 \\ & \hat{Q}_1 & \\ & & \hat{Q}_2 \\ 0 & \hat{Q}_1 & \hat{Q}_2 & 0 \end{bmatrix}, \quad (r = 1, \dots, 8),$$

$$r = 1; \quad \hat{Q}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \hat{Q}_2 = 0$$

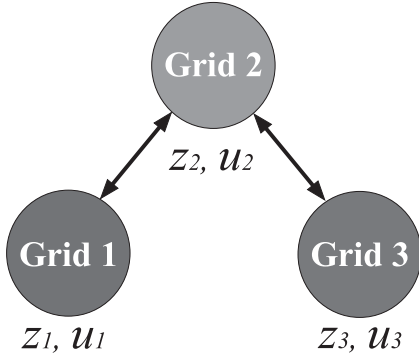


Fig. 3 Interconnections of micro grid subsystems.

$$r = 2; \quad \hat{Q}_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \hat{Q}_2 = 0$$

$$r = 3; \quad \hat{Q}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \hat{Q}_2 = 0$$

$$r = 4; \quad \hat{Q}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad \hat{Q}_2 = 0$$

$$r = 5; \quad \hat{Q}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \hat{Q}_2 = 0$$

$$r = 6; \quad \hat{Q}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad \hat{Q}_2 = 0$$

$$r = 7; \quad \hat{Q}_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \hat{Q}_1 = 0$$

$$r = 8; \quad \hat{Q}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \hat{Q}_1 = 0$$

$$\begin{bmatrix} x(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} z(t) \\ \frac{w(t-1)}{w(t-2)} \\ u(t) \end{bmatrix} \quad (24)$$

We next derive a model used for state estimation. Let us now denote by $\mathcal{Y}_i(t)$ available measurement information of subsystem Grid i at time t . Then, $\mathcal{Y}_i(t)$ is given by

$$\begin{aligned} \mathcal{Y}_1(t) &= (\bar{y}_1(t), \bar{y}_2(t-1), \bar{y}_3(t-2)), \\ \mathcal{Y}_2(t) &= (\bar{y}_1(t-1), \bar{y}_2(t), \bar{y}_3(t-1)), \\ \mathcal{Y}_3(t) &= (\bar{y}_1(t-2), \bar{y}_2(t-1), \bar{y}_3(t)), \end{aligned} \quad (25)$$

where $\bar{y}_i(t) := (y_i(t), y_i(t-1), \dots, y_i(0))$. Equation (25) is replaced by

$$\begin{aligned} \mathcal{Y}_1(t) &= (\bar{y}(t-2), y_1(t), y_1(t-1), y_2(t-1)), \\ \mathcal{Y}_2(t) &= (\bar{y}(t-1), y_2(t)), \\ \mathcal{Y}_3(t) &= (\bar{y}(t-2), y_2(t-1), y_3(t), y_3(t-1)), \end{aligned} \quad (26)$$

where $\bar{y}(t) := (y(t), y(t-1), \dots, y(0))$ and $y(t) := (y_1(t), y_2(t), y_3(t))$. This means that, for example, Grid 1 has the following covariance constraints.

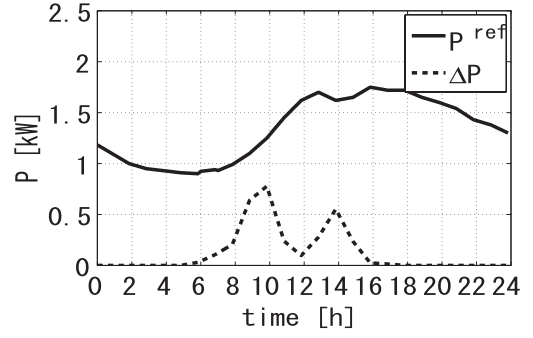
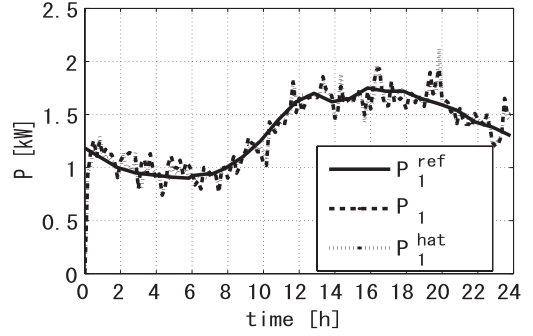


Fig. 4 References and photo-voltaic power generation.

Fig. 5 Time response of z_1 .

$$\mathbf{E} y_1(t) v_2^T(t) = 0,$$

$$\mathbf{E} y_1(t) v_3^T(t) = 0,$$

$$\mathbf{E} y_1(t-1) v_3^T(t-1) = 0.$$

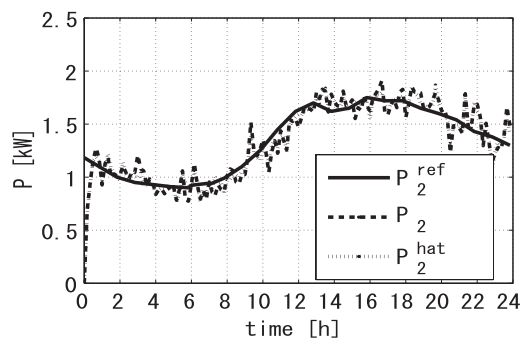
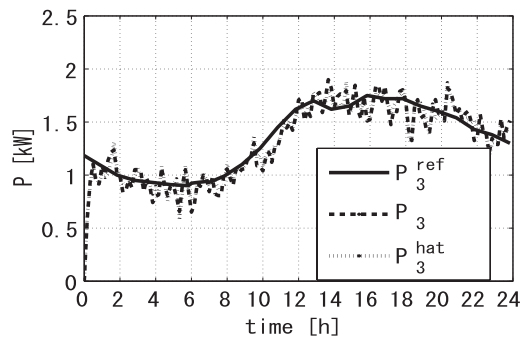
In summary, the micro grid system has been reduced to the system in Section 2. Accordingly, we formulate Problem 1 and hence the convex optimization problem Problem 3. It is thus possible to implement the present predictive control scheme for the micro grid system.

4.2 Verification

In this section, we demonstrate the effectiveness of the present control law through the system in Fig. 3 with $a_i = 0.3 \forall i \in \{1, 2, 3\}$, $f_i = 1 \forall i \in \{1, 2, 3\}$ and $a_{ij} = 0.3 \forall i, j \in \{1, 2, 3\}$. Let the parameters be $Q = I$, $P = I$, $P_{N_c} = 0$, $N_c = 2$ and $\gamma = 1$. Note that we do not impose the linear constraints (12d) nor (12e). The sampling time is 10 [minute].

In order to compute the power by photo-voltaic generators, the data of daylight at Tokyo on 30th August, 2008 is employed [25] (see Fig. 4). We also let the reference power P_i^{ref} be the power consumption pattern [26] and all P_i^{ref} are common in this case.

The simulation results of the present predictive control scheme are shown in Figs. 5–7, where we let the initial values of P_i be 0 for all $i \in \{1, 2, 3\}$. In the simulation, we set $R_{ww} = R_{vv} = 0.01$ and random signals generated according to the probability measure are added to the system. The solid curves illustrate the references P_i^{ref} , the dash-dotted curves the responses of the actual total power $P_i(t)$, and the dotted curves the estimates of $P_i(t)$ produced by the moving horizon estimator. We see from these figures that the states are successfully estimated and our predictive control scheme achieves a good tracking of $z_i(t)$ to the reference $z_i^{ref}(t)$ in spite of the effects of disturbances. This simulation demonstrates the effectiveness of the present control scheme.

Fig. 6 Time response of z_2 .Fig. 7 Time response of z_3 .

5. Conclusion

In this paper, we have proposed a predictive control scheme for systems with information structures motivated by control of micro grid systems. In order to formulate the constrained finite-time optimal control problem solved online in the predictive control scheme, we have used the modeling method of [5], where the information structures are described by covariance constraints. We then have proved that the constrained finite-time optimal control problem is reduced to a deterministic convex programming problem which means that the optimal control problem can be solved efficiently. Furthermore an estimation scheme for systems with information structure has been presented. Finally the effectiveness of the proposed control law and the estimator have been demonstrated through a numerical simulation of a simplified micro grid.

It was shown in this paper that predictive control reflects a lot of unique characteristics of energy networks including micro grid systems. The authors believe that this work can be used as a starting point in dealing with this kind of control problem. However, a lot of problems are left as future works of this research. A direction is to include batteries into the energy network in order to buffer the effects of uncertainty in renewable energy generation and to clarify the value of the device. Another direction is to extend the result to distributed control. To deal with such an issue, it is required to use highly matured techniques in cooperative control and network games by viewing components of the network or grids as agents.

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Masayuki FUJITA (Member)



He received the Dr. of Eng. degree in Electrical Engineering from Waseda University, Tokyo, in 1987. He is presently a Professor with the Department of Mechanical and Control Engineering at the Tokyo Institute of Technology. Prior to his appointment at the Tokyo Institute of Technology, he held faculty appointments at Kanazawa University and Japan Advanced Institute of Science and Technology. He also held a visiting position at the Technical University of Munich, Germany. His research interests include passivity-based visual feedback, cooperative control, and robust control with its industrial applications. He has served as the General Chair of the 2010 IEEE Multi-conference on Systems and Control. He is currently a Vice President of IEEE Control Systems Society (CSS) and a member of the IEEE CSS Board of Governors, and was a Director of the Society of Instrument and Control Engineers (SICE). He has served/been serving as an Associate Editor for the IEEE Transactions on Automatic Control, the IEEE Transactions on Control Systems Technology, Automatica, Asian Journal of Control, and an Editor for the SICE Journal of Control, Measurement, and System Integration. He is a recipient of the 2008 IEEE Transactions on Control Systems Technology Outstanding Paper Award. He also received the Outstanding Paper Awards from the SICE and the Institute of Systems, Control and Information Engineers (ISCIE) in Japan. Recently he has received the 2010 SICE Education Award.

Toru NAMERIKAWA (Member)



He received the B.E., M.E., and Dr. of Engineering degrees in electrical and computer engineering from Kanazawa University, Japan, in 1991, 1993 and 1997, respectively. From 1994 until 2002 he was with Kanazawa University as an Assistant Professor. From 2002 until 2005, he was with the Nagaoka University of Technology as an Associate Professor. From 2006 until 2009, he was with Kanazawa University again. In April 2009, he joined Keio University, where he is currently an Associate Professor at the Department of System Design Engineering. His main research interests are robust control, nonlinear control, cooperative control theories and their application to power network systems and robotic systems. He is a member of ISCIE and IEEE.

Takeshi HATANAKA (Member)



He received the B.Eng. degree in informatics and mathematical science, the M.Inf. and Ph.D. degrees in applied mathematics and physics all from Kyoto University, Japan in 2002, 2004 and 2007, respectively. From 2006 to 2007, he was a research fellow of the Japan Society for the Promotion of Science at Kyoto University. He is currently an Assistant Professor in the Department of Mechanical and Control Engineering, Tokyo Institute of Technology, Japan. He received the Best Paper Award from the Society of Instrument and Control Engineers (SICE) in 2009. His research interests include cooperative control, vision based control and estimation, and predictive control.