SICE Journal of Control, Measurement, and System Integration, Vol. 4, No. 2, pp. 089–096, March 2011

Multiple Cooperative Bilateral Teleoperation with Time-Varying Delay

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Abstract : This paper deals with a passive-decomposition based control of bilateral teleoperation between a single master robot and multiple cooperative slave robots with time varying delay. First, we decompose the dynamics of multiple slave robots into two decoupled dynamics by using the passive-decomposition: the shape-system describing dynamics of the cooperative works and the locked-system representing the overall behavior of the multiple slave robots. Second, we propose a PD control method for bilateral teleoperation to guarantee asymptotic stability of the system with time varying delay. Finally, experimental results show the effectiveness of the proposed teleoperation.

Key Words : bilateral teleoperation, SMMS, time-varying delay, Lyapunov stability.

1. Introduction

Teleoperation systems allow persons to extend their sense and manipulation capabilities to remote places. In general, a slave robot is controlled to do some real tasks at the remote place by controll signals that are sent from a master side. Communication channels are composed to connect the robots and the remote environment. In bilateral control, contact information is fed back to the master side when the salve robot interacts with the remote environment, therefore the manipulation capability can be improved [1]. One absolutely unsolved problem of the control of teleoperation systems is time delay in the communication line. In some cases, the master and the slave are coupled via a communication network (e.g Internet), the time delay is incurred in the transmission of data between the master and the slave sides. The delay may destabilize and deteriorate the transparency of the teleoperation system. Therefore, it is necessary to design a control law to guarantee stability of the system under communication delays. The time delay is not only constant but also variable in many cases.

Up to now, many successful control schemes have been proposed for the teleoperation system with single master and single slave (SMSS). However, studies on teleoperation systems with multi robots are relatively rare. In [2]–[5] some control methods were proposed for the system with multiple master and multiple slave (MMMS). In this system, one human can control one slave robot to perform a separated operation in a cooperative task, thus the system may demand a large number of human operators if the task requires many slave robots. In [6]–[9] the single master and multiple slaves (SMMS) systems were considered, but the control methods were proposed only for the motion coordination. Both MMMS and SMMS systems are applied for the tasks which need the cooperation of many slave robots, such as lifting heavy objects, assembly works, etc.

In the SMMS systems, there are one master robot and two or more slave robots. One human has to operate all slave robots at the same time by using only one master robot in a cooperative task. The control scheme for this system is not easy especially in the case of movement and contact force of each slave robot are of variety. A control algorithm of only one master robot is required corresponding with the number of the slave robots. To solve above difficulties, a method based on the passive-decomposition is proposed as a technique for making two or more slave robots cooperate in the SMMS system [4]. In this work, utilizing the passive-decomposition, the dynamics of the two or more slave robots is decomposed into decoupled systems while enforcing passivity. There are two concepts: the shape-system instructs the dynamics of the cooperative work; the locked-system abstracts the overall dynamics of the multi-slave robots. To passivate the master-slave communication delay, the scattering-based communication is utilized [10]. However in the work, neither alignment error between each slave robot position nor force reflection of them is guaranteed. On the other hand, in [11],[12] PD control was used without the scattering conversion and the controller gains depend on the maximum round-trip delay, where stability is guaranteed with the communication delay.

In this paper, one control law is proposed, that is based on the technique of [10],[11] for the SMMS system with time varying delay in the communication line. This proposed control guarantees asymptotic stability. While in [10], scattering conversion uses the PD control law with constant time delay in the communication lines, by the proposed control law for the time varying communication delay, we also achieve stability. In our proposed control law, we use an individual gain for the different structures of the master and the slaves. In the independent design, a scaling power can be set at both sides of teleoperation. In addition, the teleoperation achieves asymptotic stability for any time varying communication delay, i.e., the master and slave spacing errors achieve zero, and the static reflection force is transferred when an object grasped by multiple slaves contacts with the remote environment in this control law. In experiments, two slave robots hold and carry the object to a desired position, and experimental results show the effectiveness

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(Received November 6, 2009)
(Revised July 13, 2010)

JCMSI 0002/11/0402–0089 © 2009 SICE

of our proposed control technique.

The paper is organized as follows. Problem formulation is given in Section 2. In Section 3, the passive-decomposition is presented and proposed control laws are designed. The stability analysis of the shape and locked systems are presented in Section 4. Experimental results are presented in Section 5. Section 6 contains some concluding remarks.

2. Problem Formulations

2.1 Dynamics of Teleoperation System

In this section, we show the dynamics of the SMMS system that is composed of one master and N slave robots can be expressed by a motion equation of a general robot arm. The dynamics of the master with m-DOF and the dynamics of the i slave with n_i-DOF are shown as follows:

$$\begin{cases} M_m(q_m)\ddot{q}_m + C_m(q_m,\dot{q}_m)\dot{q}_m = \tau_m + J_m^T(q_m)F_{op} \\ M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i = \tau_i + J_i^T(q_i)F_i \end{cases} \quad (1)$$

where the subscript "m" denotes the master and the subscript "i" denotes the order indexes of the slave, $q_m \in R^{m\times 1}$, $q_i \in R^{n_i\times 1}$ are the joint angle vectors, $\tau_m \in R^{m\times 1}$, $\tau_i \in R^{n_i\times 1}$ are the input torque vectors, $F_{op} \in R^{m\times 1}$ is the operational force vector, $F_i \in R^{n_i\times 1}$ are the environmental force vectors, $M_m \in R^{m\times m}$, $M_i \in R^{n_i\times n_i}$ are the symmetric and positive definite inertia matrices, $C_m(q_m,\dot{q}_m)\dot{q}_m \in R^m$, $C_i(q_i,\dot{q}_i)\dot{q}_i \in R^{n_i}$ are the centripetal and Coriolis torque vectors, $J_m(q_m) \in R^{m\times m}$, $J_i(q_i) \in R^{n_i\times n_i}$ are Jacobian matrices. However, degrees of freedom of the slave are assumed to be larger than ones of the master ($n_i \geq m$). The Jacobian matrices satisfy the assumption below:

Assumption 1. The J_m and J_i are nonsingular matrices at all times in operation.

In this paper, we propose a control law for different structural teleoperation. This control law of the system may be not possible with some parameters in the joint space, therefore it is useful to rewrite the master and slave robot dynamics directly in the task space. The end-effector velocities $\dot{x}_m \in R^{m\times 1}$ and $\dot{x}_i \in R^{n_i\times 1}$ in the task space relate to the joint velocity $\dot{q}_m$, $\dot{q}_i$ as follows:

$$\dot{x}_k(t) = J_k(q_k)\dot{q}_k(t), \ k = m, \ i. \quad (2)$$

by further differentiation of (2) as:

$$\ddot{x}_k(t) = J_k(q_k)\ddot{q}_k(t) + \dot{J}_k(q_k)\dot{q}_k^2(t), \ k = m, \ i. \quad (3)$$

where $\ddot{x}_m \in R^{m\times 1}$ and $\ddot{x}_i \in R^{n_i\times 1}$ are the end-effector acceleration vectors. Substituting (2) and 3 into (1), we can receive the master and multiple slave robots dynamics in the task space as follows:

$$\widetilde{M}_m(q_m)\ddot{x}_m + \widetilde{C}_m(q_m,\dot{q}_m)\dot{x}_m = J_m^{-T}\tau_m + F_{op} \quad (4)$$

$$\widetilde{M}_i(q_i)\ddot{x}_i + \widetilde{C}_i(q_i,\dot{q}_i)\dot{x}_i = J_i^{-T}\tau_i + F_i \quad (5)$$

where:

$$\widetilde{M}_k = J_k^{-T}M_kJ_k^{-1}, \ \widetilde{C}_k = J_k^{-T}\{C_k - M_kJ_k^{-1}\dot{J}_k\}J_k^{-1}, \quad (k = m, \ i)$$

x_i is end-effector of each slave robot in Cartesian coordinate system of multiple slaves. Let us denote the total degree of freedom of the N slave robots by: $n = \sum_i^N n_i$, hence the group dynamics of the N slave robots can be rewritten as follows:

$$\widetilde{M}(q)\ddot{x} + \widetilde{C}(q,\dot{q})\dot{x} = \tau + F \quad (6)$$

where $x = [x_1^T,\ldots,x_N^T]^T \in R^n$, $\tau = [\tau_1^TJ_1^{-T},\ldots,\tau_N^TJ_N^{-T}]^T \in R^n$, $F = [F_1^T,\ldots,F_N^T]^T \in R^n$, and $\widetilde{M}(q) = diag[\widetilde{M}_1(q_1),\ldots,\widetilde{M}_N(q_N)] \in R^{n\times n}$, $\widetilde{C}(q,\dot{q}) = diag[\widetilde{C}_1(q_1,\dot{q}_1),\ldots,\widetilde{C}_N(q_N,\dot{q}_N)] \in R^{n\times n}$ are the inertia matrices and Coriolis matrices, respectively. It is well known that the dynamics (4) and (5) have several fundamental properties under the Assumption 1 as follows:

***Property* 1.** The inertia matrices $\widetilde{M}_k(q_k)$ $(k = m, \ i)$ are symmetric and positive definite and there exist some positive constants m_{k1}, m_{k2}, c_k in [13] such as:

$$0 < m_{k1} \leq \| \widetilde{M}_k \| \leq m_{k2}$$
$$\| \widetilde{C}_k \| \leq c_k \| \dot{x}_k \| \quad (7)$$

***Property* 2.** Consider an appropriate definition of the matrices $\widetilde{C}_k(q_k,\dot{q}_k)$, the matrices $\widetilde{N}_k = \dot{\widetilde{M}}_k(q_k) - 2\widetilde{C}_k(q_k,\dot{q}_k)$ are skew symmetric as in [13] such that:

$$z^T\widetilde{N}_kz = 0 \ (k = m, \ i) \quad (8)$$

where $z \in R^{n\times 1}$ is any vector.

***Property* 3.** $\dot{x}_k, \ddot{x}_k$ $(k = m, i)$ are bounded and $\dot{\widetilde{M}}_k$, $\dot{\widetilde{C}}_k$ are also bounded [14]

Communication delay is assumed as follows:

Assumption 2. Both time varying delay $T_m(t)$ and $T_s(t)$ are continuously differentiable functions and possibly bounded as:

$$0 \leq T_h(t) \leq T_h^+ < \infty, \ |\dot{T}_h(t)| < 1, \ h = m, s \quad (9)$$

where $T_h^+ \in R$ are upper bounds of the communication delays. Moreover, the upper bound of the round trip communication delay $T_{ms}^+ = T_m^+ + T_s^+$ is known preliminarily.

Assumption 3. The delays among all slave robots are very small and they can be disregarded.

2.2 Control Objectives

In this paper, the SMMS system is shown in Fig. 1 with one master and two slave robots. The cooperative slave robot is similar to a dual-arm robot. The object is grasped to transport to a specified place according to the instruction values of a controller from the operator in the task space.

Control Objective 1. (Autonomous Grasping by Multiple Slave Robots) In this work, the achievement of grasping: "a relative position of the end-effectors of the slave robots is shaped in a certain specified form" means that the following condition is accomplished:

$$x_S = x_S^d \quad (10)$$

where $x_S \in R^{n-m}$ is the relative position of the end-effector of the slaves, $x_S^d \in R^{n-m}$ is a desired position of x_S.

Control Objective 2. (Movement of Grasped Object) When the grasping is achieved, the center position between the end-effector of the slave robots is same with the center position of the grasped object, then the movement of the grasped object is achieved as:

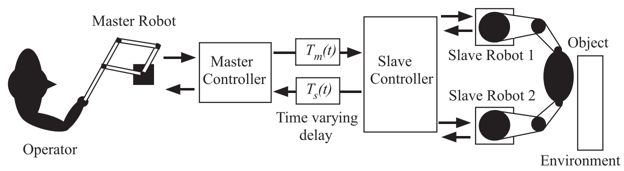


Fig. 1 SMMS teleoperation system.

$$x_L = x_m \tag{11}$$

where $x_L = \alpha x_{L0} - C$, $x_{L0} \in R^m$ and x_m are the center position of the end-effectors and the grasped object, respectively; $\alpha \in R$ is the position scale, $C \in R^m$ is shown a translation value.

Control Objective 3. (Static Force Reflection) The teleoperation with static Force Reflection is achieved as $\dot{x}_j = \ddot{x}_j = 0$ ($j = m, L$) such that:

$$F_{op} = \beta F_L \tag{12}$$

where F_L is the contact force of cooperative-slave, $\beta > 0 \in R$ is a positive scalar and it expresses a force scaling effect.

3. Control Design

In this section, to achieve above Control objectives, we propose a control law for the SMMS system.

3.1 Passive-Decomposition

First, based on the passive-decomposition, the dynamic of multiple slave robots is decomposed into two decoupled systems: the shape-system describing "movement of the multiple slaves with grasping object" and the locked-system describing "movement of the multiple slaves according to the instruction from the master". Utilizing the passive-decomposition, the velocity of multiple slave robots is rewritten with each system as follows:

$$\dot{x} = S^{-1}\begin{bmatrix}\dot{x}_S \\ \dot{x}_L\end{bmatrix} \tag{13}$$

where $\dot{x}_S \in R^{n-m}$ and $\dot{x}_L \in R^m$ are velocities of the shape-system and the locked-system, respectively. S is the non-singular decomposition matrix. The matrix S is also a positive matrix of a decoupling shape and locked system. In the following formula of $S^T\widetilde{M}S^{-1}$, the non-diagonal terms are removed as:

$$S^T\widetilde{M}S^{-1} = \begin{bmatrix}M_S & 0 \\ 0 & M_L\end{bmatrix} \tag{14}$$

where $M_S \in R^{(n-m)\times(n-m)}$, $M_L \in R^{m\times m}$ are inertia matrices of the shape-system and the locked-system, respectively. In the fact that, $\dot{x}_s$ and $\dot{x}_L$ are defined for satisfying (14). In addition, a local compensation of impedance shaping is necessary. The reflection forces from environment relate with the control input of slave dynamics of the shape-system and the locked-system as follows:

$$\begin{bmatrix}F_S \\ F_L\end{bmatrix} = S^{-T}F, \quad \begin{bmatrix}\tau_S \\ \tau_L\end{bmatrix} = S^{-T}\tau \tag{15}$$

from above definitions, we define:

$$\begin{bmatrix}C_S & C_{SL} \\ C_{LS} & C_L\end{bmatrix} = S^{-T}\widetilde{M}\frac{d}{dt}(S^{-1}) + S^{-T}\widetilde{C}S^{-1} \tag{16}$$

note (6), the passive-decomposition form is written as:

$$M_S(q)\ddot{x}_S + C_S(q,\dot{q})\dot{x}_S + C_{SL}(q,\dot{q})\dot{x}_L = \tau_S + F_S \tag{17}$$
$$M_L(q)\ddot{x}_L + C_L(q,\dot{q})\dot{x}_L + C_{LS}(q,\dot{q})\dot{x}_S = \tau_L + F_L \tag{18}$$

where the subscript "S" denotes the shape-system and the subscript "L" denotes the locked-system. Above dynamic equations include friction terms $C_{SL}(q,\dot{q})\dot{x}_L$ and $C_{LS}(q,\dot{q})\dot{x}_S$, however, ignore the remote control by the human, decoupling of the shape-system and the locked-system is desired for the slave that maybe autonomous grasping. Therefore, the decoupling control inputs are given:

$$\tau_S = C_{SL}(q,\dot{q})\dot{x}_L + \tau_S' \tag{19}$$
$$\tau_L = C_{LS}(q,\dot{q})\dot{x}_S + \tau_L' \tag{20}$$

where τ_S', τ_L' are new control inputs. Substituting (19), (20) into (17), (18), we get:

$$M_S(q)\ddot{x}_S + C_S(q,\dot{q})\dot{x}_S = \tau_S' + F_S \tag{21}$$
$$M_L(q)\ddot{x}_L + C_L(q,\dot{q})\dot{x}_L = \tau_L' + F_L \tag{22}$$

hence, two above dynamics are decoupled.

Proposition 1. The dynamics (21), (22) are similar to the normal dynamics which relate to the Properties 1-3, thus some properties of this SMMS system are given as follows:

***Property* 4.** $M_i(q)$ $(i = S, L)$ is a positive symmetric matrix, and there exist some constant parameters with the following relationship as :

$$\begin{aligned} &0 < m_{i1} \le \| M_i \| \le m_{i2} \\ &\qquad \| C_i \| \le c_i \| \dot{x}_i \| \end{aligned} \tag{23}$$

***Property* 5.** $\dot{M}_i(q) - 2C_i(q,\dot{q})$ $(i = S, L)$ is a skew-symmetric matrix.

***Property* 6.** $\dot{x}_i, \ddot{x}_i$ $(i = S, L)$ are bounded and $\dot{M}_i$, $\dot{C}_i$ are also bounded.

Proof. In the Properties 4 and 6, M_i, C_i $(i = S, L)$ are defined by (14) and (16), respectively, we can see from the Properties 1, 3 and the definition of S.

From the Property 5, we can get:

$$\begin{aligned} &\begin{bmatrix}\dot{M}_S - 2C_S & -2C_{SL} \\ -2C_{LS} & \dot{M}_L - 2C_L\end{bmatrix} \\ &= \frac{d}{dt}(S^{-T}\widetilde{M}S^{-1}) - 2S^{-T}\widetilde{M}\frac{d}{dt}(S^{-1}) - 2S^{-T}\widetilde{C}S^{-1} \end{aligned} \tag{24}$$

Using above skew-symmetric property of $\dot{\widetilde{M}} - 2\widetilde{C}$ and the symmetric property of $\widetilde{M}$, we can conclude that three terms at the right side of (24) are the skew-symmetric matrices, thus $\dot{M}_S - 2C_S$ and $\dot{M}_L - 2C_L$ should be skew-symmetric matrices and equivalence. We also obtain:

$$\begin{bmatrix}\dot{M}_S - 2C_S & -2C_{SL} \\ -2C_{LS} & \dot{M}_L - 2C_L\end{bmatrix} = -\begin{bmatrix}\dot{M}_S - 2C_S & -2C_{SL} \\ -2C_{LS} & \dot{M}_L - 2C_L\end{bmatrix}^T \tag{25}$$

where $C_{SL} = -C_{LS}^T$. The proof of Proposition 1 is completed. □

Properties 4 ~ 6 denote the feature of motion equation of normal robots, otherwise, we can applied them for the control law of abundance robots.

The following assumptions are from (1), (21), (22) and used in next stability analysis section.

Assumption 4. All signals belong to $\mathcal{L}_2$ space. The velocities $\dot{x}_m$, $\dot{x}_L$ equal zero for $t < 0$.

Assumption 5. The operator and the environment can be modeled as passive systems, where the velocities $\dot{x}_m$, $\dot{x}_L$ are system inputs, the force F_{op}, F_L are system outputs, respectively. Moreover, these forces are bounded by the functions of the velocities of the master and the locked-system.

3.2 Proposed Control Law

Concerning the control law of the shape-system (21), the Control objective of this system is: $x_S = x_S^d$, then the position tracking with this control law is shown as follows:

$$\begin{aligned}\tau_S^{'} =& M_S\{\ddot{x}_S^d(t) - K_d^S(\dot{x}_S - \dot{x}_S^d(t)) - K_P^S(x_S - x_S^d(t))\}\\ &+ C_S\dot{x}_S - F_S \end{aligned} \tag{26}$$

Substituting (26) into (21) we obtain the following closed-loop systems:

$$\begin{aligned}&\ddot{e} + K_d^S\dot{e} + K_P^S e = 0,\\ &e = x_S - x_S^d\end{aligned} \tag{27}$$

where K_d^S, K_P^S are positive definite diagonal gain matrices.

Remark 1. In the control law of this shape-system, information of grasped object and two slave robots are necessary. Otherwise, this proposed control of object grasping is only a position control, no force control in this case, although the force F_S is available in the torque input of the system. Therefore, the grasping object is assumed to be soft enough to be kept by two slave robots, then the Control objective 1 is also achieved.

Considering the coupling control of the locked-system and the master. Note the Control objective: $x_L = x_m$, the control law is defined as:

$$\tau_L^{'} = -K_d^L\dot{x}_L - K_P^L(x_L - x_m(t - T_m(t))) \tag{28}$$

$$\tau_m = J_m^T\{-K_d^m\dot{x}_m - K_P^m(x_m - x_L(t - T_s(t)))\} \tag{29}$$

Substituting above control law into the locked-system (22) and dynamic equation of the master (4), we obtain a closed-loop system as follows:

$$\begin{aligned}&M_L(q)\ddot{x}_L + C_L(q,\dot{q})\dot{x}_L\\ &\quad = -K_d^L\dot{x}_L - K_P^L(x_L - x_m(t - T_m(t))) + F_L\end{aligned} \tag{30}$$

$$\begin{aligned}&\widetilde{M}_m(q_m)\ddot{x}_m + \widetilde{C}_m(q_m,\dot{q}_m)\dot{x}_m\\ &\quad = -K_d^m\dot{x}_m - K_P^m(x_m - x_L(t - T_s(t))) + F_{op}\end{aligned} \tag{31}$$

where K_P^j, K_d^j $(j = m, L)$ are gains and defined as follows:

$$\begin{cases} K_P^m = k_m K_P\\ K_P^L = k_L K_P\end{cases},\ \begin{cases} K_d^m = k_m K_d\\ K_d^L = k_L K_d\end{cases} \tag{32}$$

where $K_P \in R^{n\times n}$, $K_d \in R^{n\times n}$ are positive definite diagonal control gains; $k_m > 0$, $k_L > 0$ are constant gains of scalar that designed separately on the master and the slave side.

4. Stability Analysis

4.1 Stability of Shape-System

The below theorem concerns the shape-system.

Theorem 1. Consider the closed-loop shape-system (27) and Assumption 3, desired value of relative position of spaces between the slave robots is conversed as follows:

$$e = x_S - x_S^d \to 0 \ \text{ as t} \to \infty \tag{33}$$

Proof. The equation (27) can be rewritten as follows:

$$\begin{bmatrix}\dot{e}\\ \ddot{e}\end{bmatrix} = \phi\begin{bmatrix}e\\ \dot{e}\end{bmatrix},\ \ \phi = \begin{bmatrix}0 & I\\ -K_P^S & -K_d^S\end{bmatrix} \tag{34}$$

where K_P^S, K_d^S are positive diagonal matrices, eigenvalues of ϕ are negative, therefore following errors of position and velocity are achieved:

$$e = x_S - x_S^d \to 0 \ \text{ as } \ \text{t} \to \infty \tag{35}$$

$$\dot{e} = \dot{x}_S - \dot{x}_S^d \to 0 \ \text{ as } \ \text{t} \to \infty \tag{36}$$

it means the Control Objective 1 is achieved and the autonomous grasping of multiple slaves is also achieved. □

4.2 Stability of Locked-System

The below theorem concerns the dynamics (30), (31).

Theorem 2. Consider the systems described by (30) and (31). Then under Assumptions 1~5, the position tracking error given by x_e is bounded; the velocities of the master and the cooperative slaves $\dot{x}_m$, $\dot{x}_L$, respectively, are asymptotically converged to origin. In a sense of matrix inequality, the control gain matrices K_P, K_d are chosen to satisfy below condition:

$$K_P < \frac{2}{T_{ms}^{+}}K_d \tag{37}$$

then the system is asymptotically stabilized.

Proof. The state vector of the locked-system is proposed as:

$$x(t) = [\dot{x}_m^T, \dot{x}_L^T, x_e^T]^T$$

where $x_e \in R^{m\times 1}$ is the position error of master and the locked-system: $x_e = x_m - x_L$. We define a Lyapunov function candidate for the system as follows:

$$\begin{aligned}V(x(t)) =& k_m^{-1}\dot{x}_m^T(t)\widetilde{M}_m\dot{x}_m(t) + k_L^{-1}\dot{x}_L^T(t)M_L\dot{x}_L(t)\\ &+ x_e^T(t)K_P x_e(t) - 2k_L^{-1}\int_0^t F_L^T(\xi)\dot{x}_L(\xi)d\xi\\ &- 2k_m^{-1}\int_0^t F_{op}^T(\xi)\dot{x}_m(\xi)d\xi\end{aligned} \tag{38}$$

where M_m, M_L, K_P are positive definite matrices, $k_m, k_L > 0$. Following the Assumption 5, the environment and the manipulator are passive, then $V(x(t))$ is the positive function. The derivative of above Lyapunov function along trajectories of the system (30), (31) with concerning Properties 2 and 5 as:

$$\begin{aligned}\dot{V} =& -2\dot{x}_m^T K_d\dot{x}_m + 2\dot{x}_m^T K_P(x_L(t - T_s(t)) - x_L)\\ &- 2\dot{x}_L^T K_d\dot{x}_L + 2\dot{x}_L^T K_P(x_m(t - T_m(t)) - x_m)\end{aligned} \tag{39}$$

applying Leibniz-Newton formula:

$$x_i(t - T_h(t)) - x_i = -\int_0^{T_h(t)} \dot{x}_h(t-\xi)d\xi, \quad (h = m, s) \tag{40}$$

substituting (40) into (39), we get:

$$\begin{aligned}\dot{V} = &- 2\dot{x}_m^T K_d \dot{x}_m - 2\dot{x}_m^T K_P \int_0^{T_s(t)} \dot{x}_L(t-\xi)d\xi \\ &- 2\dot{x}_L^T K_d \dot{x}_L - 2\dot{x}_L^T K_P \int_0^{T_m(t)} \dot{x}_m(t-\xi)d\xi\end{aligned} \tag{41}$$

The second term at the right side of (41) is transformed as follows:

$$\begin{aligned}&-2\dot{x}_m^T K_P \int_0^{T_s(t)} \dot{x}_L(t-\xi)d\xi \\ &= -\sum_{j=1}^{n} K_{Pj} 2\dot{x}_{mj} \int_0^{T_s(t)} \dot{x}_{Lj}(t-\xi)d\xi\end{aligned} \tag{42}$$

where $\dot{x}_{mj}$, $\dot{x}_{Lj}$, K_{Pj} are velocities of the master and slave (following the j axis) and positional control gains, respectively. In (42), applying Young and Schawartz inequality for the term in the right side, then note the inequality $T_s \le T_s^+$, we get:

$$\begin{aligned}&-2\dot{x}_{mj} \int_0^{T_s(t)} \dot{x}_{Lj}(t-\xi)d\xi \\ &\le T_s^+ \dot{x}_{mj}^2 + \frac{1}{T_s^+}\Big\{T_s \int_0^{T_s(t)} \dot{x}_{Lj}^2(t-\xi)d\xi\Big\} \\ &\le T_s^+ \dot{x}_{mj}^2 + \int_0^{T_s^+} \dot{x}_{Lj}^2(t-\xi)d\xi\end{aligned} \tag{43}$$

Therefore, (42) is rewritten as follows:

$$\begin{aligned}&-2\dot{x}_m^T K_P \int_0^{T_s(t)} \dot{x}_L(t-\xi)d\xi \\ &\le \sum_{j=1}^{n} K_{Pj}\Big\{T_s^+ \dot{x}_{mj}^2 + \int_0^{T_s^+} \dot{x}_{Lj}^2(t-\xi)d\xi\Big\} \\ &= T_s^+ \dot{x}_m^T K_P \dot{x}_m + \int_0^{T_s^+} \dot{x}_L^T(t-\xi) K_P \dot{x}_L(t-\xi)d\xi\end{aligned} \tag{44}$$

Similar to (42), the fourth term in the right side can also be rewritten. We receive below inequality from (41) as:

$$\begin{aligned}\dot{V} \le &- 2\dot{x}_m^T K_d \dot{x}_m - 2\dot{x}_L K_d \dot{x}_L \\ &+ T_s^+ \dot{x}_m^T K_P \dot{x}_m + \int_0^{T_s^+} \dot{x}_L^T(t-\xi) K_P \dot{x}_L(t-\xi)d\xi \\ &+ T_m^+ \dot{x}_L^T K_P \dot{x}_L + \int_0^{T_m^+} \dot{x}_m^T(t-\xi) K_P \dot{x}_m(t-\xi)d\xi\end{aligned} \tag{45}$$

here, integrating both sides of above inequality [0, t], we get:

$$\begin{aligned}\int_0^t \dot{V} d\tau \le &- 2\int_0^t \dot{x}_m^T K_d \dot{x}_m d\tau - 2\int_0^t \dot{x}_L^T K_d \dot{x}_L d\tau \\ &+ \int_0^t T_s^+ \dot{x}_m^T K_P \dot{x}_m d\tau + \int_0^t T_m^+ \dot{x}_L^T K_P \dot{x}_L d\tau \\ &+ \int_0^t \int_0^{T_s^+} \dot{x}_L^T(\tau-\xi) K_P \dot{x}_L(\tau-\xi)d\xi d\tau \\ &+ \int_0^t \int_0^{T_m^+} \dot{x}_m^T(\tau-\xi) K_P \dot{x}_m(\tau-\xi)d\xi d\tau\end{aligned} \tag{46}$$

here, the fifth and sixth terms of right side in (46) can be transformed by a simple calculation as follows:

$$\begin{aligned}&\int_0^t \int_0^{T_s^+} \dot{x}_L^T(\tau-\xi) K_P \dot{x}_L(\tau-\xi)d\xi d\tau \\ &\quad \le T_s^+ \int_0^t \dot{x}_L^T(\tau) K_P \dot{x}_L(\tau)d\tau\end{aligned} \tag{47}$$

$$\begin{aligned}&\int_0^t \int_0^{T_m^+} \dot{x}_m^T(\tau-\xi) K_P \dot{x}_m(\tau-\xi)d\xi d\tau \\ &\quad \le T_m^+ \int_0^t \dot{x}_m^T(\tau) K_P \dot{x}_m(\tau)d\tau\end{aligned} \tag{48}$$

Substituting (47), (48) into (46), we obtain:

$$\begin{aligned}\int_0^t \dot{V} d\tau \le &- \int_0^t \dot{x}_L^T\{2K_d - T_{ms}^+ K_P\}\dot{x}_L d\tau \\ &- \int_0^t \dot{x}_m^T\{2K_d - T_{ms}^+ K_P\}\dot{x}_m d\tau\end{aligned} \tag{49}$$

and then, we receive:

$$\dot{V} \le -\dot{x}_L^T\{2K_d - T_{ms}^+ K_P\}\dot{x}_L - \dot{x}_m^T\{2K_d - T_{ms}^+ K_P\}\dot{x}_m \tag{50}$$

From above inequality, we can choose the gains K_P, K_d to satisfy the condition (37), thus the derivative of the Lyapunov function $\dot{V}$ is negative semi-definite with denoting the Assumption 4: $\dot{x}_m, \dot{x}_L \in \mathcal{L}_2$. To show the uniformly continuity of $\dot{V}$, we consider the derivative of $\ddot{V}$ as follows:

$$\ddot{V} \le -2\ddot{x}_L^T\{2K_d - T_{ms}^+ K_P\}\dot{x}_L - 2\ddot{x}_m^T\{2K_d - T_{ms}^+ K_P\}\dot{x}_m \tag{51}$$

The $\dot{V}$ is uniformly continuity if the $\ddot{x}_m$, $\ddot{x}_L$, $\dot{x}_m$, $\dot{x}_L$ are bounded. Since V is lower-bounded by zero and $\dot{V}$ is negative semi-definite, we can conclude that the signals $\dot{x}_m$, $\dot{x}_L$ and x_e are bounded. Moreover, applying Properties 1, 4 and the Assumption 5 for the dynamics of system (30), (31), we have the signal $\ddot{x}_m, \ddot{x}_L \in \mathcal{L}_\infty$. Thus, using lemma of [14], this implies that $\lim_{t\to\infty} \dot{x}_m = \lim_{t\to\infty} \dot{x}_L = 0$, and using Properties 3, 6, we also can conclude $\dddot{x}_m, \dddot{x}_L \in \mathcal{L}_\infty$. Hence, invoking Barbalat's lemma [15], $\ddot{x}_m$, $\ddot{x}_L$ are uniformly continuous; $\lim_{t\to\infty} \dot{x}_m = \lim_{t\to\infty} \dot{x}_L = 0$ and $\lim_{t\to\infty} \ddot{x}_m = \lim_{t\to\infty} \ddot{x}_L = 0$. Therefore the system is asymptotic stable. □

In addition, two below corollaries that relate above theorem as:

Corollary 1. Assume that the teleoperation system described by (4), (22) satisfy the Theorem 2. When $F_L = 0$, the master and slaves spacing error achieve to zero as below:

$$x_e = x_m - x_L \to 0 \;\; as \;\; t \to \infty \tag{52}$$

Proof. when $F_L = 0$, equation (30) as:

$$K_P(x_L - x_m(t - T_m(t))) = 0 \tag{53}$$

Moreover, using Leibniz-Newton formula, following equation is achieved:

$$K_P\Big\{x_e - \int_{t-T_m}^{t} \dot{x}_m dt\Big\} = 0 \tag{54}$$

where $\lim_{t\to\infty} \dot{x}_m = 0$, K_P is a positive symmetric matrix,

$$\lim_{t\to\infty} x_e = 0 \qquad (55)$$

hence the position error of the master and the slave robots is to zero. Thus, the Control Objective 2 is achieved. □

Corollary 2. Assume that the teleoperation system described by (4), (22) satisfies Theorem 2. We obtain that the scaled reflection force from remote environment is accurately transmitted to the slave robot side as follows:

$$F_{op} = -\beta F_L, \quad (\beta = \frac{k_m}{k_L}) \qquad (56)$$

Proof. From Theorem 2, $\lim_{t\to\infty} \ddot{x}_m = \lim_{t\to\infty} \ddot{x}_L = \lim_{t\to\infty} \dot{x}_m = \lim_{t\to\infty} \dot{x}_L = 0$, and concerning about (30), (31) we can obtained:

$$\begin{cases} F_{op} = K_P^m(x_m - x_L) = k_m K_P(x_m - x_L) \\ F_L = K_P^L(x_L - x_m) = -k_L K_P(x_m - x_L) \end{cases} \qquad (57)$$

From equation (57), we get above expression (56)

$$F_{op} = -\beta F_L, \quad (\beta = \frac{k_m}{k_L})$$

we should choose the design parameters of scalar k_m, k_L for power scaling. Therefore, the static reflection force is achieved. □

5. Evaluation by Control Experiments

5.1 Impedance Shaping

In this paper, the SMMS system was constructed with two of 2-DOF serial-link arm of slave robots. Some parameters x_S, x_S^d, x_L are defined as follows:

$$x_S = \bar{x}_1 - \bar{x}_2 = \begin{bmatrix} x_1 - x_2 \\ y_1 - y_2 \end{bmatrix} \qquad (58)$$

$$x_S^d = \begin{bmatrix} d \\ 0 \end{bmatrix} \qquad (59)$$

$$x_L = \alpha \frac{\bar{x}_1 + \bar{x}_2 - C}{2} = \frac{\alpha}{2} \begin{bmatrix} x_1 + x_2 - c \\ y_1 + y_2 \end{bmatrix} \qquad (60)$$

where $C = [c \ \ 0]^T$, $\bar{x}_1 = [x_1 \ y_1]^T$, $\bar{x}_2 = [x_2 \ y_2]^T$; from (58) and (60) we get:

$$\begin{bmatrix} \dot{x}_S \\ \dot{x}_L \end{bmatrix} = \begin{bmatrix} \dot{\bar{x}}_1 - \dot{\bar{x}}_2 \\ \frac{\alpha}{2}(\dot{\bar{x}}_1 + \dot{\bar{x}}_2) \end{bmatrix} = \begin{bmatrix} I & -I \\ \frac{\alpha}{2}I & \frac{\alpha}{2}I \end{bmatrix} \begin{bmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \end{bmatrix} \qquad (61)$$

We define the decomposition matrix S as follows:

$$S = \begin{bmatrix} I & -I \\ \frac{\alpha}{2}I & \frac{\alpha}{2}I \end{bmatrix} \qquad (62)$$

However, the non-diagonal and coupling terms between the shape-system and the Locked System still exist even by using this decomposition matrix S. Thus, a linearization technique with the impedance shaping is then introduced as:

$$\tau_i = J_i^T\{M_i H^{-1}(\tau_i^{'} + F_i) - F_i + C_i \dot{\bar{x}}_i\} \ \ (i = 1, 2) \qquad (63)$$

where $\tau_i^{'}$ is a new control input, H is an inertia matrix of a robot. To satisfy (14), by a simple calculation, we can receive the slaves 1 and 2 with same inertia matrix. Therefore, substituting $M_1 = M_2 = H$ into slave dynamics (63), we obtain:

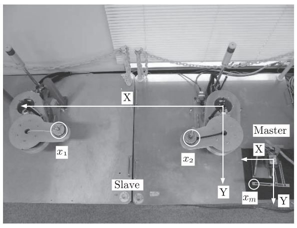


Fig. 2 Experimental setup.

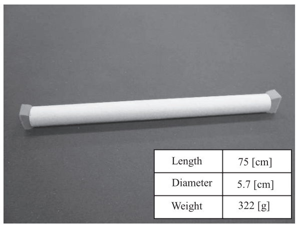


Fig. 3 Grasping object.

$$\begin{bmatrix} H & 0 \\ 0 & H \end{bmatrix} \begin{bmatrix} \ddot{\bar{x}}_1 \\ \ddot{\bar{x}}_2 \end{bmatrix} = \begin{bmatrix} \tau_1^{'} \\ \tau_2^{'} \end{bmatrix} + \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \qquad (64)$$

from (14), we get:

$$\begin{aligned} S^{-T} M S^{-1} &= \begin{bmatrix} \frac{1}{2}I & -\frac{1}{2}I \\ \frac{1}{\alpha}I & \frac{1}{\alpha}I \end{bmatrix} \begin{bmatrix} H & 0 \\ 0 & H \end{bmatrix} \begin{bmatrix} \frac{1}{2}I & \frac{1}{\alpha}I \\ -\frac{1}{2}I & \frac{1}{\alpha}I \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{2}H & 0 \\ 0 & \frac{2}{\alpha^2}H \end{bmatrix} \\ &= \begin{bmatrix} M_S & 0 \\ 0 & M_L \end{bmatrix} \end{aligned} \qquad (65)$$

In addition, since (14) is satisfied, it is easy to be seen that the shape-system and the locked-system to be decoupling. If the passive-decomposition is denoted by (64), we receive:

$$\begin{bmatrix} M_S & 0 \\ 0 & M_L \end{bmatrix} \begin{bmatrix} \ddot{x}_S \\ \ddot{x}_L \end{bmatrix} = \begin{bmatrix} \tau_S^{'} \\ \tau_L^{'} \end{bmatrix} + \begin{bmatrix} F_S \\ F_L \end{bmatrix} \qquad (66)$$

Therefore, by the definition of x_S, x_L mentioned above, the Shape- System and the locked-system are decoupling by the impedance shaping only.

5.2 Evaluation by Control Experiments

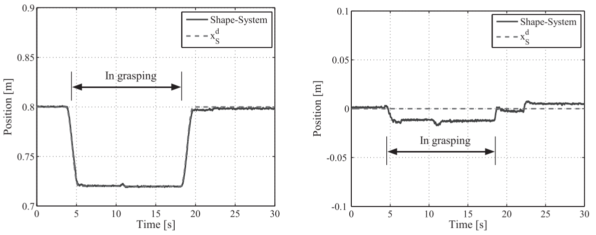
In this section, the effectiveness of the proposed methodology is verified by the control experiments. In the experiments, the SMMS system is constructed by one master with two DOFs parallel link type arm and two slaves with two-two DOFs series link type arms. The experimental setup is shown in Fig. 2. The cylindrical grasping object is used and shown in Fig. 3. We can measure the operational force F_{op} and environment reflecting force F_L by using force sensors at the end-effector of each robot. For implementation of the controllers and communication lines, we utilise a dSPACE digital control system (dSPACE Inc.). All experiments have been done with the artificial time varying communication delays and the sampling time is $1[ms]$:

$$\begin{cases} T_m(t) = 0.1 \sin t + 0.14 \ [s] \\ T_s(t) = 0.05 \sin t + 0.1 \ [s] \end{cases} \qquad (67)$$

From above equation, maximum round-trip delay is $0.39[s]$. To satisfy (37) the controller gains are chosen as: $K_P = diag(30, 35)$, $K_d = diag(6, 7)$, $k_m = 1, k_L = 10$, $K_P^S = diag(400, 400)$, $K_d^S = diag(50, 50)$. Two kinds of experimental conditions are given as follows:

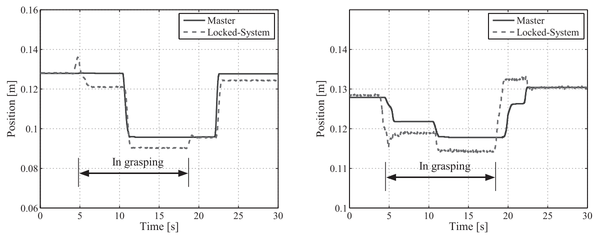
Case 1: Control the grasping object without any contact with remote environment.
Case 2: Control the grasping object in contact with remote environment.

However, in actual experiments, it is difficult for entirety time synchronization on master and slave side in the system configuration. The data that received from master and the data of slave



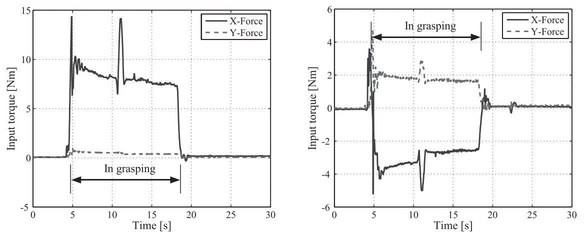
(a) X-position (b) Y-position

Fig. 4 Position of shape-system and x_S^d (Case 1).



(a) X-position (b) Y-position

Fig. 5 Position of master and locked-system (Case 1).



(a) Shape-system (b) Locked-system

Fig. 6 Force (Case 1).

that measured from slave side need be compared, especially the position data on the slave side. In addition, the force data is not sent and received, then the measurement value is used. Therefore, the gap of the time axis is caused for the force data to be not same at the both sides of teleoperation. Moreover, there is not sensor in the parallel link type arm of the master robot, thus the value of human force F_{op} is presumed from the input torque ($F_{op} = J_m^{-T}\tau_m$).

The experimental results of Case 1 are shown in Figs. 4–6. Figure 4 shows time responses of end-effector position of slave of the shape-system, Fig. 5 shows the time responses of end-effector of the master of the locked-system. In Fig. 4, we can conclude that the relative position between slaves following a target trajectory with grasping object is achieved. And in Fig. 5, we also conclude that the grasping object at the center position of slaves is able to transported following the end-effector of the master. The object is presumed to mix with closed links of slaves. When grasping, the distance between slaves is narrowed. However, this distance narrowed by each slave robot is different when the object is held deflection. The force of the shape-system and the locked-system in this case are shown in Fig. 6. We can see that Fig. 6 (b) shows the force data when the object is transferred without contact with the remote environment.

The experimental results of Case 2 are shown in Figs. 8–9. The object comes and contacts with the remote environment following vertical Y axis as shown in Fig. 7. Figure 8 shows the time responses of end-effector position of the locked-system

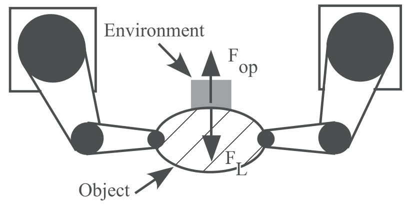
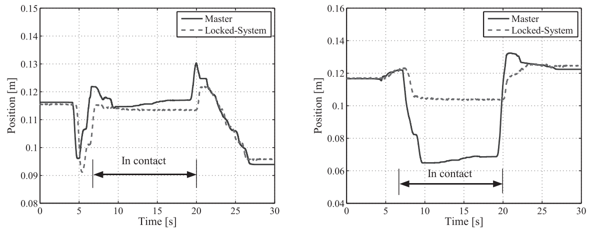
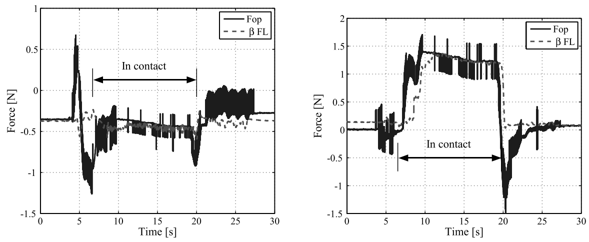


Fig. 7 Experimental setup in Case 2.



(a) X-position (b) Y-position

Fig. 8 Position of master and locked-system (Case 2).



(a) X-force (b) Y-force

Fig. 9 Force of operator and scale locked-system (Case 2).

with the master, Fig. 9 shows the time responses of reflection force from environment. In Fig. 9, the grasping object comes and contacts with environment in case of the master and the slave are stationary states. Moreover, the reflecting force is transmitted in scale environment with $F_{op} = -\beta F_L$ ($\beta = 1/10$).

6. Conclusions

In this paper, we proposed a control method that guarantees asymptotic stability of the SMMS system with time varying delay in the communication lines. The proposed control law shows that the system is asymptotically stabilized under the communication with time varying delay by using PD control and applying the passive-decomposition. This method resolves the dynamics of multiple slave systems as the shape-system dynamic and the locked-system dynamic of the control law. Moreover, the proposed control law can be used to achieve an autonomous object grasping by multiple slaves and the transportation of the object by the control equipments. In this work, the slaves can hold even if objects are unknown or width-extendable as long as it can be held by the force control. The force information on the grasped object is necessary for the position control law to keep the object to be held.

Finally, several experimental results show the effectiveness of the proposed control method.

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